physics of gravitational waves

workshop for b-physics, Neckarzimmern

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outline

gravity: not just a force

- universal, does not depend on the mass of the test particle
- common property with centrifugal and Coriolis-forces (inertial forces, Scheinkräfte): no difference?
- all gravitational and inertial accelerations disappear in a freely falling frame
- equivalence principle
- Newtonian gravity: (weak) static fields only, description with a field $\vec{g} = -\nabla\Phi$ resulting from a potential Φ
- field equation: Gauß-type law $\mathrm{div}\vec{g} = -4\pi G\rho$ leads to $\Delta\Phi = 4\pi G\rho$ with matter density ρ as a source

gravity: regimes

	strong	weak
static	black holes	Newton gravity
dynamic	FLRW-cosmologies	gravitational waves

- dynamic side of gravity: gravitational waves

gravity: theory

- let's wiggle our way towards gravitational waves (pun intended) by using intuition for relativity
- Poisson-equation

$$\Delta\Phi = 4\pi G\rho$$

- Euclidean Laplace-operator $\rightarrow$ Minkowskian box-operator

$$\Delta = \delta_{ij}\partial^i\partial^j \quad \rightarrow \quad \Box = \eta_{\mu\nu}\partial^\mu\partial^\nu = \partial_{ct}^2 - \Delta$$

- Poisson-equation $\rightarrow$ wave equation

$$\Delta\Phi = 4\pi G\rho \quad \rightarrow \quad \Box\Phi = -4\pi G\rho$$

- new field equation allows for
  - waves in the field Φ with propagation speed c
  - source: time-evolving $\rho(t)$

gravity: classical solutions

- field equation for Φ
$$\Box\Phi = -4\pi G\rho$$
- 3 particular cases
  - vacuum situation $\rho = 0$:
    $\Box\Phi = 0$: plane gravitational wave
  - static solution, field does not depend on t:
    $\Delta\Phi = 4\pi G\rho$: Newtonian gravity
  - spatially homogeneous Φ, field does not depend on x, y, z:
    $\partial_{ct}^2\Phi = -4\pi G\rho$: FLRW-cosmology

gravity: solutions

	black holes	grav. waves	FLRW-cosmologies	white dwarfs
homogeneous	t	$r \pm ct$	r	t
isotropic	yes	no	yes	yes
varies along	r	r,t	t	r
gravity	strong	weak	strong	weak...strong
scales	$r_S = \frac{2GM}{c^2}$	linear physics	$\rho_{crit} = \frac{3H_0^2}{8\pi G}$	eqn. of state
curvature	Weyl	Weyl	Ricci	Weyl + Ricci
sources	vacuum	vacuum	p, ρ (ideal fluid)	p, ρ (ideal fluid)

dynamics of gravity

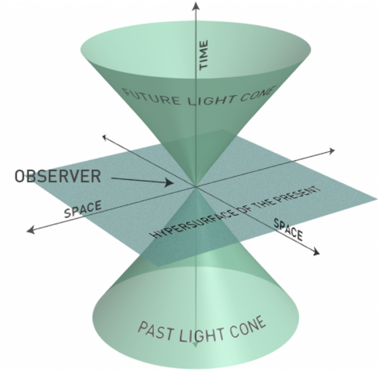
- field equation for Φ
$$\Box\Phi = -4\pi G\rho$$
- propagation of excitations in Φ with c
- possible solution:
$$\Phi \propto \exp\left(\pm i\eta_{\alpha\beta}k^\alpha x^\beta\right)$$
with wave vector k^α
- substitution into the wave equation yields dispersion
$$\eta_{\mu\nu}k^\mu k^\nu = 0 = (\omega/c)^2 - k^2 \quad \rightarrow \quad \omega = \pm ck$$
null-condition for the wave vector
- phase and group velocity:
$$\upsilon_{gr} = \frac{d\omega}{dk} = c = \frac{\omega}{k} = \upsilon_{ph}$$
no dispersion, wave forms are perfectly conserved

hyperbolicity

- Poisson-equation $\Delta\Phi = 4\pi G\rho$ or wave equation $\Box\Phi = 0$
  - partial differential equations (PDE)
  - inhomogeneous with ρ
  - second order $\rightarrow$ time-reversible and parity-even
  - linear $\rightarrow$ superposition holds
- hyperbolic vs. elliptical: signature of differential operator
  - Poisson equation $\Delta\Phi = 4\pi G\rho$: signature $(+++)$, elliptical
  - wave equation $\Box\Phi = 0$: signature $(+---)$, hyperbolic
- solution is unique if
  - elliptical PDE: boundary conditions are specified
  - hyperbolic PDE: **initial** conditions are specified

wave forms depend only on the dynamics of the source

causality and light cones (or gravity cones)

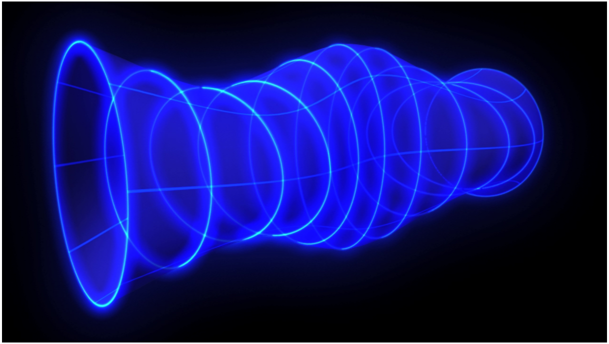


causality cone

properties of gravitational waves

	gravitational wave	electromagnetic wave
propagation speed	c	c
dispersion	no	no
type	transverse	transverse
polarisation modes	2	2
source	quadrupole	dipole

polarisation



gravitational wave

- two polarisation modes for gravitational waves
- choice: linear polarisation or circular polarisation

geodesic deviation

- Newton equation of motion: $\ddot{x}^i = -\partial^i\Phi$
- two test particles fall through space, relative distance $\delta^i = y^i - x^i$
- relative acceleration

$$\ddot{\delta}^i = \ddot{y}^i - \ddot{x}^i = -\partial^i\Phi(y) + \partial^i\Phi(x)$$

- Taylor-expand $\Phi(y)$ around x:

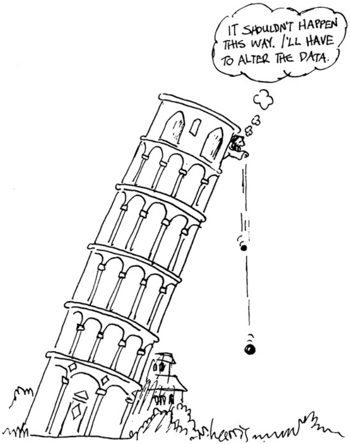
$$\Phi(y) = \Phi(x) + \partial_j\Phi(x)\,(y^j - x^j) = \Phi(x) + \partial_j\Phi(x)\,\delta^j$$

- geodesic deviation: relative acceleration depends on tidal field $\partial^i\partial_j\Phi$

$$\ddot{\delta}^i = -\partial^i\partial_j\Phi(x)\,\delta^j$$

- relativistic version: tidal field $\rightarrow$ Riemann curvature
- test particles inside the field of a gravitational wave **change their physical distance by δx** in an oscillating way

Galilei's experiment, more interesting

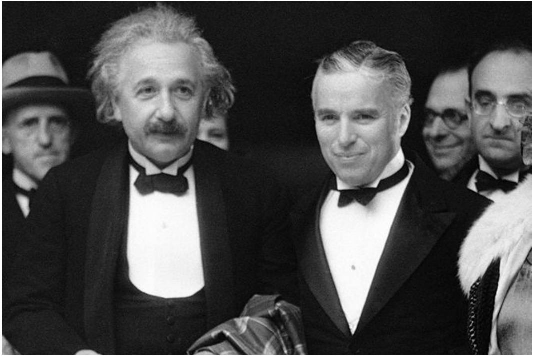


Galilei, first try at geodesic deviation

Ricci and Weyl curvature

- evolution of freely falling **extended** test bodies
- formally: Raychaudhuri-equation
- two modes in the time evolution of the test bodies
  - change in volume, conserved shape:
    field generating mass **inside** the cloud
    Ricci-curvature (always $\propto$ energy density)
    typical for FLRW-universes
  - change in shape, conserved volume:
    field generating mass **outside** the cloud
    Weyl-curvature
    typical for vacuum solutions (gravitational waves, black holes)

Galilei's experiment, but with volumes



unlikely but genuine friendship

energy transport

- electromagnetic waves carry energy and momentum: Poynting-law
- gravitational waves as well!
- two intuitive arguments:
  - Feynman's curtain rail $\rightarrow$ awesome!
  - geometry: gravitational waves change the semi-axes of a circle by $1 \pm \varepsilon$ due to their Weyl-curvature. The enclosed area changes by $(1+\varepsilon) \times (1-\varepsilon) \simeq 1-\varepsilon^2 < 1$, so there must be Ricci-curvature, implying that there is energy and momentum
- BTW: that's a major problem for quantum gravity: plane waves have anomalous dispersion due to their own gravity, they propagate in an energy-dependent way

black hole mergers: estimate frequency

- "centrifugal balance"

$$\frac{\upsilon^2}{r} = \frac{GM}{r^2}$$

- multiply with $1/c^2$

$$\frac{\upsilon^2}{c^2} = \frac{GM}{c^2 r}$$

- identify Schwarzschild-radius $r_S = 2GM/c^2$

$$\frac{\upsilon^2}{c^2} = \frac{r_S}{r}$$

  if $r_s = r$, υ reaches c

- frequency ν

$$\nu = \frac{\upsilon}{r} = \frac{c}{r_S} = \frac{c^3}{2GM}$$

  $\nu \sim$ kHz for Solar mass black holes, $M_\odot \sim 10^{30}$ kg, $r_S = 10^3$ m

black hole mergers: estimate amplitude

- let's assume the amplitude is of order unity at the source, i.e. at $r = r_S$. for a Solar-mass black hole $r_S \simeq 10^3$ meters
- amplitudes decrease with $1/r$, luminosity with $1/r^2$ (luminosity is amplitude^2)
- then,

$$\frac{\delta x}{x} \simeq \frac{r_S}{r}$$

- typically at a few 100 Mpc ($\simeq 10^{24}$ m) the signal is

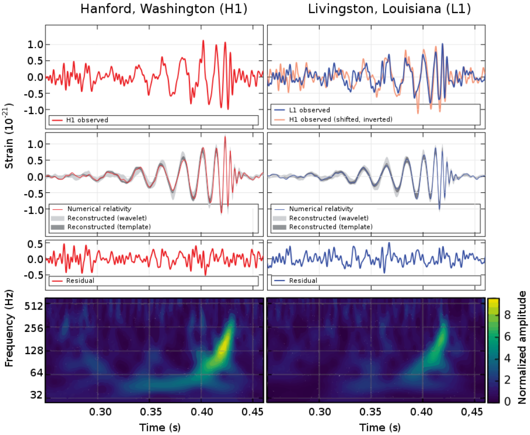
$$\frac{\delta x}{x} \simeq \frac{10^3}{10^{24}} \simeq 10^{-21}$$

- $\delta x = 10^{-24}x$: a distance of $x = 10^3$ meters is changed by 10^{-18} meters: that's 10^{-3} of the diameter of a proton

Newtonian signal: principal characteristics

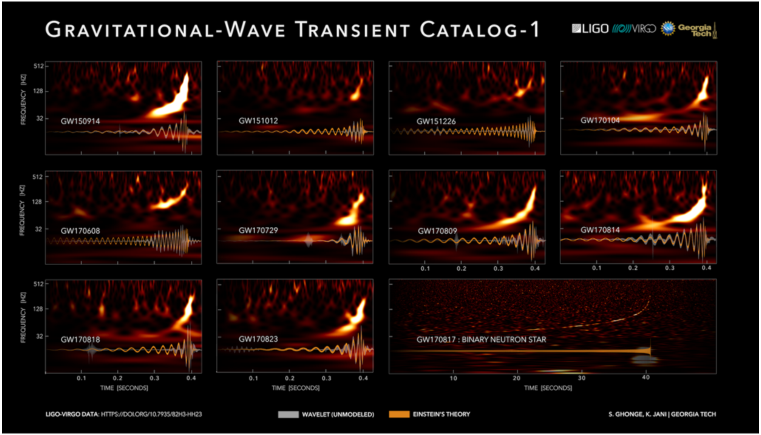
- two objects orbit each other around their common centre of mass
- oscillating quadrupole in the matter distribution
- generate gravitational waves at the orbital frequency
- gravitational wave carries away energy
- system retracts inside its own gravitational well
- deeper down, orbital periods are faster
- gravitational wave signal typically **increases** in frequency $\rightarrow$ **chirp**

first direct signal from LIGO



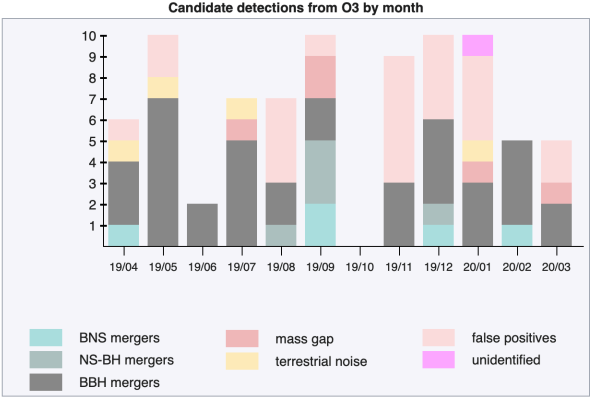
discovery of gravitational waves

typical gravitational wave signal



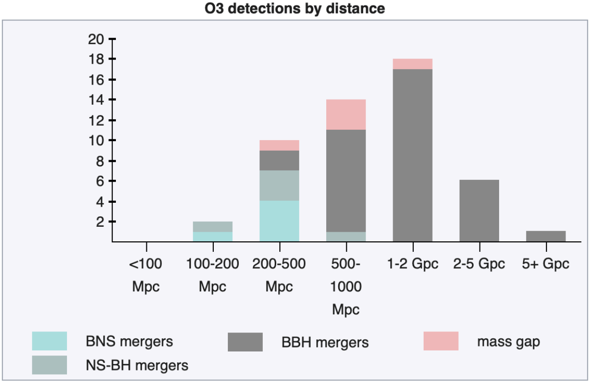
typical, strong signals

gravitational wave signal demographics



detections and their classification

gravitational wave signal demographics

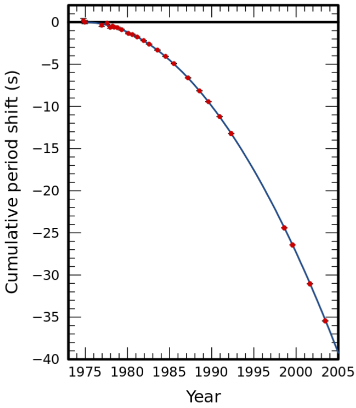


distances to source

Taylor-Hulse pulsar PSR B1913+16

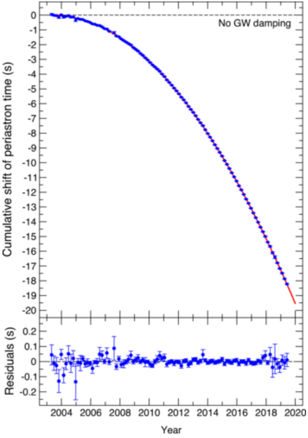
- first hint of gravitational waves: orbital decay of a binary pulsar
- pulsar orbiting a neutron star
- gravitational waves carry away energy, system becomes more tightly bound
- **increasing** orbital frequency, determined by radio interferometry

Taylor-Hulse pulsar PSR B1913+16



Taylor-Hulse pulsar, orbital decay

double pulsar PSR J0737-3039



double pulsar: better determination of the orbital elements

interferometry

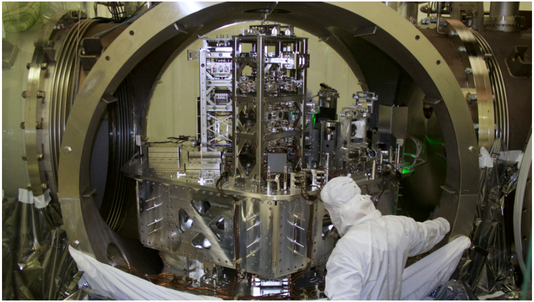
- measure the relative acceleration of two test particles
- geodesic deviation

$$\ddot{\delta}^i = -\partial^i\partial_j\Phi(x)\,\delta^j$$

relative acceleration measures tidal field $\partial^i\partial_j\Phi$ or curvature

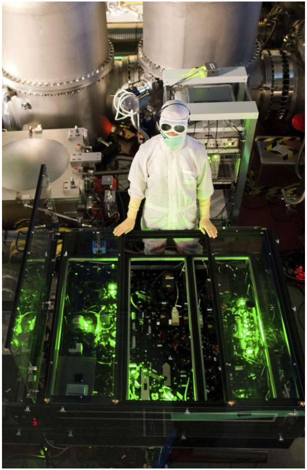
- signal is proportional to the distance δ:
  need a **big** interferometer, typically, $\delta x/x \simeq 10^{-21}$
- interferometry idea:
  - mirrors are the freely falling test particles: they're suspended on strings
  - difference in travel time to the mirrors and back is measured by interference
- interferometers measure amplitude

LIGO picture book



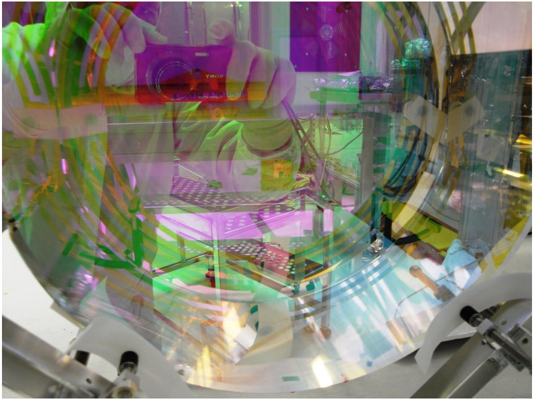
beamsplitter

LIGO picture book



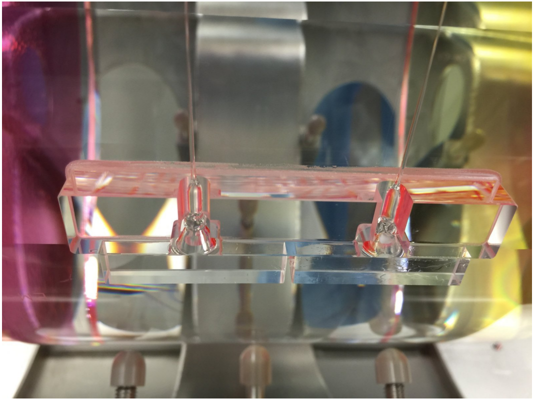
laser assembly (few 100 watts of power)

LIGO picture book



mirror

LIGO picture book



mirror suspension: really freely falling!

funky idea: minimal size of an interferometer

- sensible size of an interferometer: light travel time t should be inverse frequency: then, the mirrors have maximally changed their position
- $t \simeq 1/\nu$, but then $ct \simeq c/\nu \simeq \lambda$
- but $c/\nu = r_S$, so wave length $\lambda \simeq r_S$
- LIGO is about the same size as the binary black holes systems it tries to detect!

Weber-type detectors

- gravitational wave: relative acceleration of test particles:

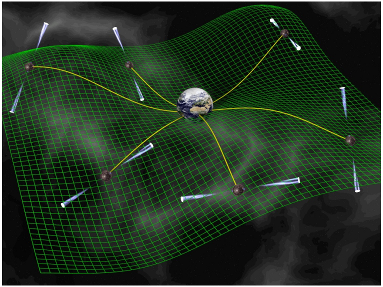
$$\ddot{\delta}^i = -\partial^i\partial_j\Phi(x)\,\delta^j$$

- join two test particles with a spring
- resonance with the wave: transfer of energy from the wave to the detector
- narrow resonance window, widens for stronger damping, but signal smaller
- Weber-detectors measure energy flux
- typically, $\nu \simeq$ kHz: imagine a big metal cylinder (rings like a bell!)

Weber-type detectors

Weber-type resonant detectors

pulsar timing arrays



pulsars inside the Milky Way

- natural gravitational wave detector!
- jitter in arrival time distribution of pulses
- radial positional uncertainty of 100m due to the gravitational wave background

quadrupole formula and radiative efficiency

- quadrupole formula for energy emitted by a time-evolving quadrupole Q_{ij}

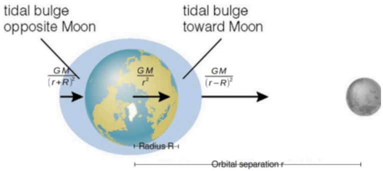
$$\frac{dE}{dt} = \frac{G}{c^5}\sum_{ij}\left(\frac{d^3Q_{ij}}{dt^3}\right)^2$$

- prefactor G/c^5 is very, very small
- need to make up with large d^3Q/dt^3: rapid evolution...
- quadrupole Q_{ij}

$$Q_{ij} = \int d^3r\rho\left(r_ir_j - \frac{r^2}{3}\delta_{ij}\right)$$

...and high-density, large systems are efficient in generating gravitational waves

quadrupole formula: water world



tides on an ocean form a quadrupole

- imagine a planet with a planet-wide ocean
- gravitational wave: produces tides 2 mountains and 2 valleys, opposite sides, i.e. **quadrupole**
- turn situation around: a moving quadrupole should generate a gravitational wave!

bounds on modified gravity

- kilonovae generate a gravitational wave signal with associated burst of photons
- few seconds time difference of a source at few hundred Mpc
- photons and gravitational waves travel at the same speed
- incredibly strong constraints on graviton mass or on modified gravity theories

bounds on modified gravity

- wave-equation with a mass-term:

$$\mathcal{L} = \frac{1}{2}\eta_{\mu\nu}\partial^\mu\Phi\partial^\nu\Phi - \frac{m^2}{2}\Phi^2 - 4\pi G\rho$$

- variation of the action $S = \int d^3x\, \mathcal{L}$ yields:

$$(\Box - m^2)\Phi = 4\pi G\rho$$

- consider vacuum $\rho = 0$ and try a plane wave ansatz:

$$\Phi \propto \exp(\pm i\eta_{\alpha\beta}k^\alpha x^\beta) \quad \rightarrow \quad \eta_{\mu\nu}k^\mu k^\nu = m^2 > 0$$

- dispersion relation

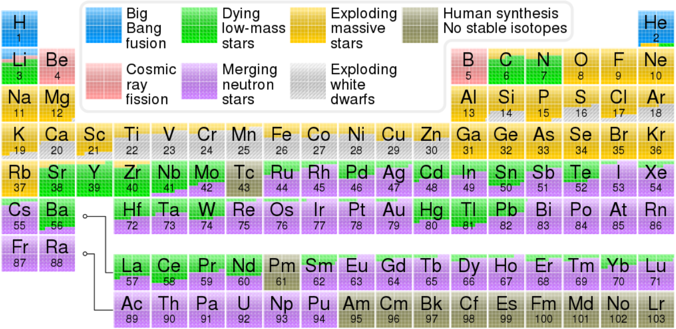
$$\omega = c\sqrt{k^2 + m^2} \quad \rightarrow \quad \upsilon_{gr} = \frac{d\omega}{dk} = c\frac{k}{\sqrt{k^2 + m^2}} < c$$

  slower than speed of light, waves would be dispersive
- similar: upper bound on coupling strength of Φ to other fields

neutron star mergers: kilonovae

- classification:
  - novae: runaway nuclear fusion in the surface of ordinary stars
  - supernovae: nuclear fusion triggered in a mass-accreting white dwarf (type I) or collapse of a massive star (type II)
  - hypernova: collapse of an extremely massive star, possibly with GRB
  - kilonova: merger of two neutron stars
- kilonova
  - merging process of two neutron stars $\rightarrow$ gravitational waves
  - associated optical signal, GRB
  - explosive nucleosynthesis: r-process neutron capture with subsequent decay into the valley of stability, detectable as afterglow
  - GW-signal and element yield depend on nuclear equation of state, relativistic fluid mechanical simulations necessary

nucleosynthesis processes

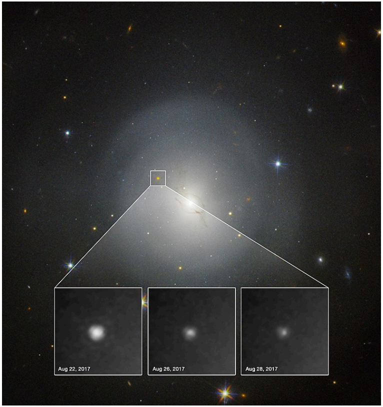


origin of chemical elements

kilonova AT 2017gfo with associated GRB 170817A

- detection
  - 100 seconds of gravitational wave signal, starting at $\nu = 24$ Hz
  - detected by both LIGO and VIRGO, positional fix by triangulation within 30 square degrees
  - high-energy transient detected by INTEGRAL and FERMI
  - optical, UV and X-ray follow-up
- wave form analysis: total system with about $3M_\odot$
- about 10^4 Earth masses in heavy elements produced, about 10 $M_\oplus$ in gold

kilonova AT 2017gfo



kilonova AT 2017gfo

questions for future gravitational experiments

- signal form analysis: constraints on nuclear equation of state
- standard siren technique: luminosity versus redshift for cosmology, dark energy
- modified gravity in the merging process
- gravitational wave background, phase transitions in the early universe

other aspects of gravity to be probed

- Lovelock's theorem: general relativity is unique
  - for energy-momentum conserving systems
  - in 4 dimensions
  - in a metric spacetime
  - with a second-order, local field equation
- constructive gravity: matter field theories imply relativity
  - if the QFTs are defined on a hyperbolic spacetime
  - and are quantisable
- questions for "BSM-gravity"
  - new gravitational degrees of freedom (i.e. more gravitons)
  - non-local field equations, in particular higher orders
  - new structures in field equations

summary

- natural phenomenon of gravity, due to hyperbolicity of the equations
- many analogies to electromagnetic waves
- sources: relativistic motion of heavy masses, typically merging of neutron stars or black holes
- detection in interferometers: measurement of amplitude
- signal shape: indicates details of the merging process, nuclear equation of state
- explosive nucleosynthesis
- questions for future experiments