Introduction to Flavour Physics

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What is Flavour Physics?

Fundamental matter comes in three generations carrying the same charges under the Standard Model gauge group $SU(3)_c \times SU(2)_L \times U(1)$.

Leptons

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

Quarks

$$\begin{pmatrix} uuu \\ ddd \end{pmatrix} \quad \begin{pmatrix} ccc \\ sss \end{pmatrix} \quad \begin{pmatrix} ttt \\ bbb \end{pmatrix}$$

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Quarks

$$\begin{pmatrix} uuu \\ ddd \end{pmatrix} \quad \begin{pmatrix} ccc \\ sss \end{pmatrix} \quad \begin{pmatrix} ttt \\ bbb \end{pmatrix}$$

Flavour is the feature that distinguishes the generations.

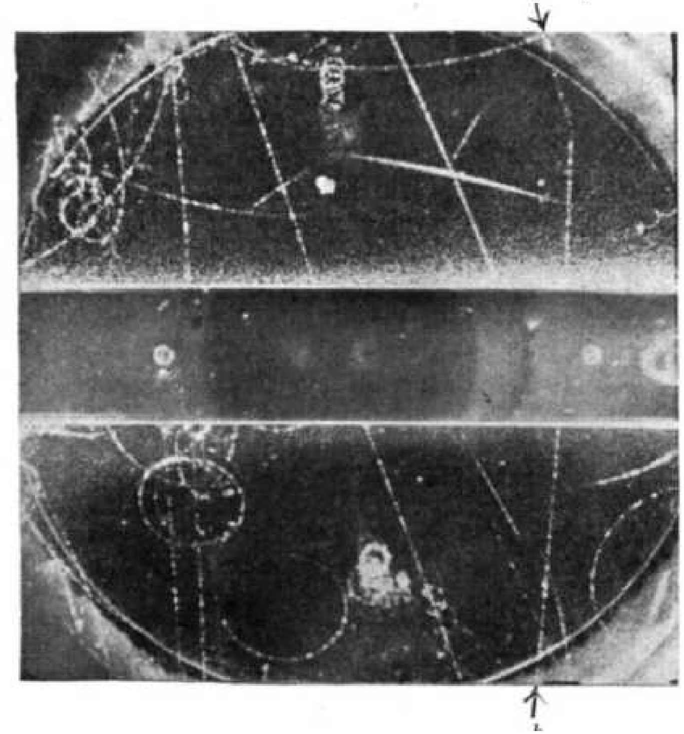
Flavour physics studies the complex phenomenology:

- masses ranging over 12 order of magnitudes
- flavour transitions (mixing)
- CP violation

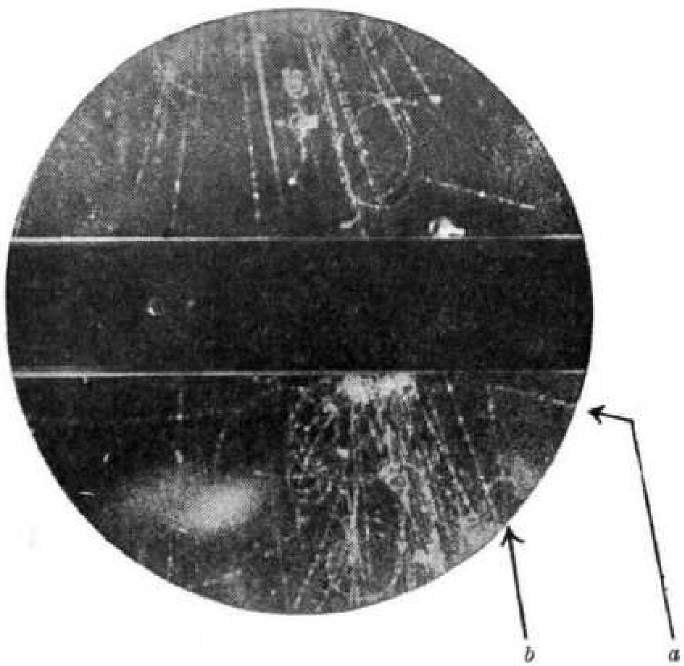
Birthday of Heavy Flavour Physics

- 1947, G. D. Rochester and C. C. Butler, discovered kaons in cloud chamber studying cosmic rays

$K^+ \to \mu^+ \nu$



$K^0 \to \pi^+ \pi^-$



- 1953: new quantum number "strangeness" (Gellmann & Pais): conserved in strong IA (production), not conserved in weak IA (decay)
$\pi + p \to \Lambda + K^0 + X$

What does CPV (experimentally) mean?

The observed rate with which a process and its CP conjugate process occur are different.

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e.g.

- $P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0)$
- $BR(B^0 \to K^+\pi^-) \neq BR(\overline{B^0} \to K^-\pi^+)$
- Different distribution in phase space of particle and anti-particle decay (e.g. different Dalitz-Plots)

Neutral Meson Mixing

K^0, $\overline{K^0}$: flavour eigenstates; clear defined quark content ($K^0 = |d\bar{s}>$, $\overline{K^0} = |\bar{d}s>$)

$$CP(K^0) = \overline{K^0} \qquad CP(\overline{K^0}) = K^0$$

K_1, K_2: CP eigenstates

$$\boldsymbol{K_1 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0}) \qquad CP(K_1) = +K_1}$$

$$\boldsymbol{K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0}) \qquad CP(K_2) = -K_2}$$

K_S, K_L: mass eigenstates

(with clear defined mass and lifetime, $\psi_{S/L}(t) = e^{-im_{S/L}t}e^{-\Gamma_{S/L}t/2}$)

$$K_S = pK^0 + q\overline{K^0} \quad K_L = pK^0 - q\overline{K^0} \quad q^2 + p^2 = 1$$

in absence of CPV: $K_S = K_1$, $K_L = K_2 \quad \rightarrow \quad q = p = \frac{1}{\sqrt{2}}$

Kaon Mixing

$$|\mathbf{K_S}>= p|\mathbf{K^0}> +q|\overline{\mathbf{K^0}}>, \quad |\mathbf{K_S(t)}>= |\mathbf{K_S}> e^{-\frac{\Gamma_S}{2}t}e^{-im_S t}$$

$$|\mathbf{K_L}>= p|\mathbf{K^0}> -q|\overline{\mathbf{K^0}}>, \quad |\mathbf{K_L(t)}>= |\mathbf{K_L}> e^{-\frac{\Gamma_L}{2}t}e^{-im_L t}$$

$|p|^2 + |q|^2 = 1$ complex coefficients; $q = p = \frac{1}{\sqrt{2}} \Leftrightarrow \mathbf{K_S} = \mathbf{K_1}, \mathbf{K_L} = \mathbf{K_2}$

Flavour eigenstates:

$$|\mathbf{K^0}>= \frac{1}{2p}(|\mathbf{K_S}> +|\mathbf{K_L}>)$$

$$|\overline{\mathbf{K^0}}>= \frac{1}{2q}(|\mathbf{K_L}> - |\mathbf{K_S}>)$$

time development of originally (at t=0) pure $\mathbf{K^0}$ and $\overline{\mathbf{K^0}}$ states:

$$|\mathbf{K^0}(\mathbf{t})>= \frac{1}{2p}(|\mathbf{K_S}(\mathbf{t})> +|\mathbf{K_L}(\mathbf{t})>)$$

$$|\overline{\mathbf{K^0}}(\mathbf{t})>= \frac{1}{2q}(|\mathbf{K_L}(\mathbf{t})> - |\mathbf{K_S}(\mathbf{t})>)$$

Kaon Mixing

$$
\begin{aligned}
& P(\mathbf{K^0} \to \overline{\mathbf{K^0}}) \\
= \;& | < \mathbf{K^0(t)} | \overline{\mathbf{K^0}} > |^2 \\
= \;& |\frac{1}{2p}(< \mathbf{K_S(t)} | \overline{\mathbf{K^0}} > + < \mathbf{K_L(t)} | \overline{\mathbf{K^0}} >)|^2 \\
= \;& |\frac{1}{2p}(< \mathbf{K_S} | \overline{\mathbf{K^0}} > e^{-\frac{\Gamma_S}{2}t} e^{-im_S t} + < \mathbf{K_L} | \overline{\mathbf{K^0}} > e^{-\frac{\Gamma_L}{2}t} e^{-im_L t})|^2 \\
= \;& |\frac{q}{2p}(e^{-\frac{\Gamma_S}{2}t} e^{-im_S t} - e^{-\frac{\Gamma_L}{2}t} e^{-im_L t})|^2 \\
= \;& \frac{1}{4}|\frac{q}{p}|^2 \cdot \left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-(\Gamma_L + \Gamma_S)t/2} \cos \Delta m t \right)
\end{aligned}
$$

Kaon Mixing

$$P(\mathbf{K^0} \to \overline{\mathbf{K^0}}) = |<\mathbf{K^0(t)}|\overline{\mathbf{K^0}}>|^2 =$$
$$\frac{1}{4}|\frac{q}{p}|^2 \cdot \left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-(\Gamma_L+\Gamma_S)t/2}\cos\Delta m t\right)$$

$$P(\overline{\mathbf{K^0}} \to \mathbf{K^0}) = |<\overline{\mathbf{K^0}}(\mathbf{t})|\mathbf{K^0}>|^2 =$$
$$\frac{1}{4}|\frac{p}{q}|^2 \cdot \left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-(\Gamma_L+\Gamma_S)t/2}\cos\Delta m t\right)$$

CP conserved: $P(\mathbf{K^0} \to \overline{\mathbf{K^0}}) = P(\overline{\mathbf{K^0}} \to \mathbf{K^0})$

$$\Leftrightarrow$$

$$|\frac{q}{p}| = 1$$

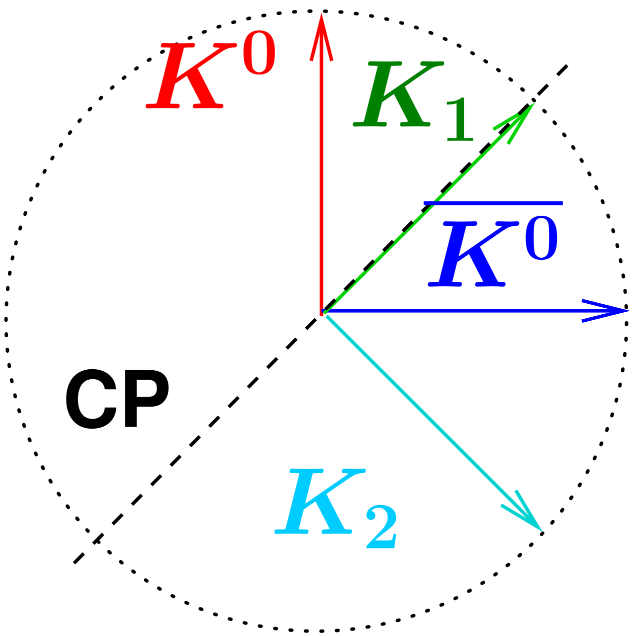
(+ normalization $q^2 + p^2 = 1$)

$$\Leftrightarrow$$

$$q = p = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow$$

$$\boldsymbol{K_S = K_1, K_L = K_2}$$



$$CP(K^0) = \overline{K^0}$$
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$$CP(K_1) = +K_1$$

$$K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0})$$
$$CP(K_2) = -K_2$$

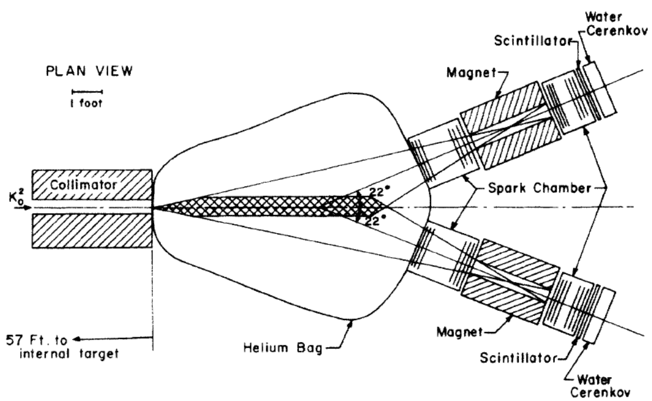
$CP\,\Psi(\pi^+\pi^-) = +\Psi(\pi^+\pi^-)$ (if relative angular momentum = 0)

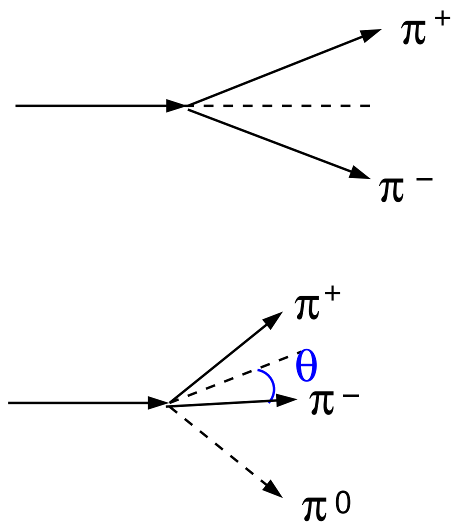
$CP\,\Psi(\pi^+\pi^-\pi^0) = -\Psi(\pi^+\pi^-\pi^0)$ (if relative angular momentum = 0)

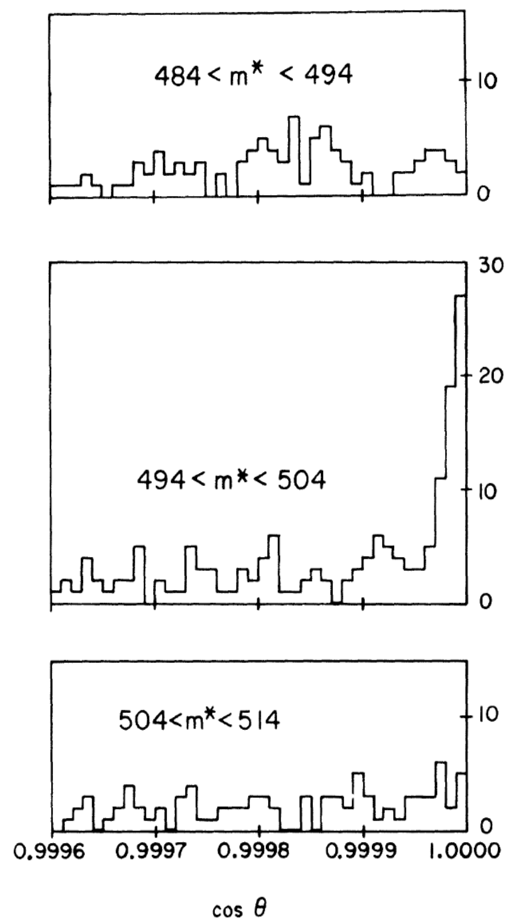
If there is no CPV in decay, then: $\mathbf{K_1} \rightarrow \pi^+\pi^-$; $\mathbf{K_2} \rightarrow \pi^+\pi^-\pi^0$

1964: Discovery of CPV

- produce K^0, wait long enough for K_S component to decay away $\rightarrow$ pure K_L beam
- search for CP violation: $K_L \rightarrow \pi^+\pi^-$
  $\rightarrow$ excess of 56 events: BR($K_L \rightarrow \pi^+\pi^-$) $\sim 2 \times 10^{-3}$







mass eigenstates $\neq$ CP eigenstates: $|\mathbf{K_L}>= \frac{1}{\sqrt{1+|\epsilon^2|}}(|\mathbf{K_2}> + \epsilon|\mathbf{K_1}>)$

CP=-1 CP=+1

Nobel prize for Cronin and Fitch in 1980

Good guessing ..... ?

three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as $K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0)]$ then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$ where τ_1 and τ_2 are the K_1^0 and K_2^0 mean lives and R_T is the branching ratio including decay to

in this paper they call K_2 what we call nowadays K_L ....

In my opinion one could not conclude from this experiment if the observed CPV is CPV in mixing or in decay.

CPV in mixing:

$$|\mathbf{K_L}> = \frac{1}{\sqrt{1+|\epsilon^2|}}(|\mathbf{K_2}> + \epsilon|\mathbf{K_1}>)$$

CPV in decay:

$\mathbf{K_1} \to \pi\pi\pi$ and $\mathbf{K_2} \to \pi\pi$

How does flavour (or the CKM matrix) enter the Standard Model Lagrangien?

Quarks and Leptons

Left handed quarks and leptons are weak isospin doublets under $SU(2)_L$,

right handed quarks and leptons are weak isospin singlets under $SU(2)_L$

Quarks: $Q_L = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right)$

$$U_R = (u_R, c_R, t_R) \quad D_R = (d_R, s_R, b_R)$$

Leptons: $L_L = \left(\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix} \right)$

$$E_R = (e_R, \mu_R, \tau_R)$$

All up-type quarks, all down-type quarks, all charged and all neutral leptons have the same quantum numbers. They only differ in their mass.

The Standard Model Lagrangian

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$\mathcal{L}_{Gauge}$: kinetic and interaction terms describe dynamics of fermions

$$\mathcal{L}_{Gauge} = \sum_f i\overline{\Psi_f}\gamma_\mu D^\mu \Psi_f \quad \Psi_f = Q_L, U_R, D_R, L_L, E_R$$

with the covariant derivative:

$$D^\mu = \delta^\mu + ig_s G_a^\mu T_a + igW_b^\mu \tau_b + ig^{'} B^\mu Y$$

a=1,..,8: index of gluon fields, T_a generator of $SU(3)_C$

($T_a = \lambda/2$, λ, Gell-Mann matrices)

b=1,..,3: index of weak boson fields, τ_b generator of $SU(2)_L$

($\tau_b = \sigma/2$, Pauli matrices)

The Higgs-Field

Skalar doublet of complex fields: $\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$

Higgs-Potential: $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2$

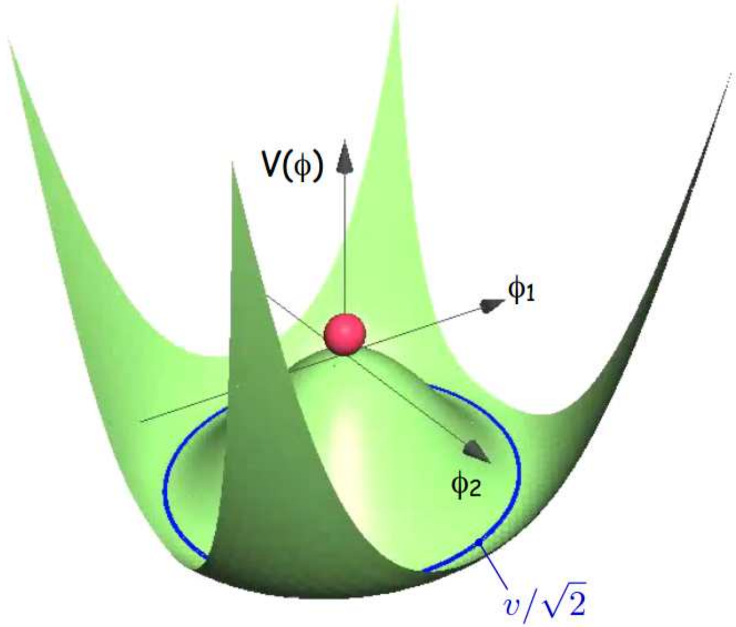
$$\mathcal{L}_{Higgs} = (D_\mu \Phi^\dagger)(D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4}(\Phi^\dagger \Phi)^2$$

$$\mathcal{L}_{Yukawa} \propto Y_U \overline{Q_L} U_R \Phi + Y_D \overline{Q_L} D_R \overline{\Phi} + Y_E \overline{L_L} E_R \overline{\Phi} + h.c.$$

Y_U, Y_D, Y_E are 3×3 matrices in flavour space and describe the coupling to the Higgs field.

Massterm and Higgs Interaction Term

$$\Phi \rightarrow \Phi' = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{pmatrix}$$



After symmetry breaking the interaction with the VEV of the Higgs fields generates the fermion masses:

$$\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{mass} + \mathcal{L}_{higgs\,IA}$$

$$\mathcal{L}_{mass} = -\frac{v}{\sqrt{2}}\left(\overline{U_L}Y_U U_R + \overline{D_L}Y_D D_R + \overline{E_L}Y_L E_R + h.c.\right)$$

$$\mathcal{L}_{higgs\,IA} = -\frac{1}{\sqrt{2}}\left(\overline{U_L}Y_U U_R H + \overline{D_L}Y_D D_R H + \overline{E_L}Y_L E_R H + h.c.\right)$$

Standard Model Lagrangian

The gauge term of the Lagrangian is flavour symmetric (same QN for all three families). The structure of the Lagrangian would not change if we would introduce e.g. a rotation in the space of charged left-handed leptons.

The Yukawa matrices describing the Yukawa interaction are in general complex and non-diagonal $\rightarrow$ flavour structure of the Standard Model

It is conveniet to choose a flavour basis for the fermion fields in which the mass term from $\mathcal{L}$ are diagonal. This can be achieved by unitary transformations:

$$U_R \rightarrow V_{u_R} u_R, \quad U_L \rightarrow V_{u_L} u_L, \quad D_R \rightarrow V_{d_R} d_R \quad D_L \rightarrow V_{d_L} d_L$$

with $V_{u_R}^\dagger V_{u_R} = 1, V_{u_L}^\dagger V_{u_L} = 1, V_{d_R}^\dagger V_{d_R} = 1, V_{d_L}^\dagger V_{d_L} = 1,$

and $V_{u_L}^\dagger Y_U V_{u_R} = \hat{Y}_U$ and $V_{d_L}^\dagger Y_D V_{d_R} = \hat{Y}_D$, where $\hat{Y}_U$ and $\hat{Y}_D$ are diagonal

Standard Model Lagrangian

One thus obtains for the Quark part of $\mathcal{L}_{mass}$:

$$\mathcal{L}_{mass} = -\frac{v}{\sqrt{2}} \left(\overline{u}_L \hat{Y}_U u_R + \overline{d}_L \hat{Y}_D d_R \right)$$

One can identify the quark masses:

$$M_u = \frac{v}{\sqrt{2}} \hat{Y}_u = \frac{v}{\sqrt{2}} V^\dagger_{u_L} Y_U V_{u_R} = diag(m_u, m_c, m_t)$$
$$M_d = \frac{v}{\sqrt{2}} \hat{Y}_d = \frac{v}{\sqrt{2}} V^\dagger_{d_L} Y_D V_{d_R} = diag(m_d, m_s, m_b)$$

with $m_t \sim$ 173 GeV and $v \sim$ 246 GeV on finds:

$y_t \sim 1$ and the other yukawa couplings way smaller

The fact that the Yukawa couplings are so different is not understood and often referred as the flavor hierarchy problem (mass spectrum of the fermions).

Quark mixing in charge current interactions

As U_L and D_L have been independently transformed, there is a non-vanishing effect for the charged current terms in $\mathcal{L}_{gauge}$ where U_L and D_L enter both.

After electroweak symmetry breaking the charged current terms for the quarks are:

$$\mathcal{L}^{CC}_{gauge} = \frac{g}{\sqrt{2}} \left(\overline{U}_L \gamma_\mu W^{+\mu} D_L + \overline{D}_L \gamma_\mu W^{-\mu} U_L \right)$$
$$W^{\pm,\mu} = \frac{1}{\sqrt{2}} (W_1^\mu \mp i W_2^\mu)$$

In the basis of the mass eigenstate u_L and d_L one obtains:

$$\mathcal{L}^{CC}_{gauge} = \frac{g}{\sqrt{2}} \left(\overline{u}_L V^\dagger_{u_L} V_{d_L} \gamma_\mu W^{+\mu} d_L + \overline{d}_L V^\dagger_{d_L} V_{u_L} \gamma_\mu W^{-\mu} u_L \right)$$

$$V^\dagger_{u_L} V_{d_L} = V_{CKM} \quad V^\dagger_{d_L} V_{u_L} = V^\dagger_{CKM}$$

Remarks

What about neutral current terms (gluon, Z^0, photon exchange)?

How are they affected by the difference of mass and flavur eigenstates?

What about charged current terms in the lepton sector?

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What about charged current terms in the lepton sector?

- Neutral current terms (gluon, Z^0, photon exchange) are not affected as there are only $\overline{U}_L U_L$, $\overline{U}_R U_R$, $\overline{D}_L D_L$ or $\overline{D}_R D_R$ terms entering. These terms are flavor diagonal.

  $\rightarrow$ At tree-level there are no FCNC terms in the SM.

- Charged currents $\overline{\nu}_L E_L$ are affected as well.

  $\rightarrow$ PNMS matrix

CP violation

Example for the charge current interation for a $u \to b$ transition:

$$\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}}(\overline{u}_L \gamma^\mu W_\mu^+ V_{ub} b_L + \bar{b}_L \gamma^\mu W_\mu^- V_{ub}^* u_L)$$

$$\mathcal{L}_{CC}^{CP} = -\frac{g_2}{\sqrt{2}}(\bar{b}_L \gamma^\mu W_\mu^- V_{ub} u_L + \overline{u}_L \gamma^\mu W_\mu^+ V_{ub}^* b_L)$$

Lagrangian is invariant under CP transformation if $V_{ub} = V_{ub}^*$.

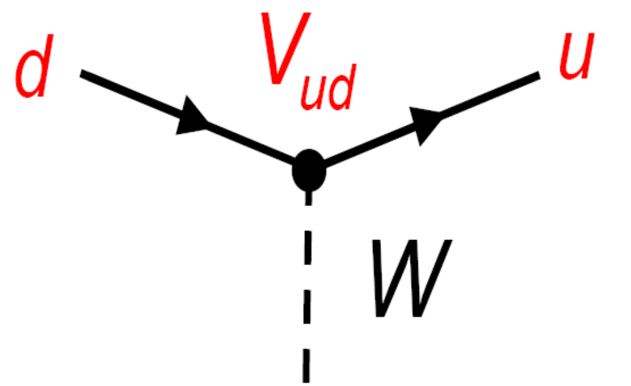
If not CPV might be observable.

CKM Matrix

Vertex current: $J^{\mu} \propto \left(\bar{u}\bar{c}\bar{t}\right) \gamma_{\mu} \left(\frac{1-\gamma^5}{2}\right) V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \times \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

flavour CKM matrix mass



18 parameters (9 complex elements)

-5 relative quark phases (unobservable)

-9 unitary conditions

——————————-

= 4 independent parameters 3 Euler angles and 1 Phase

Phase is only source of CPV in SM, requires third quark family (Nobel Prize 2008)

5 relative phases

$$J^\mu = \left(\bar{u}, \bar{c}, \bar{t}\right) \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$1 = \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\phi_u} & 0 & 0 \\ 0 & e^{-i\phi_c} & 0 \\ 0 & 0 & e^{-i\phi_t} \end{pmatrix}$$

Lagrangian insensitive to phases of quark fields, possible redefinition:

$$u \to e^{i\phi_u} u \quad c \to e^{i\phi_c} c \quad t \to e^{i\phi_t} t$$

$$d \to e^{i\phi_d} d \quad s \to e^{i\phi_s} s \quad b \to e^{i\phi_b} b$$

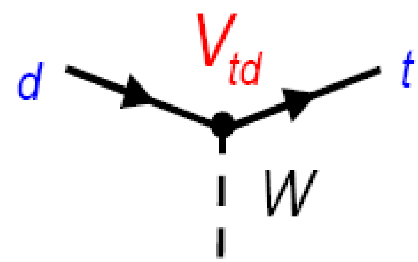
$$V_{CKM} \to \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & 0 & 0 \\ 0 & e^{-i\phi_s} & 0 \\ 0 & 0 & e^{-i\phi_b} \end{pmatrix}$$

or $V_{\alpha\beta} \to e^{\phi_\beta - \phi_\alpha} V_{\alpha\beta} \to$ 5 relative phase differences $\phi_\beta - \phi_\alpha$.

CKM under CP Transformation

Quarks

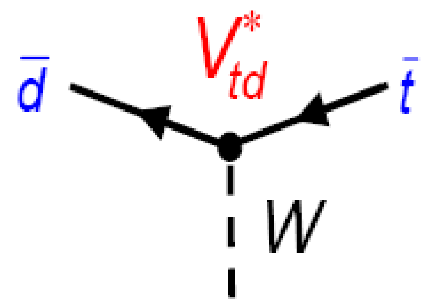
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



CP

Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



Weak (CKM) phases change sign under CP transformation!

Wolfenstein Parametrization

Reflects the hierarchical structure of the CKM matrix.

$$|V_{ub}|e^{-\gamma}$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$|V_{td}|e^{-\beta}$$

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + A^2\lambda^5(\frac{1}{2}-\rho-i\eta) & 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8}(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 + A\lambda^4(\frac{1}{2}-\rho-i\eta) & 1-A^2\frac{\lambda^4}{2} \end{pmatrix} + \mathcal{O}(\lambda^6)$$

$$|V_{ts}|e^{-\beta_s}$$

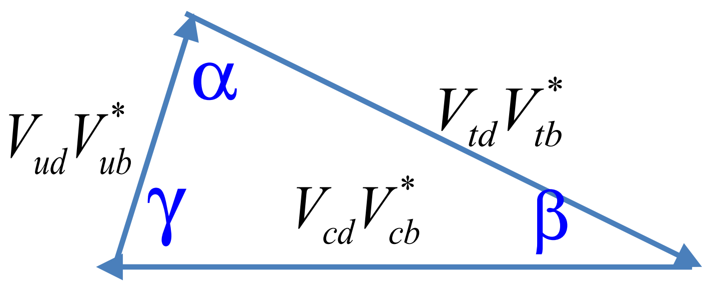
λ, A, ρ, η with λ=0.22

Unitarity of CKM Matrix $V_{CKM}^{\dagger} V_{CKM} = 1$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

(bd) triangle:



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \quad \alpha \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

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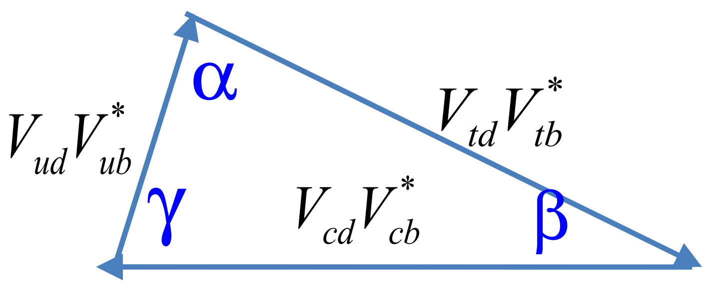
(bd) triangle:



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

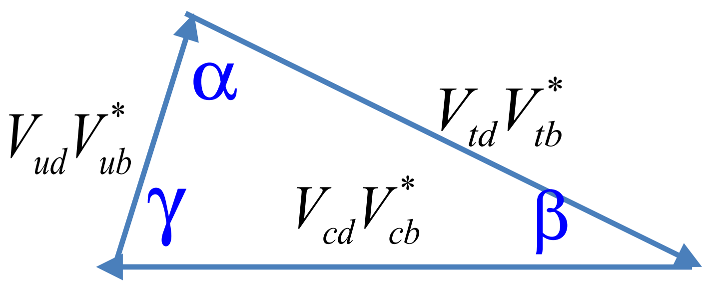
Two definitions of β and γ. Which one is the correct one? Why?

Unitarity of CKM Matrix $V^{\dagger}_{CKM} V_{CKM} = 1$

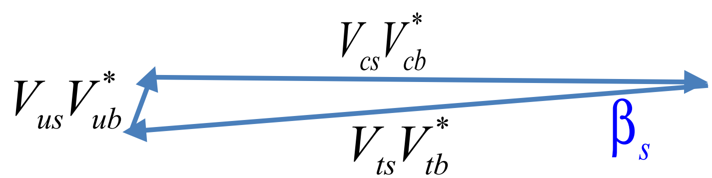
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud} V^*_{ub} + V_{cd} V^*_{cb} + V_{td} V^*_{tb} = 0$$

(bd) triangle:



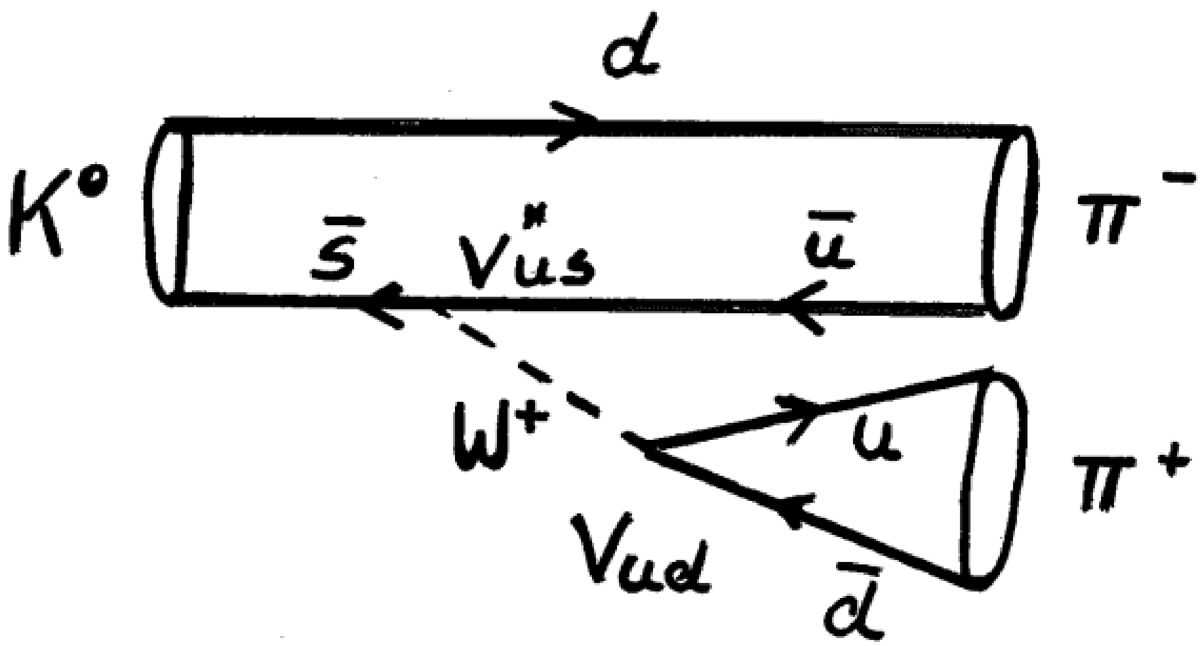
(bs) triangle:



$$\alpha \equiv \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) \quad \alpha \equiv \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) \quad \gamma \equiv \arg\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right)$$

How to connect complexe phases at the level of Lagrangien or Matrix elements to observable difference in rates for processes and their CP conjugated process?

Weak and Strong Phases



Weak phases are related to involved CKM elements: $\phi_{weak} = arg(V_{us}^{*} V_{ud})$

Strong phases δ comes often (but not always) from the hadronisation.

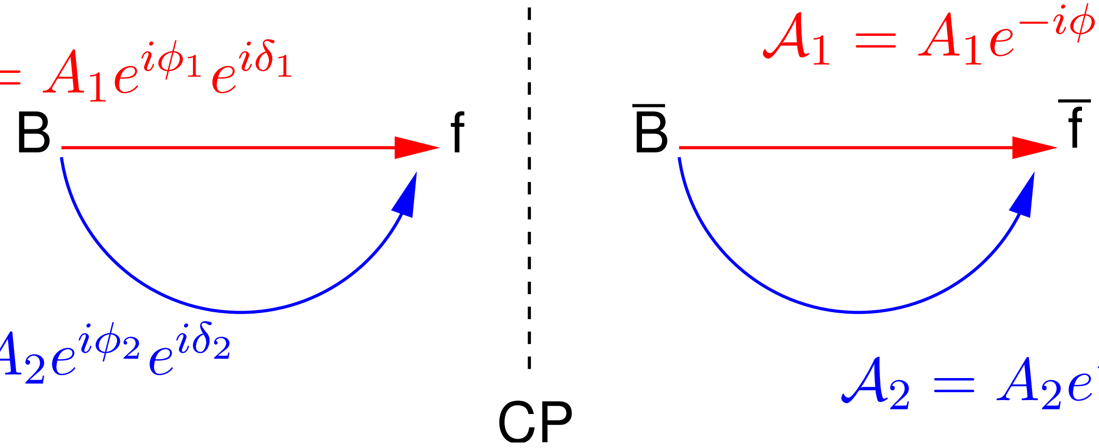
Definition of strong phase:

phase which doesn't change sign under CP transformation.

CP Violation

$$\mathcal{A}_1 = A_1 e^{i\phi_1} e^{i\delta_1}$$

$$\mathcal{A}_1 = A_1 e^{-i\phi_1} e^{i\delta_1}$$



$$\mathcal{A}_2 = A_2 e^{i\phi_2} e^{i\delta_2}$$

$$\mathcal{A}_2 = A_2 e^{-i\phi_2} e^{i\delta_2}$$

$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\phi + \Delta\delta)$$

$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(-\Delta\phi + \Delta\delta)$$

$\mathcal{A}_1$ and $\mathcal{A}_2$ need to have different weak phases ϕ and different strong phases δ.

For sizable (measurable) effects both amplitudes should have about same size, and both phase differences have to be sizable.

To conclude on weak phases, strong phases need to be known/measured.

CPV in Kaon System

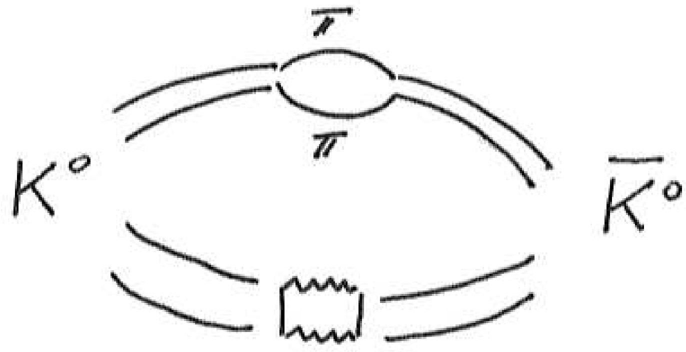
Which amplitudes contribute to CPV in kaon mixing?

Which amplitudes contribute to CPV in kaon decay?

CPV in Kaon System

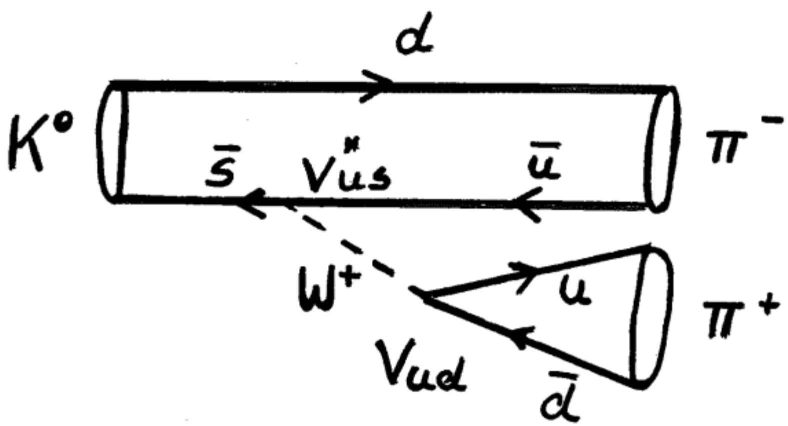
Interfering amplitudes which cause CPV in mixing:

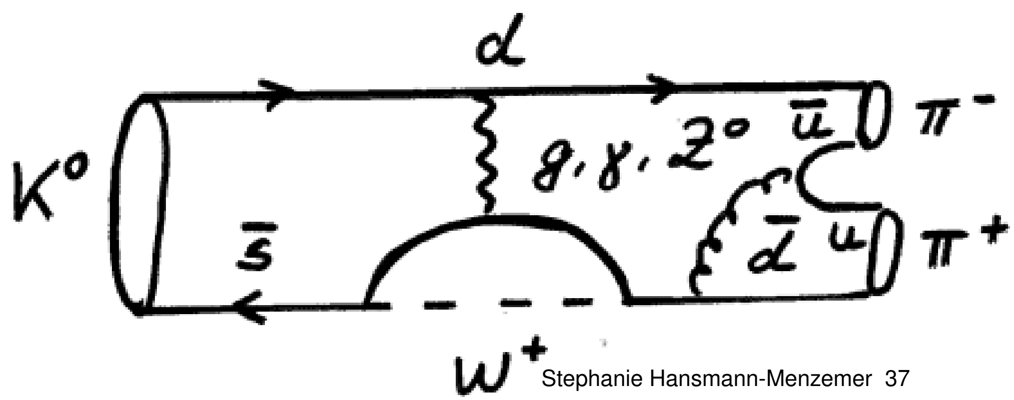
long range contribution $\Delta\Gamma$



short range contribution Δm

Interfering amplitudes which cause CPV in decay:





Neutral Meson Mixing

Assume no CPV in mixing ($|\frac{q}{p}|$=1)

$$P(\mathbf{K^0} \to \overline{\mathbf{K^0}}) = | < \mathbf{K^0(t)} | \overline{\mathbf{K^0}} > |^2 =$$
$$\frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-(\Gamma_L+\Gamma_S)t/2} \cos \Delta m\, t\right)$$

$$P(\mathbf{K^0} \to \mathbf{K^0}) = | < \mathbf{K^0(t)} | \mathbf{K^0} > |^2 =$$
$$\frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-(\Gamma_L+\Gamma_S)t/2} \cos \Delta m\, t\right)$$

$$\mathcal{A}(t) \quad = \quad \frac{N_{unmixed}(t) - N_{mixed}(t)}{N_{unmixed}(t) + N_{mixed}(t)} = 2\frac{e^{\frac{-(\Gamma_S+\Gamma_L)t}{2}} \cos(\Delta m\, t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

Neutral Meson Mixing

Assume no CPV in mixing ($|\frac{q}{p}|$=1)

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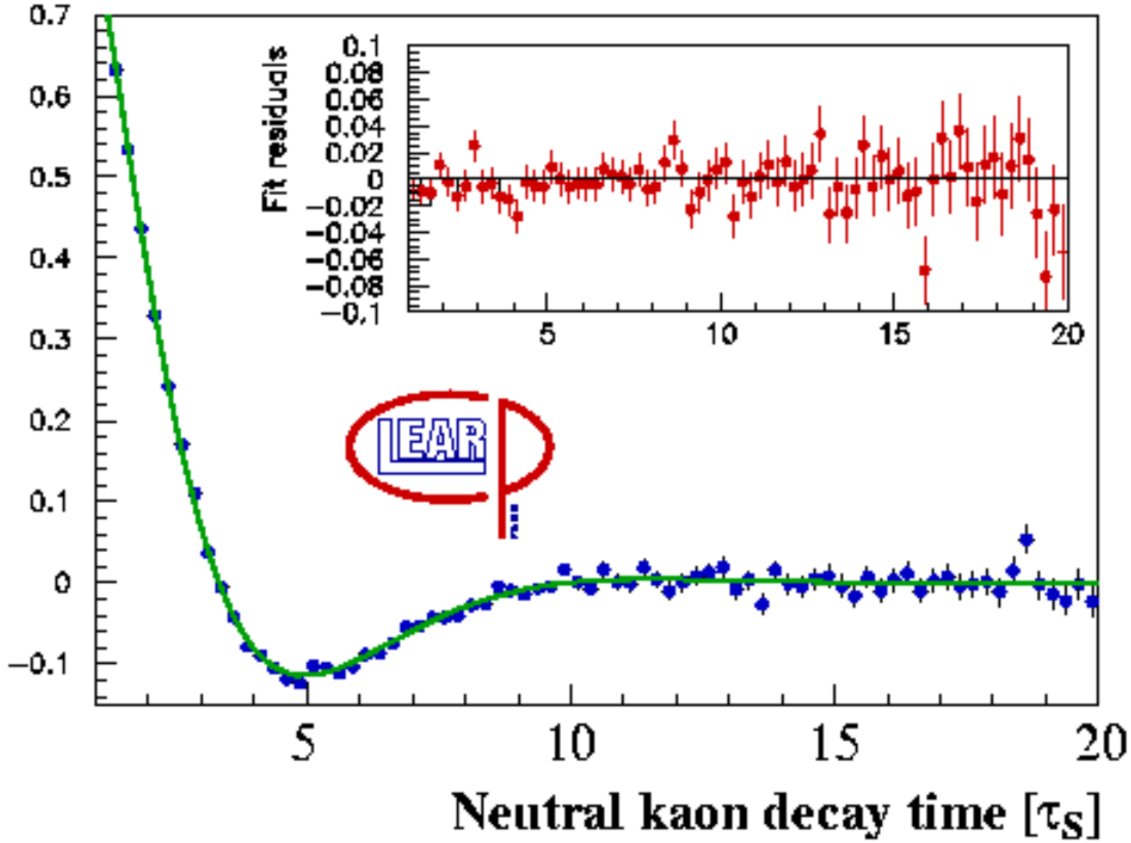
If $\Gamma_L \sim \Gamma_S \rightarrow \mathcal{A} = cos(\Delta m)\, t$ — e.g. B system.

If $\Gamma_L << \Gamma_S \rightarrow \mathcal{A} \sim e^{-\frac{\Delta\Gamma\, t}{2}}\cos(\Delta m\, t)$ — e.g. kaon system

Neutral Kaon Mixing

$$\mathcal{A}(t) = \frac{N_{unmixed}(t) - N_{mixed}(t)}{N_{unmixed}(t) + N_{mixed}(t)} = 2\frac{e^{\frac{-(\Gamma_S+\Gamma_L)t}{2}}\cos(\Delta m\, t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$



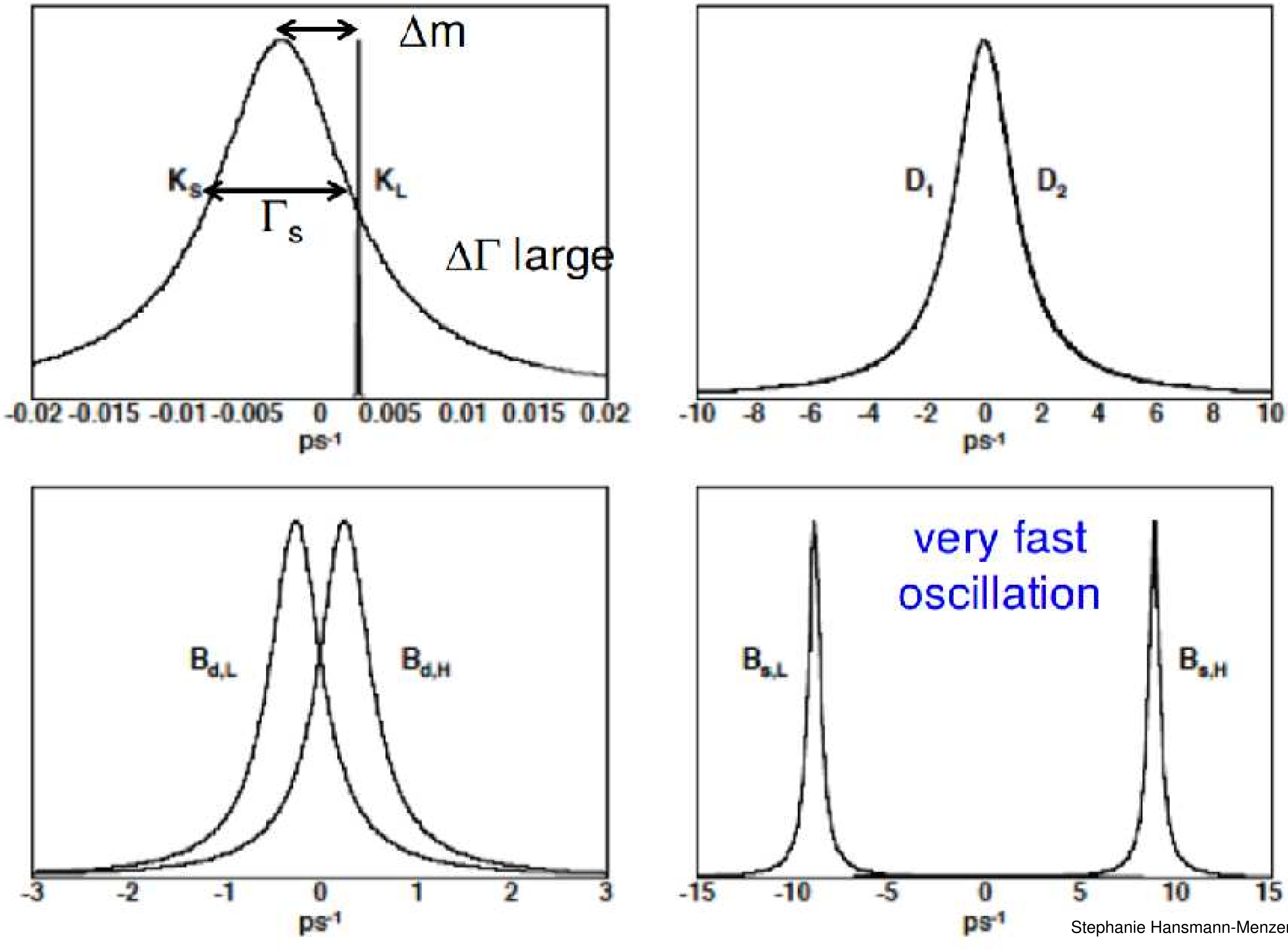
$$\Delta m = (529.5 \pm 2.0 \pm 0.3) \times 10^{-7} \hbar s^{-1}$$

Summary of Mixing I

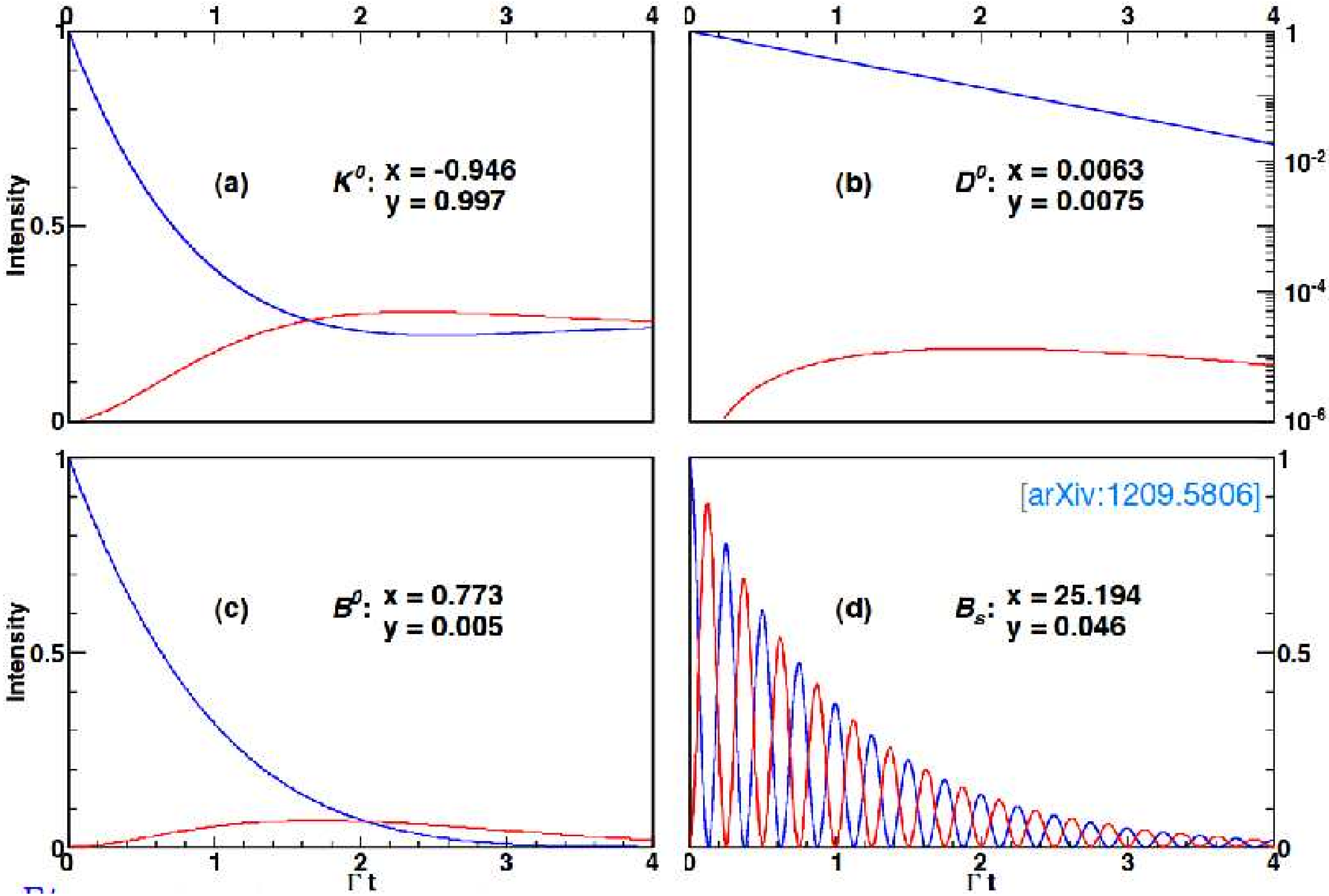
	$K^0/\overline{K^0}$	$D^0/\overline{D^0}$	$B^0/\overline{B^0}$	$B_s^0/\overline{B_s^0}$
τ [ps]	89.3	0.415	1.564	1.47
	51700			
Γ [ps^{-1}]	5.61$\cdot$10^{-3}	2.4	0.643	0.62
y=$\frac{\Delta\Gamma}{2\Gamma}$	0.9966	0.008	0.0075	0.059
Δm [ps^{-1}]	5.301$\cdot$10^{-3}	0.16	0.506	17.8
x=$\frac{\Delta m}{\Gamma}$	0.945	0.010	0.768	26.1

Depending if the decay width difference or the mass difference is the dominant criteria to distinguish the both eigenstates, they are called K_{Short} and K_{Long} or B_{Heavy} and B_{Light}.

Summary of Mixing II



Summary of Mixing III



B^0 mixing

$$\mathcal{A} = \frac{N_{unmixed} - N_{mixed}}{N_{unmixed} + N_{mixed}}$$

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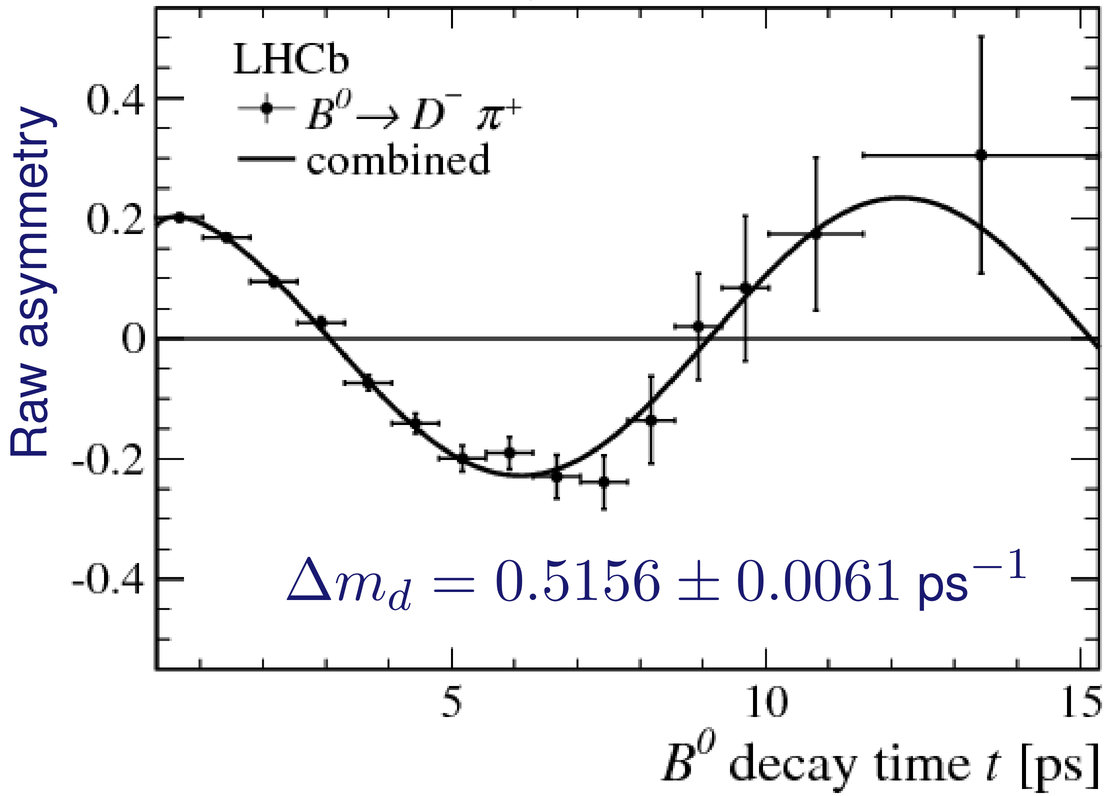
B^0

$\Updownarrow$

$\overline{B^0}$

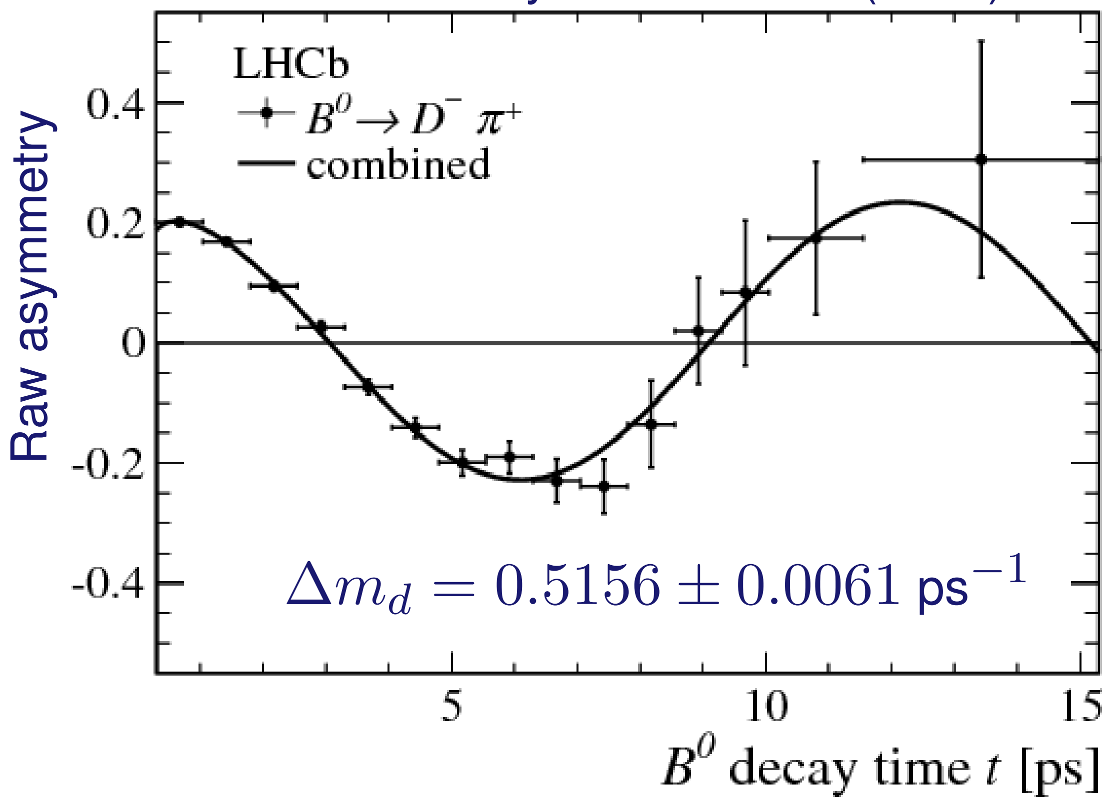


B^0 mixing

$$\mathcal{A} = \frac{N_{unmixed} - N_{mixed}}{N_{unmixed} + N_{mixed}}$$

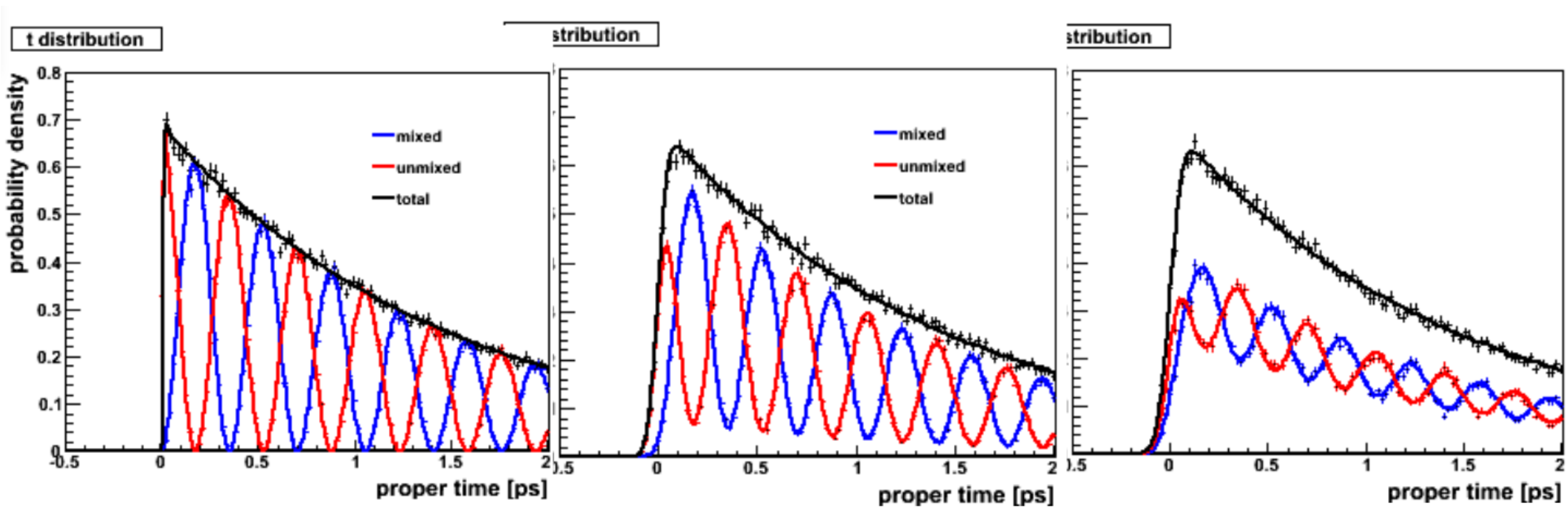
Phys. Lett. B 719 (2013) 328

$B^0 \Updownarrow \overline{B^0}$



Why is the amplitude of the oscillation not 1?

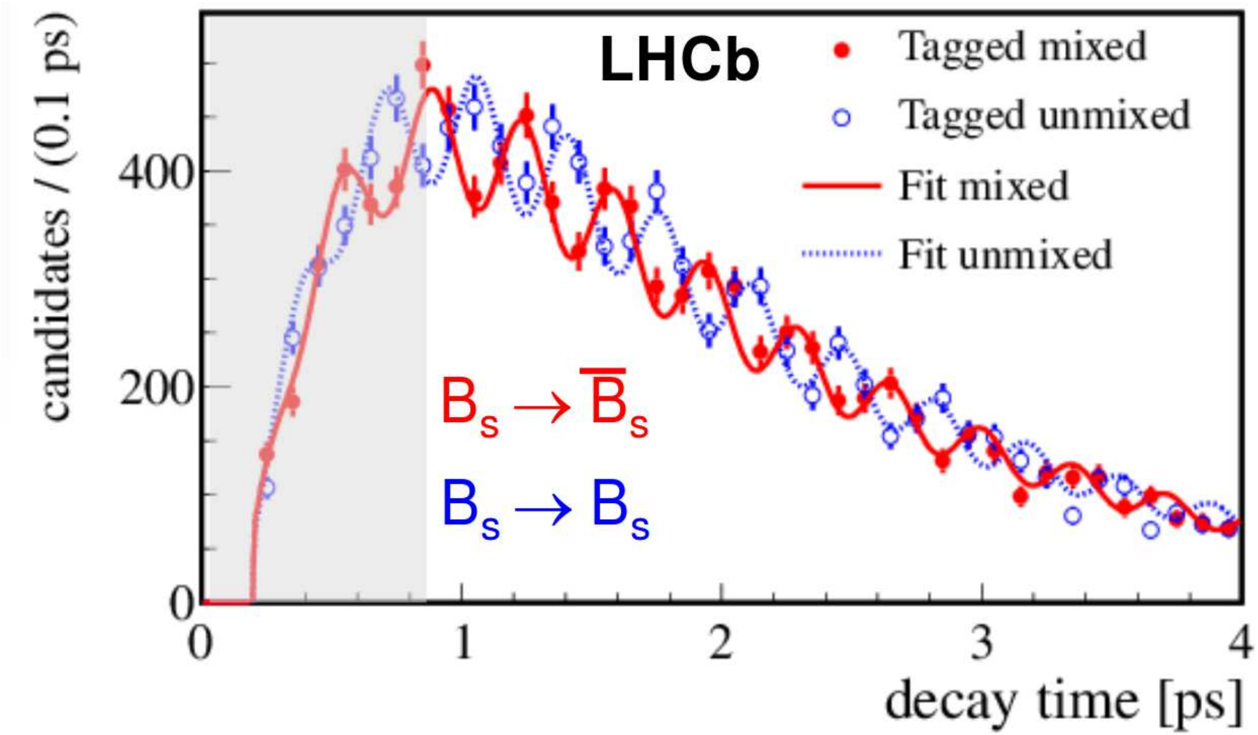
Detector effects on B_s oscillation



⇒ Finite time resolution: 44 fs

⇒ Realistic tagging

B_s mixing at LHCb



turn on, due to trigger cuts

washed out signature due to decay time resolution and flavour tagging

Δm_s = 17.768 $\pm$ 0.023 (stat) $\pm$ 0.006 (syst)

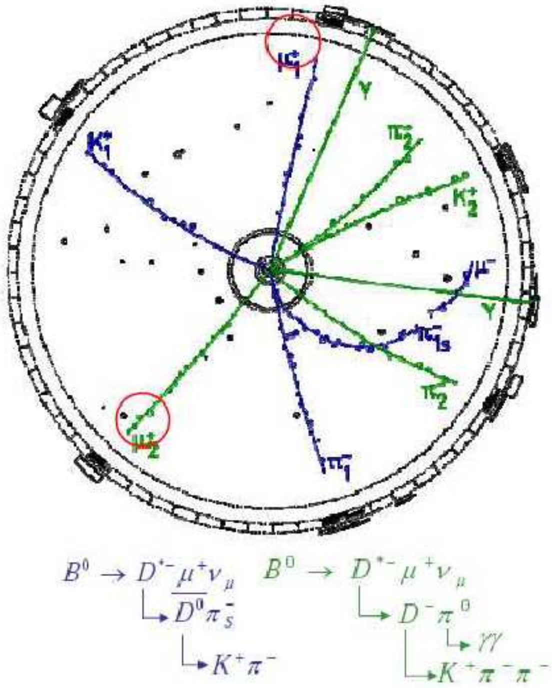
Theorie: $\Delta m_s = 18.3 \pm 2.7$ ps^{-1}

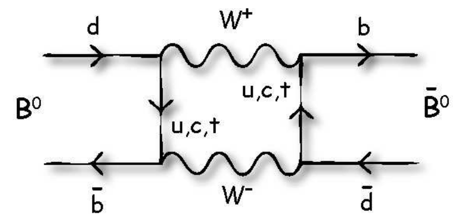
Precision tests of the Standard Model difficult:

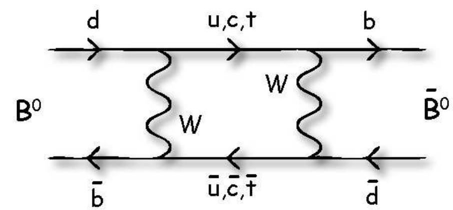
Hadronic uncertainties limit the precision of the theoretical predictions.

1986: B^0 Oscillation at ARGUS

$$e^+e^- \rightarrow Y(4S) \rightarrow B^0\overline{B^0}$$







Time integrated mixing rate: $\chi_d = \int P_{mixed}(t) \cdot e^{-t/\tau} dt = 0.17 \pm 0.05$

25 mixed events:

$B^0\overline{B^0} \rightarrow \ell^-\ell^-$

$B^0\overline{B^0} \rightarrow \ell^+\ell^+$

250 unmixed events:

$B^0\overline{B^0} \rightarrow \ell^+\ell^-$

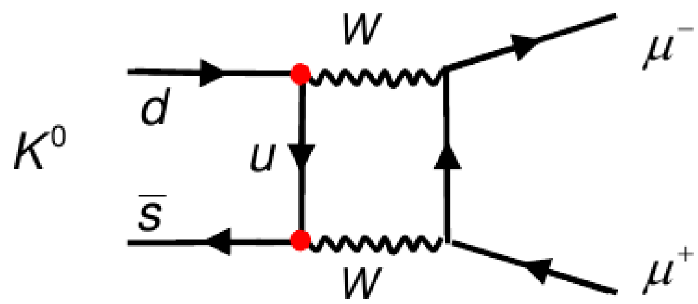
First indication for a heavy top quark $m_t > 50$ GeV! - How?

What is GIM Mechanism ?

1970: Rare Kaon Decays

Observed branching ratio $K_L \rightarrow \mu^+\mu^-$

$$\frac{BR(K_L \rightarrow \mu^+\mu^-)}{BR(K_L \rightarrow all)} = (7.2 \pm 0.5) \times 10^{-9}$$

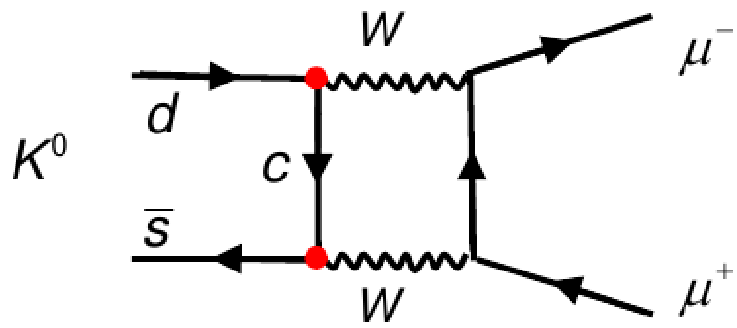


$M \sim \sin\theta_c \cos\theta_c$

In contradiction with theoretical expectations in the 3 quark model $(d' = d\cos\theta_c + s\sin\theta_c)$

➡ Glashow, Iliopolus, Maiani (1970):

Prediction of a 2^{nd} up type quark, additional Feynman graph cancels the “u box graph”



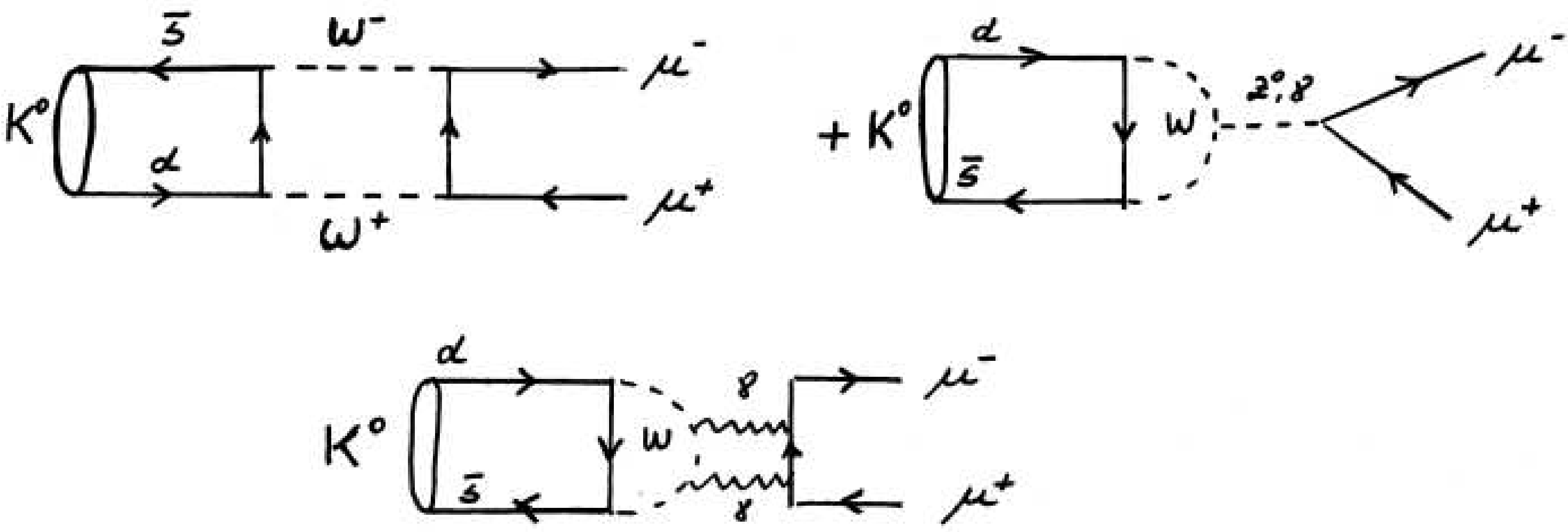
$M \sim -\sin\theta_c \cos\theta_c$

GIM mechanism

The study of this rare decay resulted in accidentally correct prediction of $m_c \sim 1.5 GeV$

Additional Diagrams

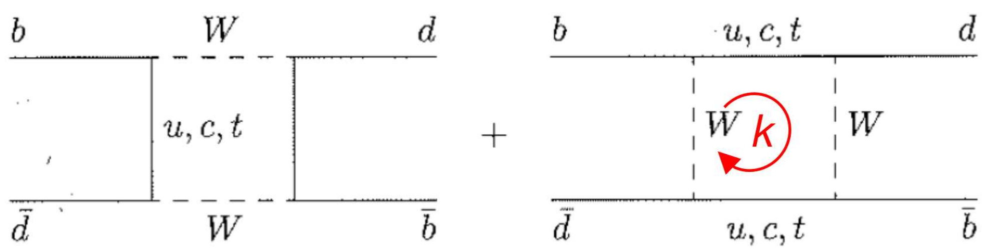
short range contribution



+ long range contributions

Standard Model Predictions

B mixing is dominated by the off-shell box diagramms:



Intergral over all possible internal loop momenta k

$$\begin{aligned} M \quad \propto \quad & \int_k V_{tb}V_{td}^*\Pi_t(V_{tb}V_{td}^*\Pi_t + V_{cb}V_{cd}^*\Pi_c + V_{ub}V_{ud}^*\Pi_u) \\ & + V_{cb}V_{cd}^*\Pi_c(V_{tb}V_{td}^*\Pi_t + V_{cb}V_{cd}^*\Pi_c + V_{ub}V_{ud}^*\Pi_u) \\ & + V_{ub}V_{ud}^*\Pi_u(V_{tb}V_{td}^*\Pi_t + V_{cb}V_{cd}^*\Pi_c + V_{ub}V_{ud}^*\Pi_u) \end{aligned}$$

$\Pi_q(k) \propto \frac{\gamma_\mu k^\mu + m_q}{k^2 - m_q^2}$

(all other factors are the same for all diagrams)

Standard Model Predictions

Since m_u, m_c are much smaller than m_t, treat them as 0.

$$\begin{aligned} M \quad \propto \quad & \int_k V_{tb}V_{td}^*\Pi_t(V_{tb}V_{td}^*\Pi_t + (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0) \\ & +V_{cb}V_{cd}^*\Pi_0(V_{tb}V_{td}^*\Pi_t + (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0) \\ & +V_{ub}V_{ud}^*\Pi_0(V_{tb}V_{td}^*\Pi_t + (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0) \end{aligned}$$

Exploit unitarity relation: $-V_{tb}V_{td}^* = V_{cb}V_{cd}^* + V_{ub}V_{ud}^*$

$$M \propto (V_{tb}V_{td}^*)^2 \left(\Pi_t\Pi_t - \Pi_t\Pi_0 - \Pi_0\Pi_t + \Pi_0\Pi_0\right)$$

Effect of inner-quark propagators are described by the Inami-Lim function

$$S(m_q^2/M_W^2) \propto \frac{m_q^2}{M_W^2}: \quad S(m_t^2/M_W^2) \propto 2.5 \quad S(m_c^2/M_W^2) \propto 3.5 \cdot 10^{-4}$$

For B-mixing, top quark is dominated due to CKM favored matrix elements and the loop intergration.

Inam-Lim function $S(x) = x\left(\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right) - \frac{3}{2}\left(\frac{x}{1-x}\right)^3 \ln x$

Standard Model Predictions

If one concludes the calculation one obtains for the M_{12} ($\approx 2\Delta m$):

$$M_{12} \approx \frac{G_F^2 M_W^2}{12\pi^2} \eta_{QCD} B_B f_B^2 m_b S(m_t^2/M_W^2) |V_{td} V_{tb}^*|^2$$

η_{QCD}: Perturbative QCD corrections

B_B: Bag-parameter;

f_B: decay constant – describe the non-perturbative effects of the bound quarks

Remark:

For neutral B mesons there exist reliable calculation of the hadronization effects.

For neutral D and K mesons more difficult.

$\Delta m_d^{SM} = 0.543 \pm 0.091$ ps^{-1} exp: $\Delta m_d = 0.515 \pm 0.002$ ps^{-1}

$\Delta m_s^{SM} = 17.3 \pm 2.6$ ps^{-1} exp: $\Delta m_s = 17.77 \pm 0.006$ ps^{-1}

Difference is effect of the CKM elements: $\Delta m_s / \Delta m_d = |V_{ts}|^2 / |V_{td}|^2$

Standard Model Predictions

For D-mesons (up-type quark system):

Mass of the most heaviest internal quark (d-type: b-quark) is not large enough to compensate the large CKM suppression ($|V_{ub}V_{cb}^*|^2$)

As a result, the light s-quark dominate the short range mixing:

$$\Delta m_D \propto |V_{us}V_{cs}^*|^2 S(m_S^2/M_W^2) \approx \lambda^2 m_s^2/M_W^2$$

experimental: $\Delta m_D \approx 0.0024\, ps^{-1}$

Mixing parameters of the neutral D mesons are very small (very slow mixing): most of the D mesons decay before they mix (lifetime of D mesons $\approx$ 1 ps is much shorter than the one of neutral koans).

D mixing was observed with high significance by LHCb – interpretation is difficult.

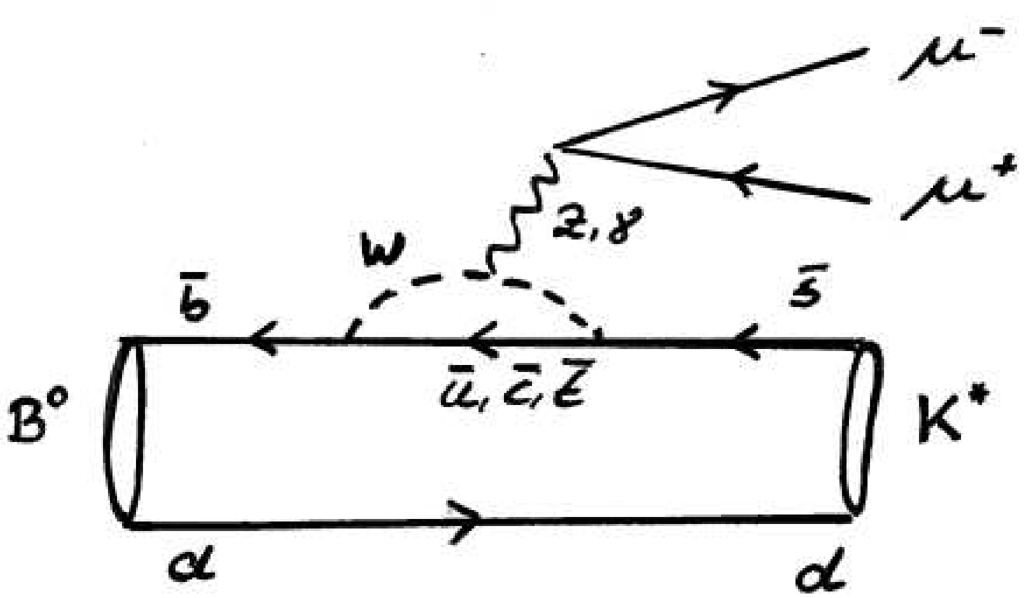
GIM & Neutral Meson Mixing

box diagram of K^0 mixing is dominated by c charm quark

box diagram of $B^0_{(s)}$ mixing is dominated by t quark

box diagram of D^0 system is dominated by s quark

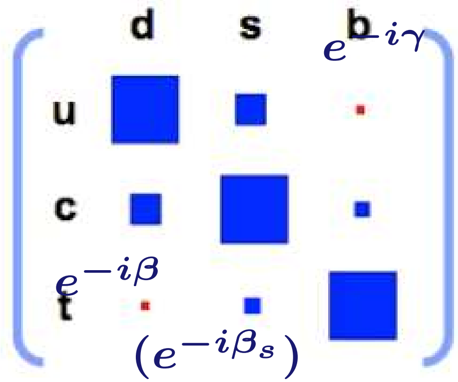
Same conclusion is true for any loop diagram:



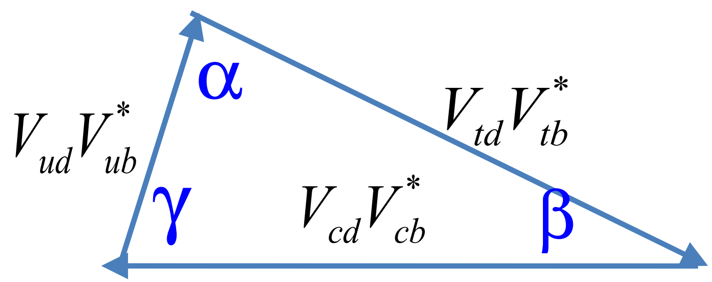
CKM Matrix and Angles

size of box, illustrates absolute value

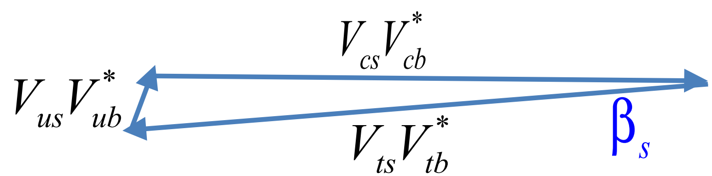
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim$$



B_d triangle:



B_s triangle:



CP violation is caused by phases of CKM matrix

$\rightarrow$ how can we use CP violation to extract CKM angles?

CP Violation in one Page

Mass eigenstates:

$$B_L = p|B^0> + q|\overline{B^0}> \text{ w. } m_L, \Gamma_L$$

$$B_H = p|B^0> - q|\overline{B^0}> \text{ w. } m_H, \Gamma_H$$

$|p^2| + |q^2| = 1,$ complex coefficients

Flavour eigenstates:

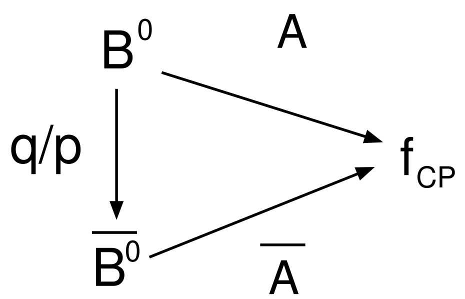
$$B^0 = \frac{1}{2p}(|B_L> + |B_H>)$$

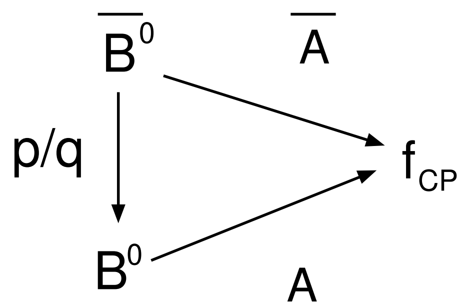
$$\overline{B^0} = \frac{1}{2q}(|B_L> - |B_H>)$$

- CP violation in decay $|A(B \to f)| \neq |\overline{A}(\overline{B} \to \overline{f})|$
- CP violation in mixing

  If $|\frac{q}{p}| \neq 1$; mass eigenstates are no CP eigenstates;

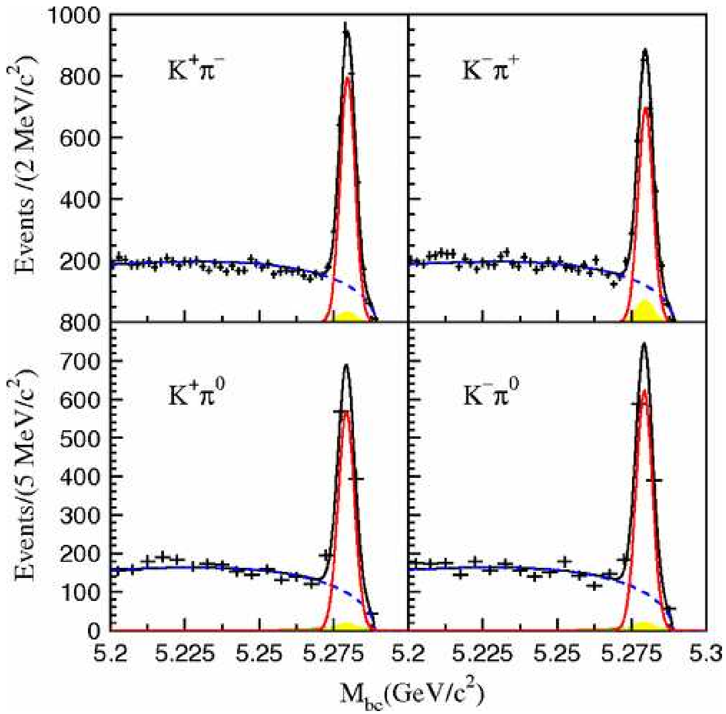
  $\to P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0)$
- CP violation in interference of mixing and decay: $Im(\frac{q}{p}\frac{\overline{A}}{A}) \neq 0$





First direct CPV in ...

$B^0 \to K^+\pi^- / \overline{B^0} \to K^-\pi^+$ (Belle)

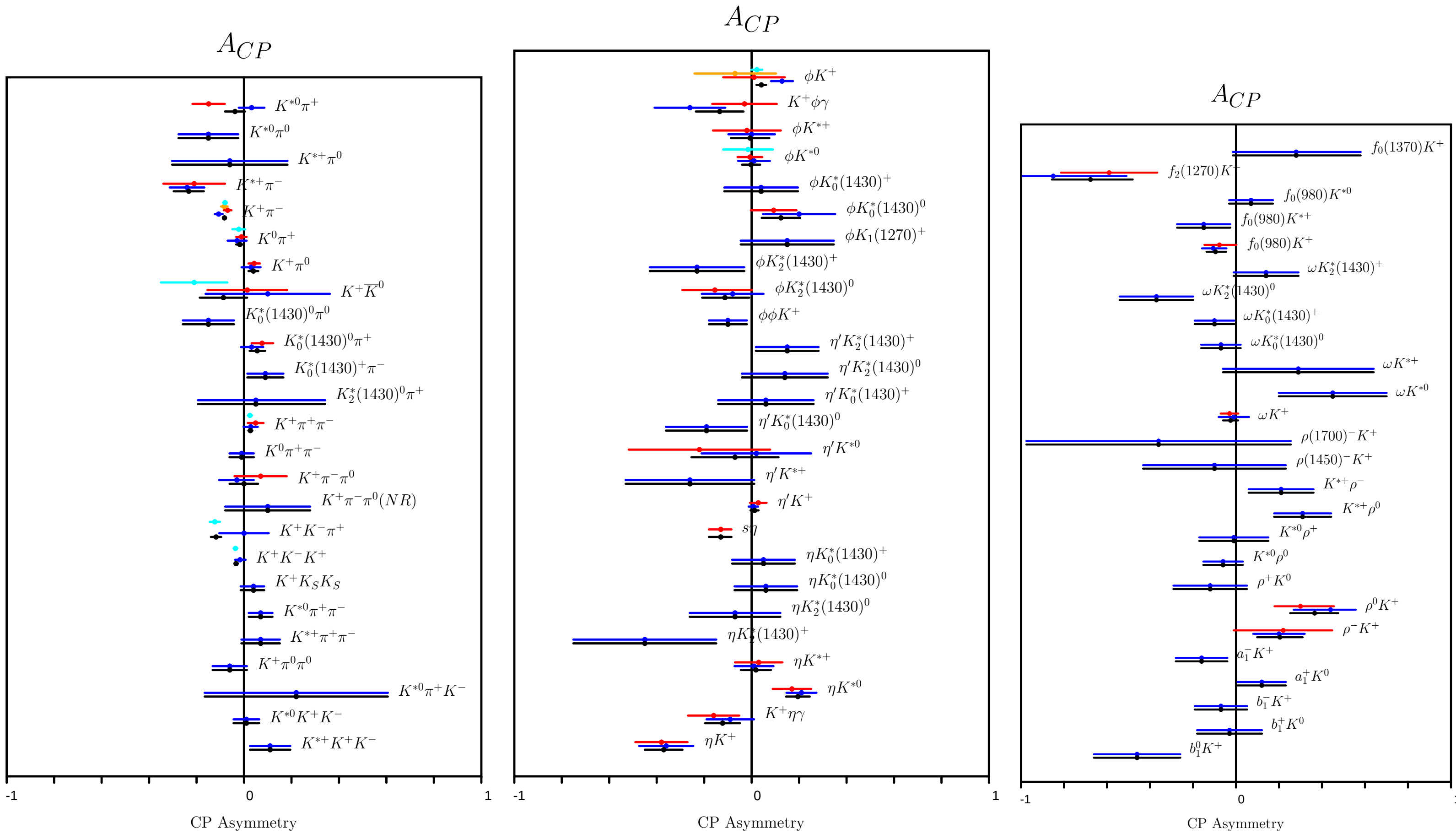


WA: $A_{CP} = -0.082 \pm 0.006$

CPV in decays is as well estabished in B_s and $B^{\pm}$ decays.

While we get different partial decay widths for individual particle B and antiparticle $\overline{B}$ decays the total decay width ($\Gamma_{tot} = \frac{1}{\tau}$) is the same for both (CPT)!

Lot's of direct CPV ...



Due to unknown strong phases, hard to relate CPV directly to CKM parameters.

Simulataneous analysis of muliple cannels needed to constraint strong phases (e.g. γ measurement).

CPV in B_s mixing?

▶ $P(B \to \overline{B}) \neq P(\overline{B} \to B)$

semileptonic asymmetry

$(B^0 + B_s)$

$B \longrightarrow \overline{B} \longrightarrow \mu^-$

$\overline{B} \longrightarrow \mu^-$

$\overline{B} \longrightarrow B \longrightarrow \mu^+$

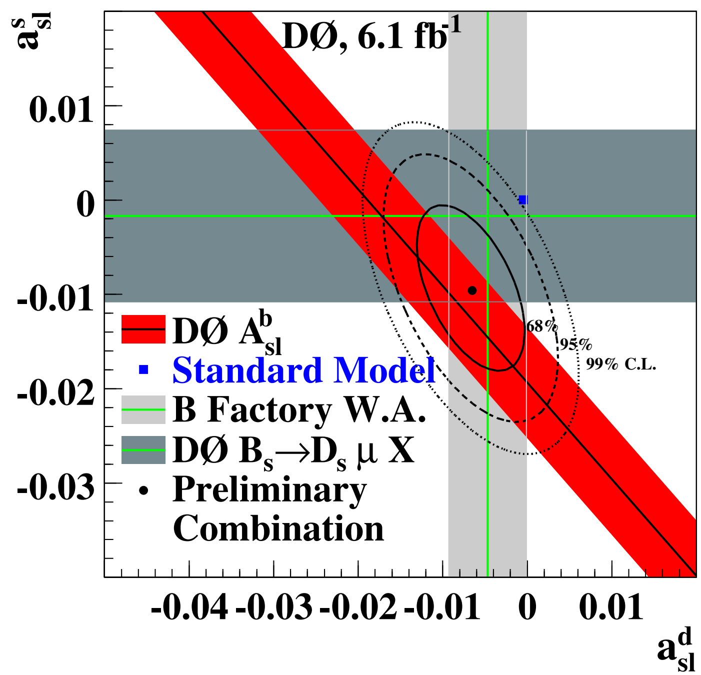
$B \longrightarrow \mu^+$

$$A = \frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)}$$

$$a = \frac{N(\mu^+) - N(\mu^-)}{N(\mu^+) + N(\mu^-)}$$

SM: A^b_{sl} = (-0.20 $\pm$ 0.03) $\times$ 10^{-3}

A. Lenz, U. Nierste, (2006/2011)



A^b_{sl} = -0.957 $\pm$ 0.251 (stat) $\pm$ 0.14 (syst) %

(Phys. Rev. Lett 105, 081802 (2010))

$\rightarrow$ 3.2σ deviation from SM

a_{sl} at LHCb

$$a_{sl} = \frac{\Gamma(\overline{B}\to B\to f)-\Gamma(B\to\overline{B}\to\overline{f})}{\Gamma(\overline{B}\to B\to f)+\Gamma(B\to\overline{B}\to\overline{f})}$$

If there is no CPV in decay $\Gamma(B \to f) = \Gamma(\overline{B} \to \overline{f})$:

$$a_{sl} \neq 0 \Leftrightarrow \Gamma(\overline{B} \to B) \neq \Gamma(B \to \overline{B})$$

However tagging the initial flavour is difficult at hadron colliders ...

(tagging power $\sim$ 4% $\to \sigma_{stat}^{tag} = \frac{1}{\sqrt{0.04}} \cdot \sigma_{stat}^{no\,tag} = 5 \cdot \sigma_{stat}^{no\,tag}$)

Untagged method:

Assuming there is no production asymmetry $(N(B, t = 0) = N(\overline{B}, t = 0))$

and no CPV in decay:

$$\frac{N(f)(t)-N(\overline{f})(t)}{N(f)(t)+N(\overline{f})(t)} = \frac{a_{sl}}{2} \cdot [1 - \frac{\cos\Delta mt}{\cosh\frac{\Delta\Gamma t}{2}}]$$

index sl: semileptonic decays ... simply due to large statistics

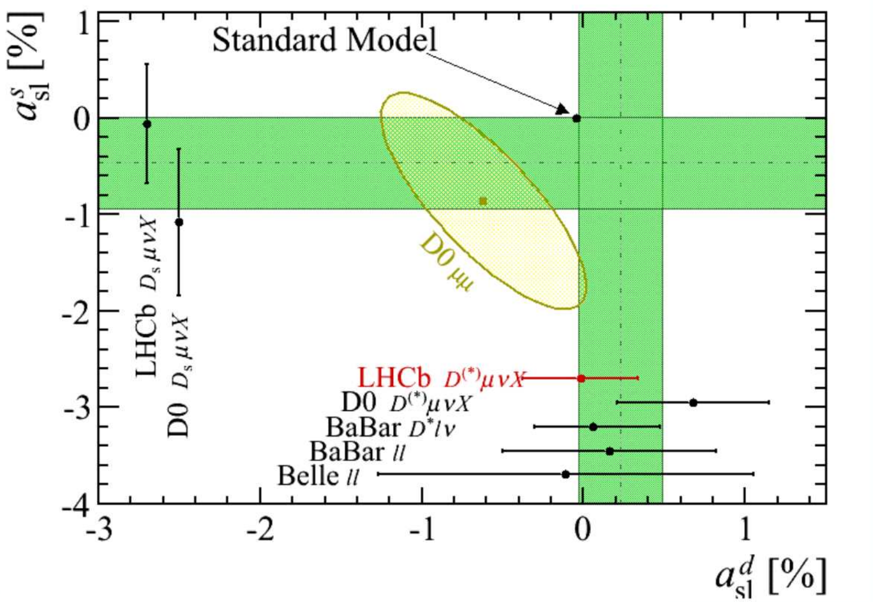
Where do production asymmetries come from at the LHC?

a_{sl} at LHCb

For B_s^0 system, can perform time integrated analysis (fast oscillation)

$$\frac{N(f)(t)-N(\overline{f}(t))}{N(f)(t)+N(\overline{f}(t))} = \frac{a_{sl}^s}{2} + [a_P - \frac{a_{sl}^s}{2}]\frac{\int e^{-\Gamma_s t}\cdot\cos\Delta m t\, dt}{\int e^{-\Gamma_s t\, dt}\cdot\cosh\frac{\Delta\Gamma_s t}{2}}] \sim \frac{a_{sl}^s}{2}$$

For B_d^0 time dependent analysis is required. Due to missing neutrino in $B_d^0 \to D^-\mu^+X$ decays need to correct for missing momentum to reconstruct B_d^0 momentum and thus the B_d^0 decay time.



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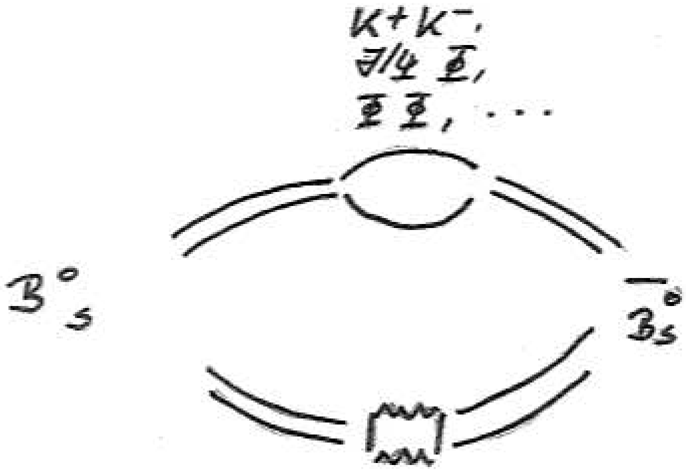
What are the interfering amplitudes?

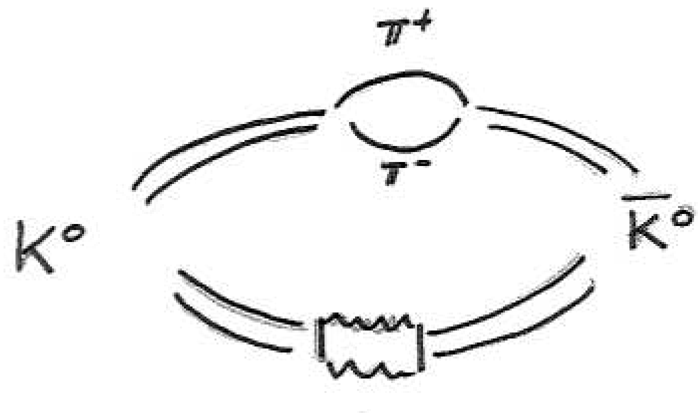
Why is CPV in B mixing so much smaller than CPV in K^0 mixing?

CPV in B Mixing

branching ratio into non-flavour specific decays

$\sim 10^{-4}$ $> 0.95\%$





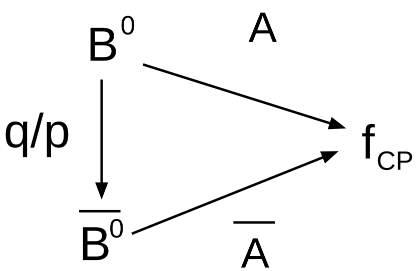
	$K^0/\overline{K^0}$	$D^0/\overline{D^0}$	$B^0/\overline{B^0}$	$B_s^0/\overline{B_s^0}$
y=$\frac{\Delta\Gamma}{2\Gamma}$	0.9966	0.008	0.0075	0.059
x=$\frac{\Delta m}{\Gamma}$	0.945	0.010	0.768	26.1

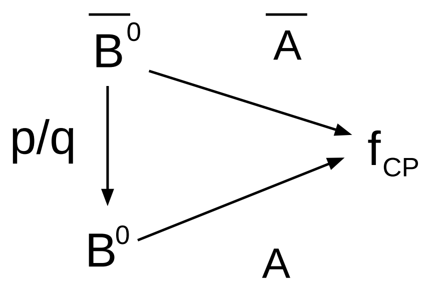
CPV in interference of mixing and decay

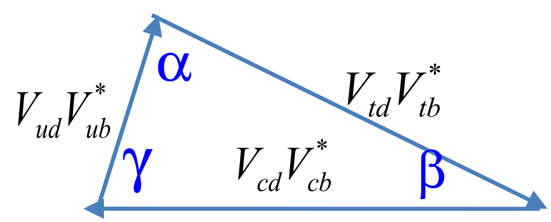
Measurement of $\sin(2\beta)$: golden channel $B_d \to J/\psi K_s$

"Golden": large statistics, easy to detect,

(almost) no CPV in decay and no CPV in mixing



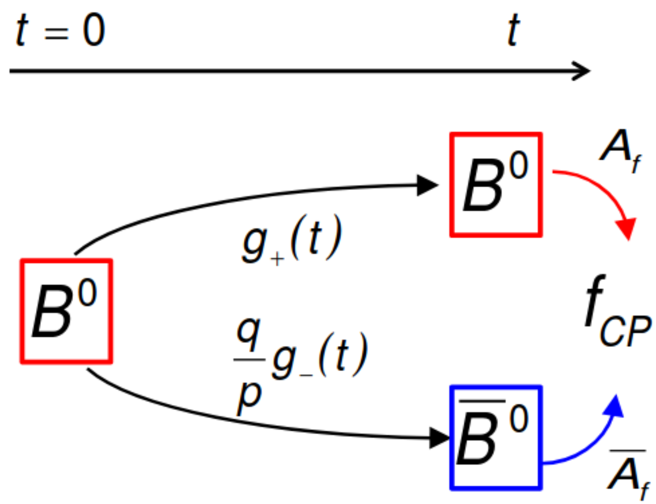




Weak phase: $Im(\frac{q}{p}\frac{\overline{A}}{A})$

$$\beta = \arg\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} \sim \arg V_{td}$$

CPV in interference of mixing and decay



$$\Gamma(B^0 \to f_{CP}) \propto |g_+(t)A_f + \frac{q}{p}g_-(t)\overline{A}_f|^2$$

$$g_+(t) = e^{-i(m-i\frac{\Gamma}{2})t}(+\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta m t}{2} - i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta m t}{2})$$
$$g_-(t) = e^{-i(m-i\frac{\Gamma}{2})t}(-\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta m t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta m t}{2})$$

For B^0 system: $\Delta\Gamma \sim 0$

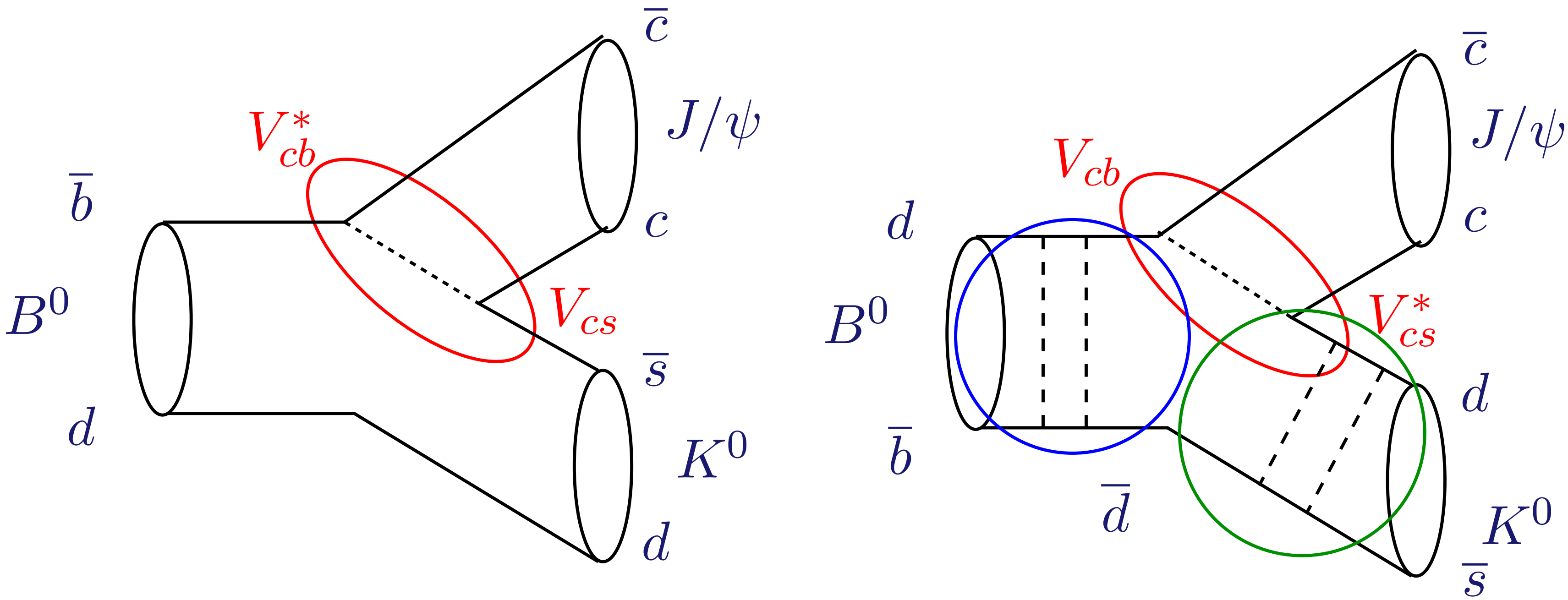
$$g_+(t) \sim e^{-i(m-i\frac{\Gamma}{2})t}(\cos\frac{\Delta m t}{2})$$
$$g_-(t) \sim e^{-i(m-i\frac{\Gamma}{2})t}(i\sin\frac{\Delta m t}{2})$$

$\rightarrow$ strong phase difference: π

$B_d \to J/\Psi K^0$

Reach same final state through decay & mixing + decay

(assume no CPV in mixing and no CPV in decay and $\Delta\Gamma \sim 0$)



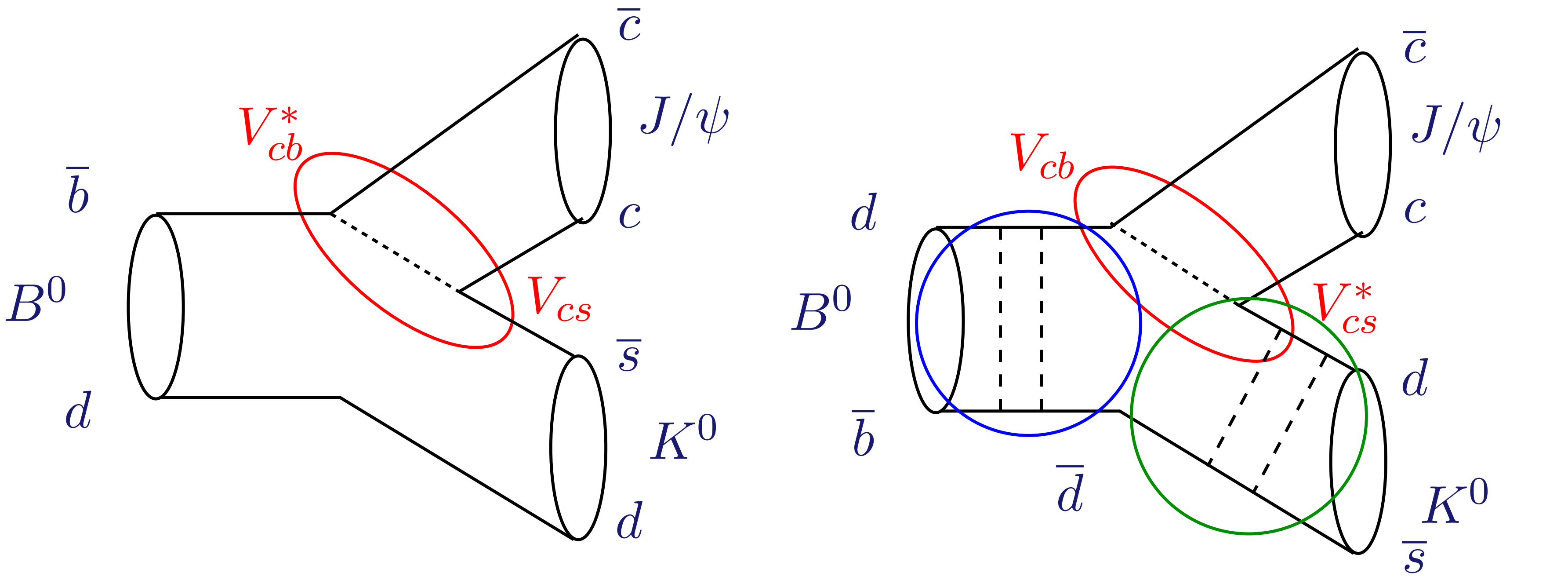
$$\mathcal{A}_1 = \mathcal{A}_{mix}(B^0 \to B^0) * \mathcal{A}_{decay}(B^0 \to J/\Psi K^0) = \cos(\frac{\Delta mt}{2}) * A * e^{i\omega}$$

$$\mathcal{A}_2 = \mathcal{A}_{mix}(B^0 \to \overline{B^0}) * \mathcal{A}_{decay}(\overline{B^0} \to J/\Psi K^0) = i\sin(\frac{\Delta mt}{2}) * e^{+i\phi} * A * e^{-i\omega} A_K * e^{+i\xi}$$

weak phase difference $\mathcal{A}_2 - \mathcal{A}_1$: $\Delta\phi = \phi - 2\omega + \xi = 2\beta$

strong phase difference $\Delta\delta = \pi \Leftarrow$ mixing introduces strong phase difference

$B_d \to J/\Psi K^0$



$$\Delta\phi = \phi - 2\omega + \xi = arg[\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}]$$

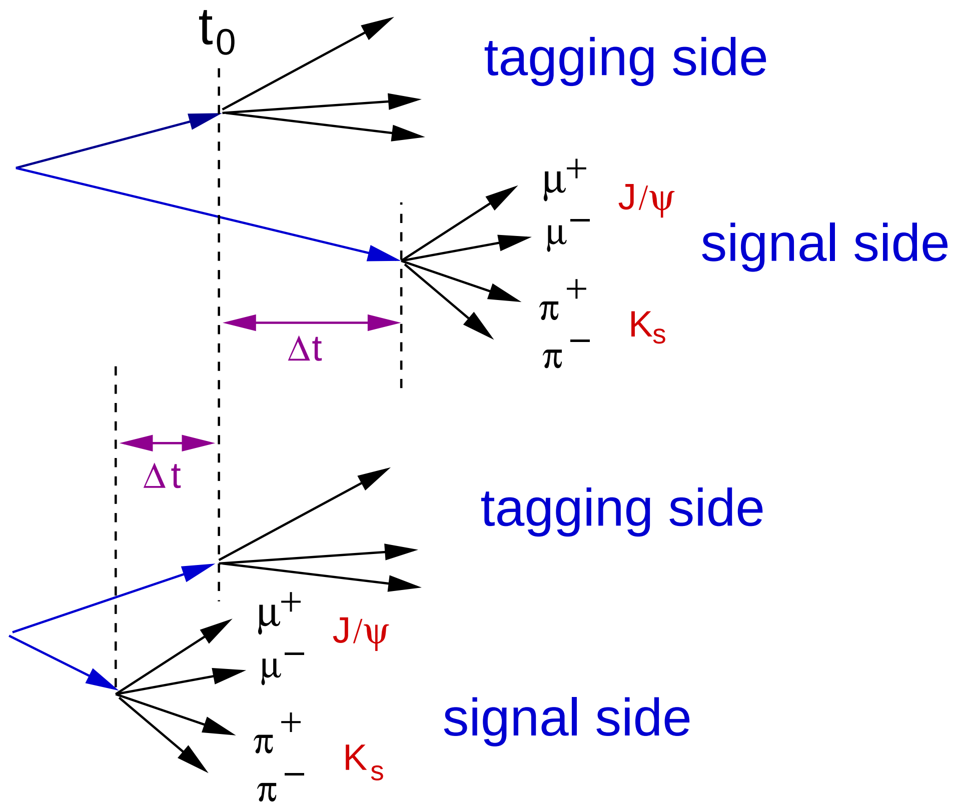
$$= arg[\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}] = 2arg[\frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*}] = 2\beta$$

t quark dominates B^0 mixing box, c quark dominates K^0 mixing box diagram

Correlated B Production

$$A(t) = \frac{N(\overline{B}\to J/\psi K_s)(t) - N(B\to J/\psi K_s)(t)}{N(\overline{B}\to J/\psi K_s)(t) + N(B\to J/\psi K_s)(t)} = \eta_{CP}\sin(2\beta)\sin\Delta m_d t$$

(for K_s η_{CP} = -1, for K_L η_{CP} = +1 ... neglecting CP in kaon mixing)

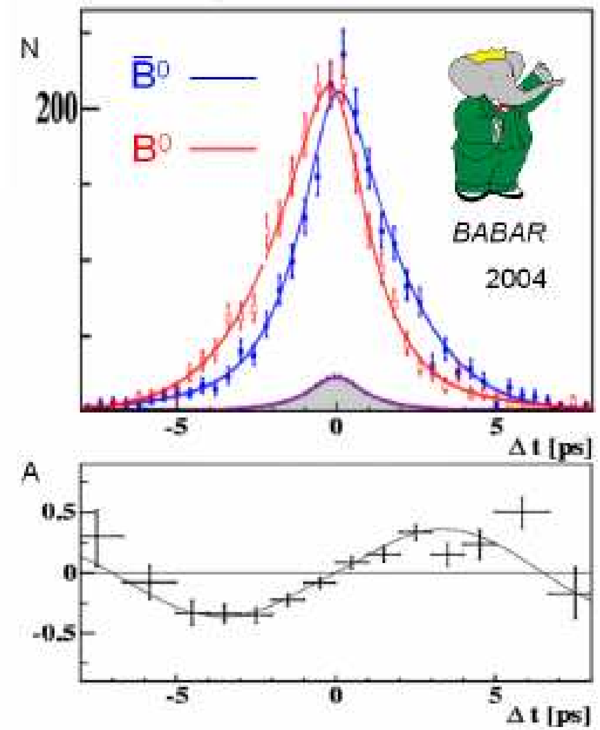


This is how it works at e^+e^- B factories

$B - \overline{B}$ pair produced on Y(4S) resonance with well defined quantum numbers.

$\rightarrow$ Correlated $B - \overline{B}$ state till the time of the decay of the first B.

$B_d \to J/\psi K_s$



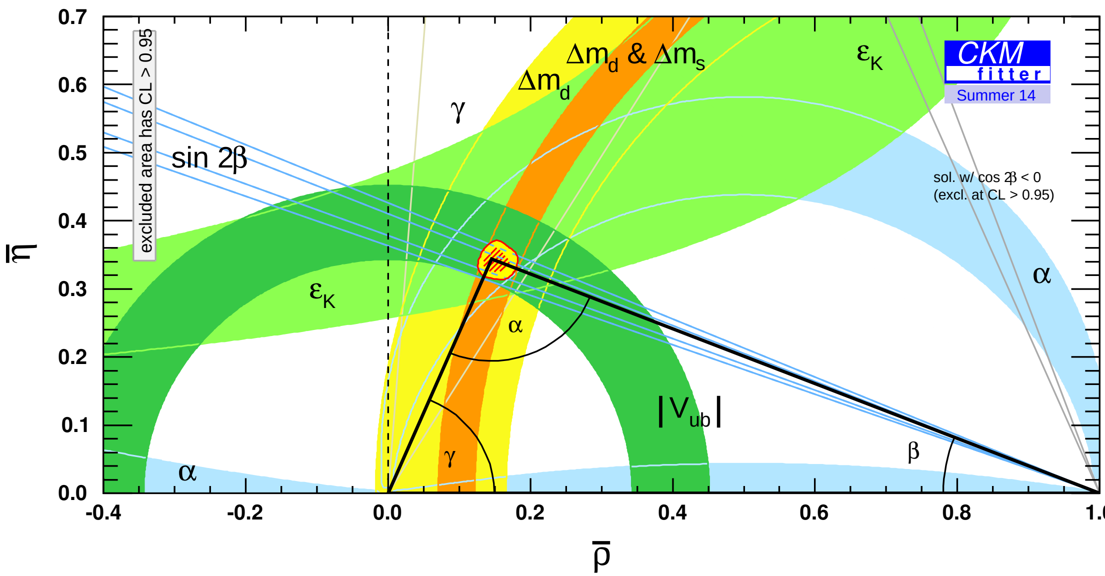
$$\mathcal{A}(t) = \frac{N(B^0)(t) - N(\overline{B^0})(t)}{N(B^0)(t) + N(\overline{B^0})(t)} = -\sin(2\beta)\sin(\Delta m_d t)$$

Babar:

$\sin(2\beta)$ = $0.722 \pm 0.040 \pm 0.023$

Belle:

$\sin(2\beta)$ = $0.652 \pm 0.039 \pm 0.020$



Nobel Prize 2008

Kobayashi & Maskawa:

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

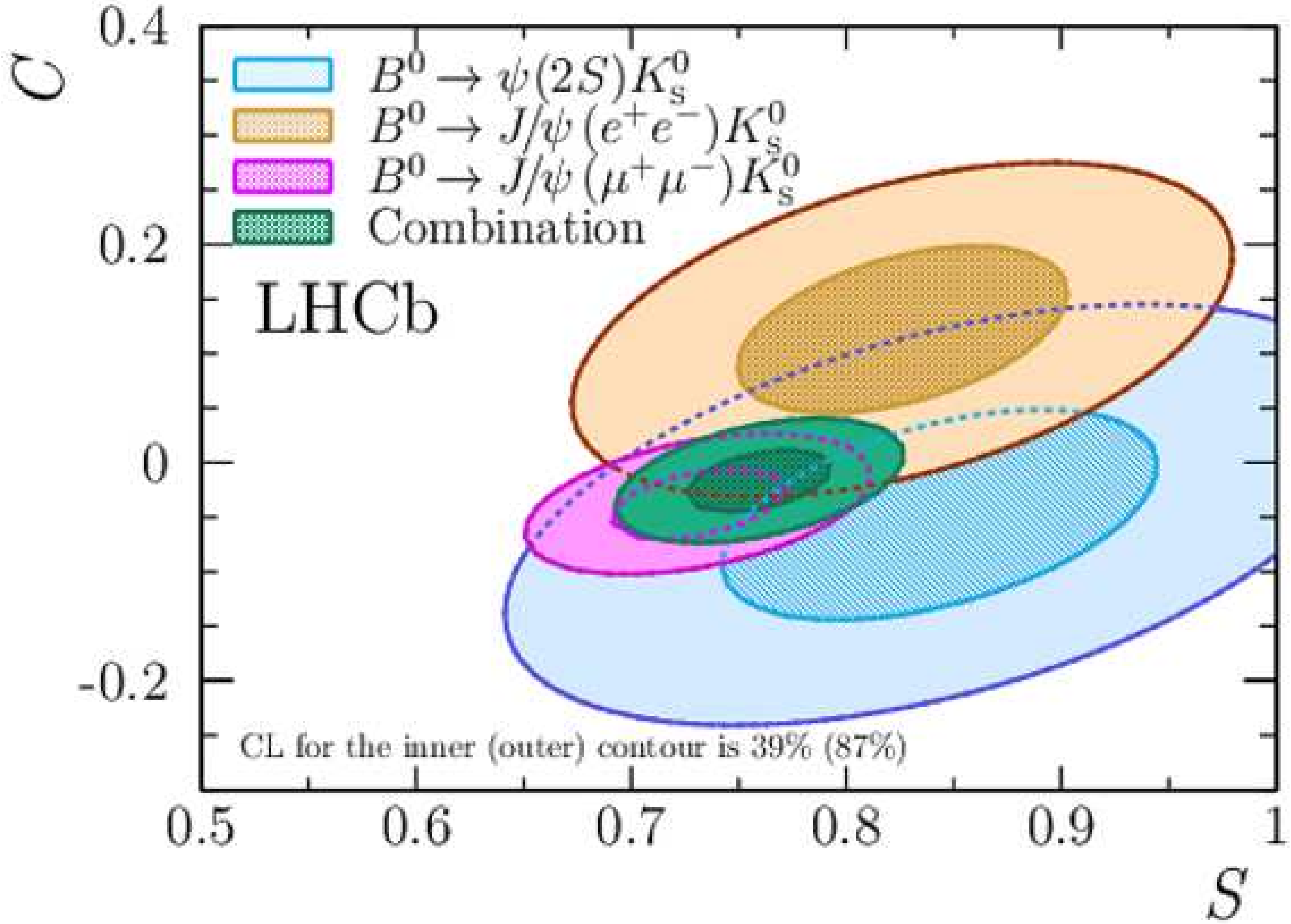
Master Formula for t-dependent CPV

$$\Gamma(B^0 \to f)(t) = |A_f|^2 (1 + |\lambda_f|)^2 \frac{e^{-\Gamma t}}{2} \cdot$$
$$(\cosh(\frac{\Delta\Gamma t}{2}) + D_f \sinh(\frac{\Delta\Gamma t}{2}) + C_f \cos(\Delta m t) - S_f \sin(\Delta m t))$$

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} \quad D_f = \frac{2\mathcal{RE}(A_f)}{1+|\lambda_f|^2} \quad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \quad S_f = \frac{2\mathcal{I}\Updownarrow(A_f)}{1+|\lambda_f|^2}$$

$$\Gamma(\overline{B}^0 \to f)(t) = |A_f|^2 (1 + |\lambda_f|)^2 \frac{e^{-\Gamma t}}{2} \cdot$$
$$(\cosh(\frac{\Delta\Gamma t}{2}) + D_f \sinh(\frac{\Delta\Gamma t}{2}) - C_f \cos(\Delta m t) + S_f \sin(\Delta m t))$$

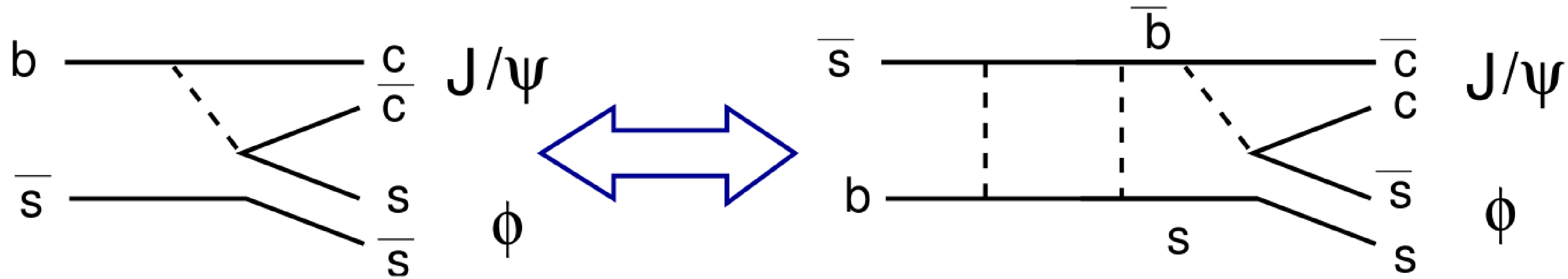
$B^0 \rightarrow J/\Psi K_s$ at LHCb



significant more statistics at LHCb overcompensates the low tagging performance

$B_s \to J/\psi\phi$

Basic idea similar to measurement of $\sin(2\beta)$:

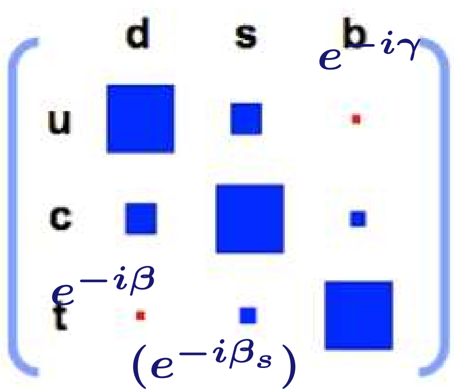


- No CP violation in mixing
- No CP violation in decay (watch out penguin pollution ..)

$$\phi_{mix} = arg((V_{ts}V_{tb}^*)^2) = -2\beta_s \approx 0.04(SM),$$ (top quark dominates the box)

$$\omega = arg((V_{cb}V_{cs}^*)^2) = 0$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$



$B_s \to J/\psi\phi$

B_s : $J^P = 0^{-1}$ (pseudo scalar)

J/ψ: : $J^{CP} = 1^{-1-1}$ (vector)

ϕ: : $J^{CP} = 1^{-1-1}$ (vector)

Angular momentum conservation:

0 = J ($J/\psi\phi$) = $|\vec{S} + \vec{L}|$; $\to$ L = 0,1,2

P($J/\psi\phi$) = P(J/ψ)*P(ϕ)*(-1)^L

CP($J/\psi\phi$) = CP(J/ψ)*CP(ϕ)*(-1)^L

L = 0,2 $\to$ CP even final state

L = 1 $\to$ CP odd final state

Final state no CP eigenstate but linear combination!

Angular analysis, to separate CP even/odd contributions.

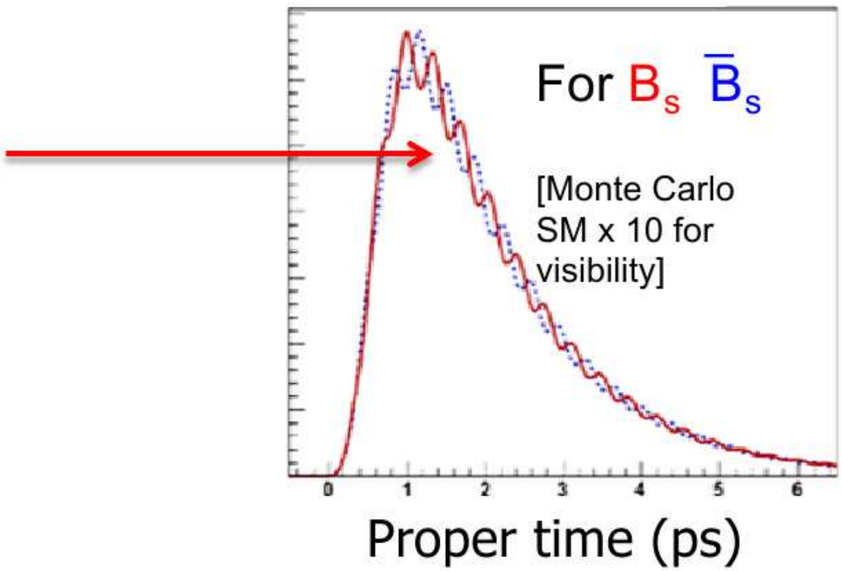
Three decay amplitudes: $|A_\perp|$ (L=1), $|A_{||}|$, $|A_0|$ (L=0,2),

+ two rel. strong phases: $\delta_1 = \arg(A_{||}(0)A_\perp)$, $\delta_2 = \arg(A_0(0)A_\perp(0))$

Measurment of ϕ_s

Measurement of modulation

in decay time distribution



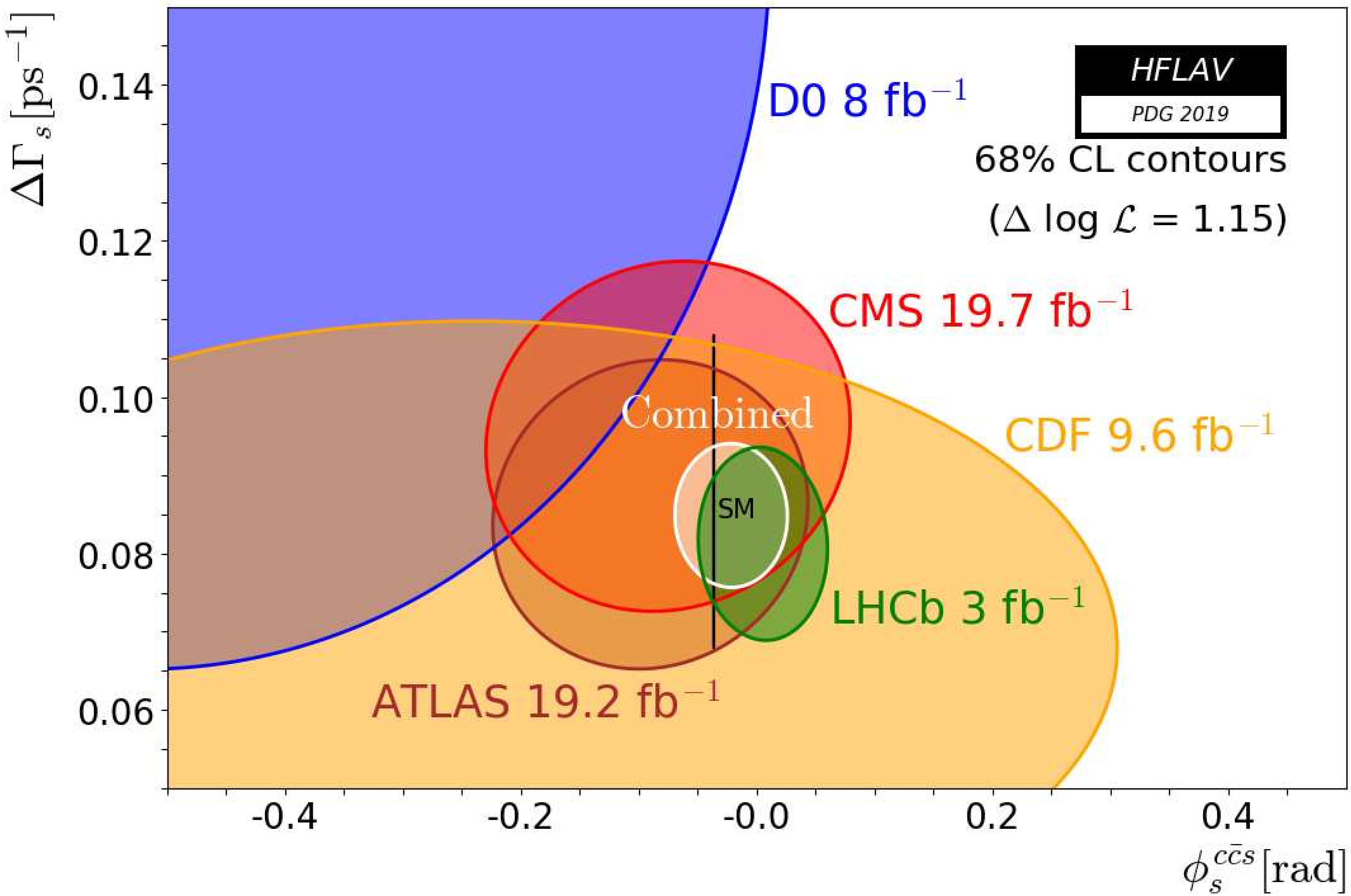
- amplitude of modulation: $D \sin \phi_s$
- sign of modulation depend on production flavour (B_s or $\overline{B_s}$) and from CP value of final state η_{CP}

Most important tools: Flavour-Tagging and decay time resolution

$J/\Psi\phi$ is combination of different CP eigenstates

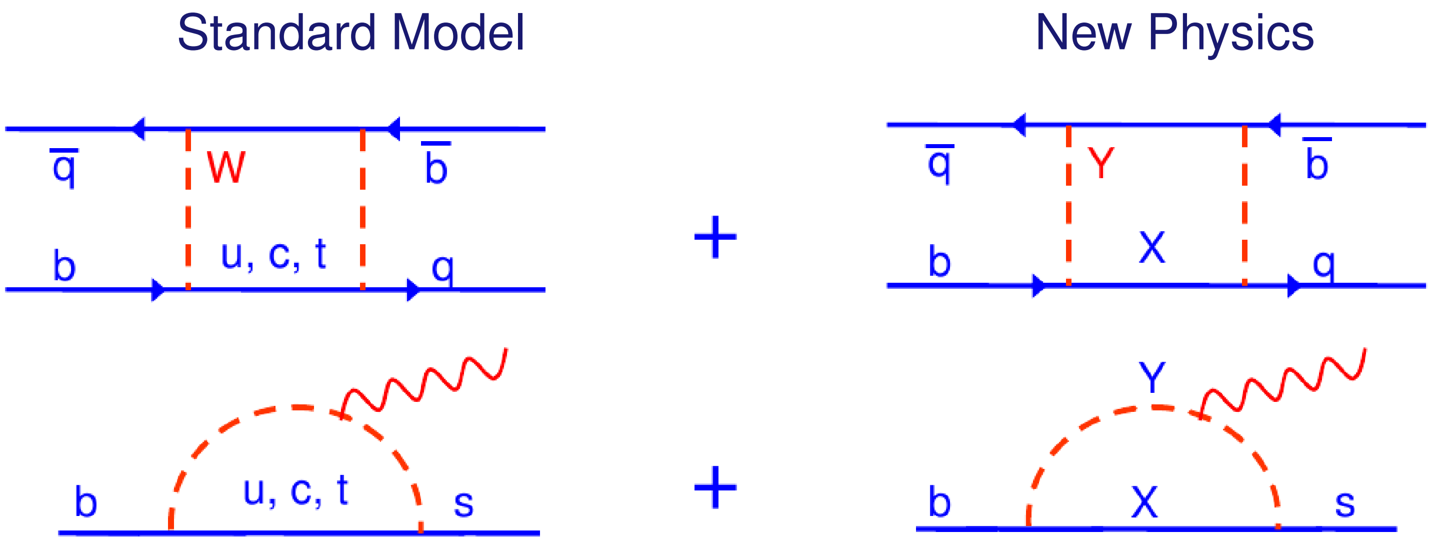
$\rightarrow$ combined measurement of Γ, $\Delta\Gamma$, Δm_s and ϕ_s possible

$B_s^0 \to J/\Psi\phi$



New Physics in B decays

New Physics effects only appear as correction to leading SM terms.



$$\mathcal{A}_{BSM} = \mathcal{A}_0 \left(\frac{C_{SM}}{m_W^2} + \frac{C_{NP}}{\lambda_{NP}^2} \right); \quad (C_{SM} = \frac{g_W^2}{4\pi} \sim \frac{1}{30}, \lambda_{NP} \sim 1 \text{ TeV (?)})$$

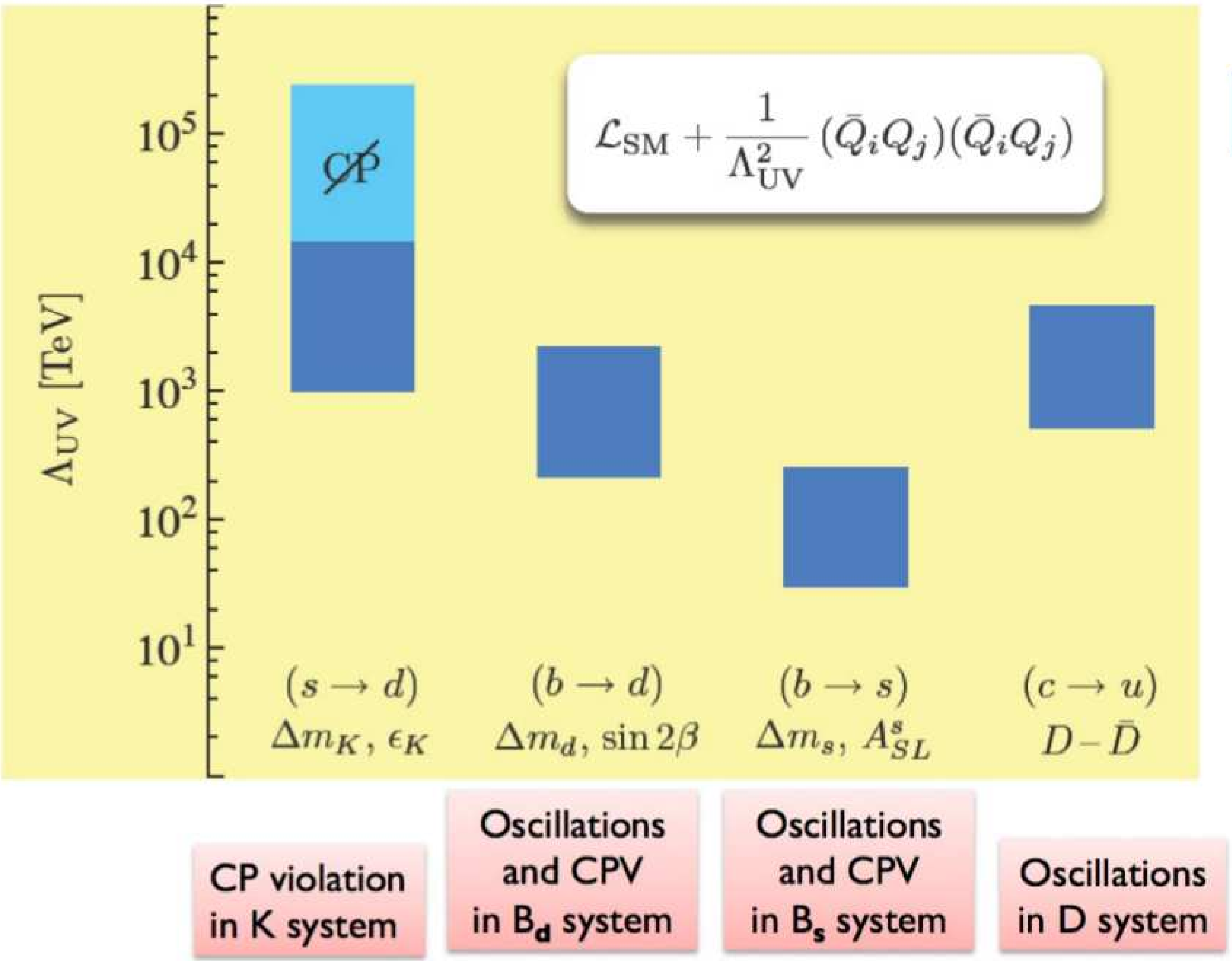
Flavour physics approach to new physics:

- study processes which are sensitive to quantum corrections:
  e.g. very rare (SM suppressed) decays, CPV

To which scales do we exclude new physics contributions via precision measurements in loop diagrams in the kaon, charm and bottom system (assuming couplings of order one)?

New Physics in the Flavour Sector?

If couplings are of order $\mathcal{O}(1)$ ...



arXiv:1302.0661