Semi-Leptonic Theory 2023

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Introduction

Levels of complexity in B decays

- Purely leptonic f_B
- Inclusive semileptonic: Heavy Quark Expansion (HQE)
- Inclusive Nonleptonic (Lifetimes, Mixing): HQE
- Exclusive semileptonic: $F^{B \to M}(q^2)$
- Inclusive FCNC $b \to s\ell\ell$ and $b \to s\gamma$: (HQE + ...)
- Exclusive FCNC $b \to s\ell\ell$ and $b \to s\gamma$: $F^{B \to M}(q^2)$ + ...
- Two-Body Non-leptonic: QCD Factorization (QCD-F)
- Multi-Body Non-Leptonic: To be developed

Make Use of the fact that $\alpha_s(m_b) \ll 1$

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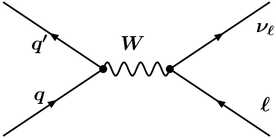
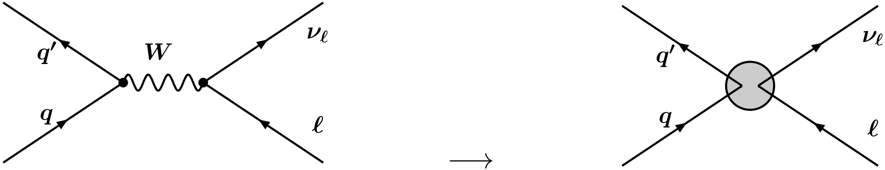
Semi-Leptonic Quark Transitions

- In the Standard Model:
  $q \to q'\ell\bar{\nu}$ or $b \to u\ell\bar{\nu}$ with $q = b$ and $q = c, u$



- The W is much heavier than the b quark: $\langle 0|T[W_\mu(x)W_\nu^*(0)]|0\rangle \sim \frac{1}{M_W^2}\delta^4(x)$



Effective Hamiltonian

It is useful to define the up and down quark fields

$$\mathcal{U}_{L/R} = \begin{bmatrix} u_{L/R} \\ c_{L/R} \\ t_{L/R} \end{bmatrix} \quad \mathcal{D}_{L/R} = \begin{bmatrix} d_{L/R} \\ s_{L/R} \\ b_{L/R} \end{bmatrix}$$

For the semi-leptonic effective Hamiltonian we get

$$H_{\rm eff}^{(sl)} = \frac{4G_F}{\sqrt{2}} \left(\bar{\mathcal{U}}_L \gamma^\kappa V_{CKM} \mathcal{D}_L\right) \left(\bar{e}_L \gamma_\kappa \bar{\nu}_{e,\rm L} + \bar{\mu}_L \gamma_\kappa \bar{\nu}_{\mu,\rm L} + \bar{\tau}_L \gamma_\kappa \bar{\nu}_{\tau,\rm L}\right) + \text{h.c.} \ ,$$

with

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad G_F = \frac{g^2}{2\sqrt{2}M_W^2}$$

For the cases at hand: $(\ell = e, \mu, \tau)$

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{cb} (\bar{c}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \nu_{\ell,L})$$

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{ub} (\bar{u}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \nu_{\ell,L})$$

- This is correct up to term of order m_b^2/M_W^2
- Largest QED corrections are of order $\frac{\alpha}{\pi} \log(M_W^2/m_b^2)$ (Sirlin)
- No QCD corrections of order $\frac{\alpha_s}{\pi} \log(M_W^2/m_b^2)$
- ... unlike in non-leptonic decays.

Exclusive Decays

Matrix Elements and Form Factors

- Leptonic part is as in the textbook
- Hadronic Matrix Elements: In general parametrized by scalar functions ("Form Factors") of $q^2 = (p_B - p')^2$
- For a pseudoscalar final state $P(p_P)$ $(p' = b_P)$

$$\langle P(p_P)|\bar{q}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_P^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu\right) + f_0(q^2)\,\frac{m_B^2 - m_P^2}{q^2} q^\mu$$

$$\langle P(p_P)|\bar{q}\gamma^\mu \gamma_5 b|B(p_B)\rangle = 0$$

- Vector final state $V(p_V, \epsilon)$ with polarization ϵ $(p' = p_V)$

$$\langle V(p_V, \epsilon)|\bar{q}\gamma^\mu b|B(p_B)\rangle = V(q^2)\, \varepsilon^{\mu\sigma}{}_{\nu\rho}\epsilon^*_\sigma \frac{2p_B^\nu p_V^\rho}{m_B + m_V},$$

$$\langle V(p_V, \epsilon)|\bar{q}\gamma^\mu\gamma^5 b|B(p_B)\rangle = i\epsilon^*_\nu \left[A_0(q^2)\, \frac{2m_V q^\mu q^\nu}{q^2} \right.$$

$$\left. + A_1(q^2)\,(m_B + m_V)\eta^{\mu\nu} - A_2(q^2)\, \frac{(p_B + p_V)_\sigma q^\nu}{m_B + m_V}\eta^{\mu\sigma}\right],$$

with $\epsilon p_V = 0$, and $\eta^{\mu\nu} = g^{\mu\nu} - \dfrac{q^\mu q^\nu}{q^2}$

- For massless leptons, f_0 and A_0 don't contribute

$B \to \pi \ell \bar{\nu}$

Straightforward calculation

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ \ell^- \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |\vec{p}_\pi|^3 |f_{B\pi}^+(q^2)|^2$$

$$|\vec{p}_\pi| = \frac{1}{2M_B} \sqrt{[(M_B^2 - (q - m\pi)^2][(M_B^2 - (q + m\pi)^2]}, \quad q = \sqrt{q^2}$$

What do we know about the form factor?

- Lattice QCD: $q^2 \sim (M_B - m_\pi)^2$
- QCD Sum rules estimates: $q^2 \sim 0$
- Interpolation between these regions.
- Form Factor Bounds from Analyticity and Unitarity

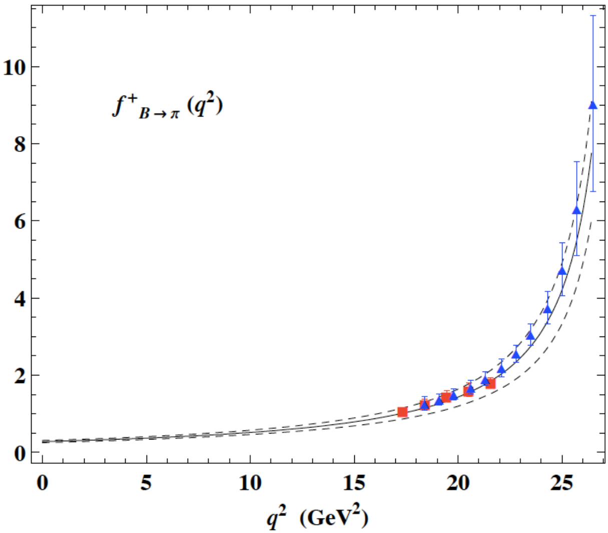
Form Factor Parametrizations

Aside form models, one uses the z parametrization
(Bourrely, Caprini Lellouch)

$$z(q^2, t_0) = \frac{\sqrt{(M_B + m_\pi)^2 - q^2} - \sqrt{(M_B + m_\pi)^2 - t_0}}{\sqrt{(M_B + m_\pi)^2 - q^2} + \sqrt{(M_B + m_\pi)^2 - t_0}}$$

$$f_{B\pi}^+(q^2) = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{k=0}^{K} b_k(t_0)(z(q^2, t_0))^k$$

very few terms (~ 2) in this expansion are sufficient for the interpolation


$f^+_{B\to\pi}(q^2)$
0
2
4
6
8
10
0
5
10
15
20
25
q^2 (GeV^2)

$B \to \rho \ell \bar{\nu}$

Only a few remarks:

- Can be computed in terms of the form factors V, A_1, A_2
- Not much is known about these form factors: Models
- Lattice as well as QCD SR fail,
  since the ρ is not a stable particle
- Study instead $B \to \pi \pi \ell \bar{\nu}$ (Faller et al.)

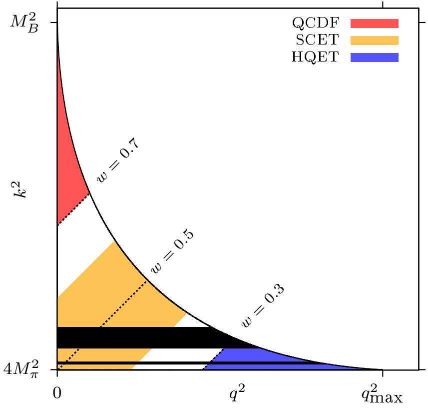
Decomposition into Form Factors:

$$
\begin{aligned}
\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu b|B^-(p)\rangle &= iF_\perp \frac{1}{\sqrt{k^2}}\, \bar{q}^\mu_{(\perp)} \\
\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu \gamma_5 b|B^-(p)\rangle &= -F_t \frac{q^\mu}{\sqrt{q^2}} \\
&+ F_0 \frac{2\sqrt{q^2}}{\sqrt{\lambda}}\, k^\mu_{(0)} + F_{||} \frac{1}{\sqrt{k^2}}\, \bar{k}^\mu_{(||)} \,.
\end{aligned}
$$

Form factors depend on the variables

$$
k = k_1 + k_2, \quad \bar{k} = k_1 - k_2 : \quad q^2,\ k^2,\ q\cdot\bar{k}
$$

$$w = (v \cdot k)/M_B$$



... work in progress, but not much progress over the last years.

$B \to D \ell \bar{\nu}$ and $B \to D^* \ell \bar{\nu}$

- Useful Limit: Heavy Quark Limit = $m_b,\ m_c \to \infty$ with fixed (four)velocity v
- In this limit we have $m_{Hadron} = m_Q$ and $p_{Hadron} = p_Q$, $v = p_{Hadron}/m_{Hadron}$
- For $m_Q \to \infty$ no recoil from the light quarks and gluons
  This is like the H-atom in Quantum Mechanics I!

Heavy Quark Symmetries for $m_Q \to \infty$

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \to 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$ Heavy Flavour Symmetry b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons: $H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B}) Q \quad \stackrel{m_Q \to \infty}{\longrightarrow} \quad 0$
  - Spin Rotations become a symmetry
  - Heavy Quark Spin Symmetry: SU(2) Rotations

- HQS imply a single form factor for Heavy→Heavy transitions:

$$\langle H^{(*)}(v)|Q_v \Gamma Q_{v'}|H^{(*)}(v')\rangle = C_\Gamma(v, v')\xi(v \cdot v')$$

  - $C_\Gamma(v, v')$: Computable Coefficient
  - $\xi(v \cdot v')$: universal non-perturbative Form Faktor, Isgur Wise Funktion
  - Normalization from HQS: $\xi(v \cdot v' = 1) = 1$

Express the form factors in terms of velocities:

$$\frac{\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_D}} = h_+(w)\,(v + v')^\mu + h_-(w)\,(v - v')^\mu, \; h_+(w) = \xi(w), \; h_-(w) = 0$$

$$\frac{\langle D^*(v', \epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = h_V(w)\,\varepsilon^{\mu\nu\rho\sigma} v_\nu v'_\rho \epsilon^*_\sigma, \; h_V(w) = \xi(w)$$

$$\frac{\langle D^*(v', \epsilon)|\bar{c}\gamma^\mu \gamma^5 b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = i h_{A_1}(w)\,(1 + w)\epsilon^{*\mu} - i\left[h_{A_2}(w) v^\mu + h_{A_3}(w) v'^\mu\right](\epsilon^* \cdot v)$$

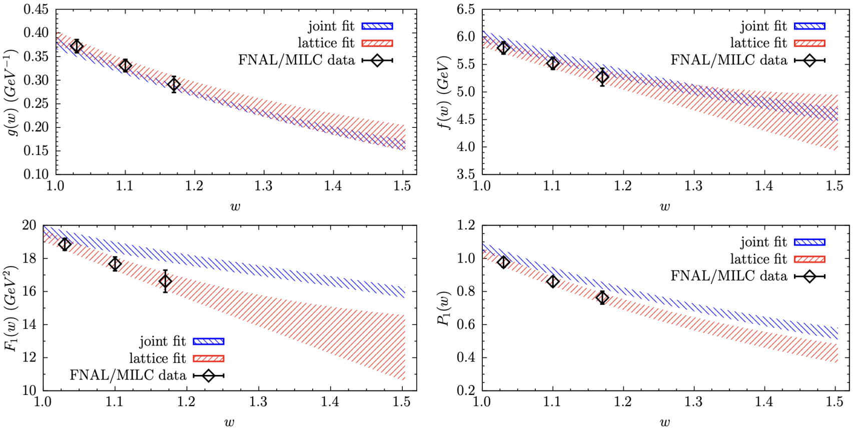
with $w = (v \cdot v')$ and $h_{A_1}(w) = h_{A_3}(w) = \xi(w)$, $h_{A_2}(w) = 0$

Lattice Calculations for the Form Factors

- Lattice calculations made enormous progress
- Heavy Quark Limit and HQS became less important
- Various Lattice calculations available: finite masses and large portions of phase space
- Combination of methods: Light Cone Sum Rules $\otimes$ Lattice Calculations
- Interpolation via the z expansion:
  - BGL: model independent parametrization based on analyticity
  - CLN: uses heavy quark limit and $1/m_b$ corrections

important news: LQCD form factors for $B \to D^* \ell \nu_\ell$ decays from **FNAL/MILC (arXiv:2105.14019)**

synthetic data points at 3 non-zero values of the recoil (w - 1)



joint fit:
BGL fit of LQCD points + Belle + BaBar exp. data

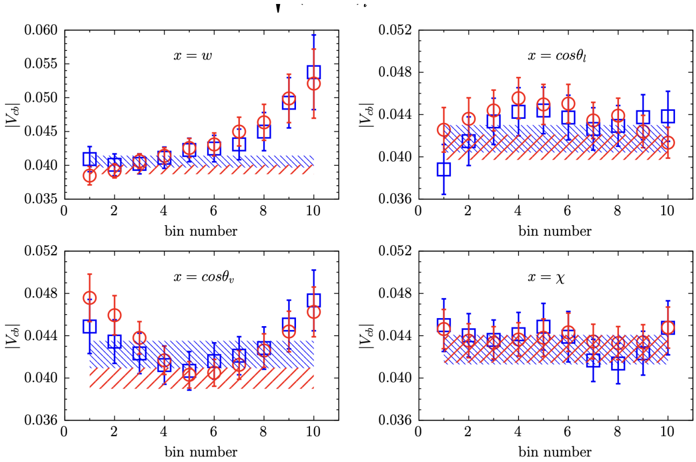
$|V_{cb}| \cdot 10^3 = 38.40 \pm 0.74$

$R(D^*) = 0.2483 \pm 0.0013$

lattice fit:
quadratic BGL fit of LQCD points only

$|V_{cb}| > |V_{cb}|^{\text{joint fit}}$?

$R(D^*) = 0.265 \pm 0.013$



- 10 bins for each variable
- total of 80 data points

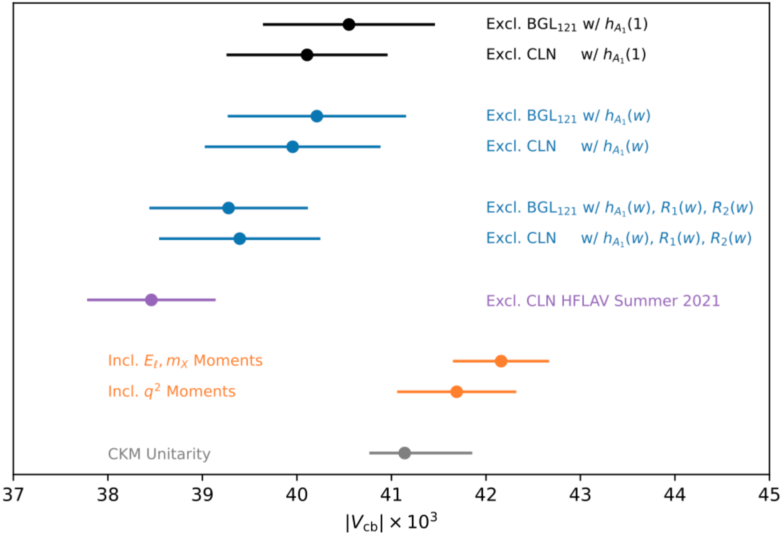
blue data: Belle 1702.01521

red data: Belle 1809.03290

bands are (correlated) weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} \,,$$

$$\sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} \,,$$


Excl. BGL_{121} w/ $h_{A_1}(1)$
Excl. CLN w/ $h_{A_1}(1)$
Excl. BGL_{121} w/ $h_{A_1}(w)$
Excl. CLN w/ $h_{A_1}(w)$
Excl. BGL_{121} w/ $h_{A_1}(w)$, $R_1(w)$, $R_2(w)$
Excl. CLN w/ $h_{A_1}(w)$, $R_1(w)$, $R_2(w)$
Excl. CLN HFLAV Summer 2021
Incl. E_ℓ, m_X Moments
Incl. q^2 Moments
CKM Unitarity
37
38
39
40
41
42
43
44
45
$|V_{\mathrm{cb}}| \times 10^3$

- HQE treatment including $1/m_c^2$ corrections (Bordone, van Dyk, Jung)

$$|V_{cb}| = (40.3 \pm 0.8) \times 10^{-3}$$

- Lattice (from Simulas talk at Barolo 2022)

decay	$\lvert V_{cb}\rvert^{\mathrm{DM}} \cdot 10^3$	inclusive [2107.00604]	exclusive [FLAG 21]
$B \to D$	41.0 ± 1.2		
$B \to D^*$	41.3 ± 1.7		
$B_s \to D_s$	42.4 ± 2.0		
$B_s \to D_s^*$	41.4 ± 2.6		
average	41.4 ± 0.8	42.16 ± 0.50	39.36 ± 0.68
difference		$\simeq 0.8\,\sigma$	$\simeq 1.9\,\sigma$

$B \to D^{**}\ell\bar{\nu}$ = Orbitally excited states

- $B \to D$ and $B \to D^*$ exhaust about 75% of the inclusive $b \to c$ rate
- Aside from non-resonant $B \to D\pi$:
  Decays into D^{**} states
- ... mainly the orbitally excited states

Make use of Heavy Quark Symmetry:

- Spin Symmetry Doublets of orbitally excited states, labelled by the total *j* of the light degrees of freedom:

$$\begin{pmatrix} |D(0^+)\rangle \\ |D(1^+)\rangle \end{pmatrix} \quad j = 1/2 \qquad \text{and} \qquad \begin{pmatrix} |D^*(1^+)\rangle \\ |D^*(2^+)\rangle \end{pmatrix} \quad j = 3/2$$

- Masses in the $m_c \to \infty$ limit:

$$M(D(0^+)) = M(D(1^+)) = m_c + \bar{\Lambda}_{1/2}$$
$$M(D^*(1^+)) = M(D^*(2^+)) = m_c + \bar{\Lambda}_{3/2}$$

- $\bar{\Lambda}_{3/2} - \bar{\Lambda}_{1/2}$ does not scale with m_c!
- Each Doublet as a new Isgur Wise Function: $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$

$B \to D^{**} \ell \bar{\nu}$

Channel	GI	VD	CCCN	ISGW
$m_c \to \infty$				
$\mathcal{B}(B^- \to D(0^+)\ell\bar{\nu})$	$4.7 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-5}$	$1.0 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D(1^+)\ell\bar{\nu})$	$6.4 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$	$4.9 \cdot 10^{-5}$	$1.4 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^*(1^+)\ell\bar{\nu})$	$4.4 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$	$4.0 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^*(2^+)\ell\bar{\nu})$	$7.4 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	$8.0 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^{**}\ell\bar{\nu})$	1.3%	0.82%	1.1%	1.5%
$\frac{\mathcal{B}(B^- \to D^*(1^+)\ell\bar{\nu})}{\mathcal{B}(B^- \to D(1^+)\ell\bar{\nu})}$	6.9	11	80	3.4
m_c finite				
$\mathcal{B}(B^- \to D_L\ell\bar{\nu})$	$3.0 \cdot 10^{-3}$	$2.1 \cdot 10^{-3}$	$3.0 \cdot 10^{-3}$	$3.2 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D_H\ell\bar{\nu})$	$2.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$
$\frac{\mathcal{B}(B^- \to D_L\ell\bar{\nu})}{\mathcal{B}(B^- \to D_H\ell\bar{\nu})}$	1.3	1.6	2.3	1.0

(R. Klein et al.)

- GI: Godfrey, Isgur (1985);
- VD: Veseli, Dunietz (1996);
- CCCN: Cea, Colangelo, Cosmai, Nardulli (1991);
- ISGW: Isgur, Scora, Grinstein, Wise (1989)

$$\frac{\mathcal{B}(B^- \to D^*(1^+)\ell\nu)}{\mathcal{B}(B^- \to D(1^+)\ell\nu)} \approx 2.2$$

Inclusive Decays

Heavy Quark Expansion

Heavy Quark Expansion = Operator Product Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$
\begin{aligned}
\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|\mathcal{H}_{eff}|B(v)\rangle|^2 \\
&= \int d^4x \, \langle B(v)|\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)|B(v)\rangle \\
&= 2\,\mathrm{Im} \int d^4x \, \langle B(v)|T\{\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)\}|B(v)\rangle \\
&= 2\,\mathrm{Im} \int d^4x \, e^{-im_b v\cdot x} \langle B(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|B(v)\rangle
\end{aligned}
$$

- Last step: $b(x) = b_v(x)\exp(-im_v vx)$,
  corresponding to $p_b = m_b v + k$
  Expansion in the residual momentum k

- Perform an “OPE”: m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\to$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\to$ Perturbation theory!
- Works also for differential rates!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$2M_H\mu_\pi^2 = -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle$$
$$2M_H\mu_G^2 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$2M_H\rho_D^3 = -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle$$
$$2M_H\rho_{LS}^3 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle$$

ρ_D: Darwin Term and ρ_{LS}: Spin-Orbit Term

- Γ_4 and Γ_5 have been computed Bigi, Uraltsev, Turczyk, TM, ...

Structure of the HQE

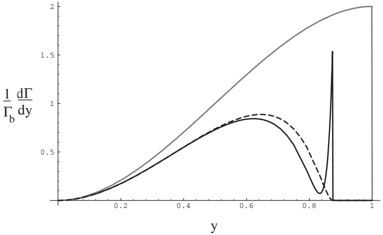
- Structure of the expansion (@ tree):

$$d\Gamma = d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4$$
$$+ d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right)$$
$$+ \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4$$

- $d\Gamma_3 \propto \ln(m_c^2/m_b^2)$
- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Determination of the HQE Parameters

- m_b, m_c, μ_π, μ_G, ρ_D etc. are determined from data
- Spectra: Hadronic invariant mass, Charegd lepton energy, Hadronic Energy
- However: There are corners in Phase Space where the OPE breaks down



Moments of the spectra can be computed in the HQE

m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{G}$$

Alberti, Healey, Nandi, Gambino arXiv 1411.6560

- Includes HQE parameters up to $1/m^3$ and full α_s/m_Q^2

QCD Corrections

For a massless final-state quark:

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_b^5 \left(1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k g_k\right) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_b^5 \left(1 + \frac{\alpha_s}{\pi} g_1 + \cdots\right)$$

What is the mass m_b?

- Start with the pole mass $m_b = m_b^{\mathrm{pole}}$
- This yields a large g_1
- In fact, this leads in general to a bad behavior of the perturbative series
- Perturbative series is "asymptotic": Looks like a convergent series, but at some order k

$$g_k \sim k!$$

Renormalon Problem (of the Pole mass)

- Problem for a precision calculation!
- Switch to a "proper mass" $m_b^{\rm kin}$:
  This has a perturbative relation to the pole mass

$$m_b^{\rm kin}(\mu) = m_b^{\rm pole}\left(1 + \sum_{k=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^k m_k(\mu)\right) = m_b^{\rm pole}\left(1 + \frac{\alpha_s}{\pi} m_1(\mu) + \cdots\right)$$

- $m_b^{\rm kin}$ is renormalon-free: The relation between the masses is asymptotic:
  at some order k

$$m_k(\mu) \sim k!$$

- Insert this

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (m_b^{\rm kin}(\mu))^5 \left(1 + \frac{\alpha_s}{\pi}(g_1 - m_1(\mu)) + \cdots\right)$$

- $m_b^{\rm kin}$ is much better known as the pole mass
- The perturbative series converges better: $|g_1 - m_1| \ll g_1$
- In general: Renormalons cancel: Better behaviour of the perturbative series.

$b \to c$ also depends on the charm mass

- The rate and the moments depend on

$$m_b^{\rm kin}(1\,{\rm GeV}) - a\,m_c \quad \text{with} \quad a \sim 0,7 - 0.8$$

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
  Bigi, Zwicky, Uraltsev, Turczyk, Vos, Milutin, ThM, ...
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate and spectra are known
  Melnikov, Czarnecki, Pak
- $\mathcal{O}(\alpha_s^3)$ to the partonic rate known (Fael, Schonwald, Steinhauser: 2011.13654)
- $\mathcal{O}(\alpha_s)$ for $1/m_b^2$ is known for rates and spectra
  Becher, Boos, Lunghi, Gambino, Pivovarov, Rosenthal, Alberti
- $\mathcal{O}(\alpha_s)$ for $1/m_b^3$ is known for rates and spectra Pivovarov, Moreno, ThM
- In the pipeline:
  - Partial Resummations
  - Estimation of Duality Violation

We are moving towards a TH-uncertainty of 1% in $V_{cb,incl}$!

Recent Development: Reducing the Number of HQE Parameters

New Idea based on an old observation: Reparametrization Invariance

ThM, Vos: 1802.09409, Fael, ThM, Vos: 1812.07472

Problem: Number of HQE parameters in higher orders!

- 4 up to $1/m^3$ 13 up to $1/m^4$ (tree level) 31 up to order $1/m^5$ (tree level)
- Factorial Proliferation

Reparametrization Invariance: (Dugan, Golden, Grinstein, Chen, Luke, Manohar...)

$$R(q) = \int d^4x\, e^{iqx}\, T[\bar{Q}(x)\Gamma q(x)\ \bar{q}(0)\Gamma^\dagger Q(0)]$$

and replace $Q(x) = \exp(-im(v \cdot x))Q_v(x)$

$$R(S) = \int d^4x\, e^{-iSx}\, T[\bar{Q}_v(x)\Gamma q(x)\ \bar{q}(0)\Gamma^\dagger Q_v(0)]$$

with $S = mv - q$.

These expressions are independent of v!

Perform the OPE $\longrightarrow$ HQE

$$R(S) = \sum_{n=0}^{\infty} \left[C^{(n)}_{\mu_1 \cdots \mu_n}(S) \right]_{\alpha\beta} \bar{Q}_{v,\alpha}(iD_{\mu_1} \cdots iD_{\mu_n}) Q_{v,\beta}$$
$$= \sum_{n=0}^{\infty} C^{(n)}_{\mu_1 \cdots \mu_n}(S) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

All this is still invariant under reparametrization of v: (as long as the sum is not truncated)

$$\delta_{\mathrm{RP}}\, v_\mu = \delta v_\mu \quad \text{with} \quad v \cdot \delta v = 0$$
$$\delta_{\mathrm{RP}}\, iD_\mu = -m \delta v_\mu$$
$$\delta_{\mathrm{RP}}\, Q_v(x) = im(x \cdot \delta v) Q_v(x) \quad \text{in particular} \quad \delta_{\mathrm{RP}}\, Q_v(0) = 0 \, .$$

The RP connects different orders in $1/m$, which yields the master relation between the coefficients $n = 0, 1, 2, \ldots$

$$\delta_{\rm RP} C^{(n)}_{\mu_1 \cdots \mu_n} = m \, \delta v^\alpha \left(C^{(n+1)}_{\alpha \mu_1 \cdots \mu_n} + C^{(n+1)}_{\mu_1 \alpha \mu_2 \cdots \mu_n} + \cdots + C^{(n+1)}_{\mu_1 \cdots \mu_n \alpha} \right)$$

Use these coefficients, integrate over phase space,
get a total rate $\Gamma = \mathrm{Im} \langle B | R | B \rangle = \mathrm{Im} \langle R \rangle$
The coeffcients of the OPE will depend only on v

$$R = \sum_{n=0}^{\infty} c^{(n)}_{\mu_1 \cdots \mu_n}(v) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

and satisfy the master relation between different orders in the HQE

Making use of RPI ...

- RPI is a consequence of Lorentz invariance of QCD
- RPI is an exact symmetry:
  the relations must hold to all order in α_s
- Resummation of towers of terms from different orders
- For Lorentz invariant observables:
  - The master relations are identical for all observables
  - “Rigid” relations between coefficients
  - Reduction of HQE parameters due to RPI

How does this happen? A Toy example without gluons

Look at the partonic result for the rate

$$
\begin{aligned}
R(p) &= R(p^2) = R((mv+k)^2) = R(m^2 + 2m(vk) + k^2) \\
&= R(m^2) + R'(m^2)(2m(vk) + k^2) + \frac{1}{2}R''(m^2)(2m(vk)+k^2)^2 \\
&= R(m^2)
\end{aligned}
$$

if there are no gluons: Equation of motion:

$$(2m(vk) + k^2) \to 2m(iv\partial) + (i\partial)^2 \to 0$$

All the fully symmetrized (zero gluon) contributions are contained in the partonic result

HQE parameters (for the total rate) to $O(1/m^4)$

$$
\begin{aligned}
2m_H\mu_3 &= \langle H(p)|\bar{Q}_v Q_v|H(p)\rangle = \langle \bar{Q}_v Q_v\rangle \\
2m_H\mu_G &= \langle \bar{Q}_v (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu}) Q_v\rangle \\
2m_H\rho_D &= \langle \bar{Q}_v \left[(iD^\mu)\,,\left[\left((ivD) + \frac{(iD)^2}{2m}\right)\,,(iD_\mu)\right]\right] Q_v\rangle \\
2m_H r_G^4 &= \langle \bar{Q}_v\, [(iD_\mu)\,,(iD_\nu)]\,[(iD^\mu)\,,(iD^\nu)]\, Q_v\rangle \\
2m_H r_E^4 &= \langle \bar{Q}_v\, [(ivD)\,,(iD_\mu)]\,[(ivD)\,,(iD^\mu)]\, Q_v\rangle \\
2m_H s_B^4 &= \langle \bar{Q}_v\, [(iD_\mu)\,,(iD_\alpha)]\,[(iD^\mu)\,,(iD_\beta)]\,(-i\sigma^{\alpha\beta}) Q_v\rangle \\
2m_H s_E^4 &= \langle \bar{Q}_v\, [(ivD)\,,(iD_\alpha)]\,[(ivD)\,,(iD_\beta)]\,(-i\sigma^{\alpha\beta}) Q_v\rangle \\
2m_H s_{qB}^4 &= \langle \bar{Q}_v\, [iD_\mu\,,[iD^\mu\,,[iD_\alpha\,,iD_\beta]]]\,(-i\sigma^{\alpha\beta}) Q_v\rangle
\end{aligned}
$$

- Less that had been identified before
- Depend on the quark mass
- Are expressed NOT in terms of $iD_\perp$, rather "full" derivatives
- Can be expressed in terms of full QCD operators via

$$ivD\, Q_v \to \left(ivD + \frac{(iD)^2}{m} \right) Q_v = \frac{1}{2m}((iD)^2 - m^2)Q$$

Thus

$$\begin{aligned} 2m_H \mu_3 &= \langle \bar{Q} Q \rangle \\ 2m_H \mu_G &= \langle \bar{Q} (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu}) Q \rangle \\ 2m_H \rho_D &= \frac{1}{2m} \langle \bar{Q} \left[(iD^\mu) \, , \, \left[(iD)^2 \, , \, (iD_\mu) \right] \right] Q \rangle \\ &\cdots \end{aligned}$$

Alternative V_{cb} Determination

The leptonic invariant mass is RPI: and so are

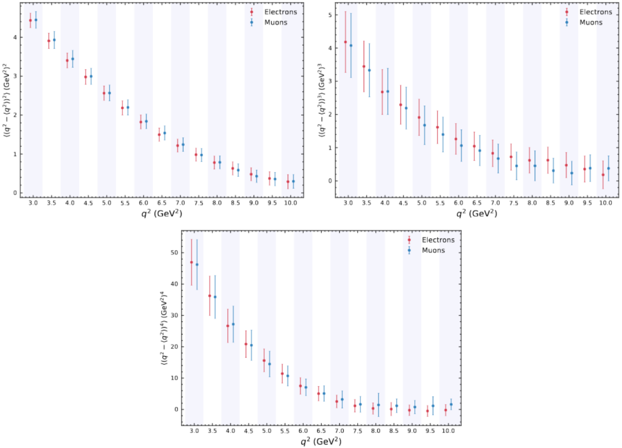
$$\frac{1}{\Gamma_0}\int d\hat{q}^2 (\hat{q}^2)^n \frac{d\Gamma}{d\hat{q}^2} \quad \text{and} \quad \frac{1}{\Gamma_0}\int_{\hat{q}^2_{\rm cut}} d\hat{q}^2 (\hat{q}^2)^n \frac{d\Gamma}{d\hat{q}^2}$$

$$\mathcal{Q}_1 = \frac{3}{10}\mu_3 - \frac{7}{5}\frac{\mu_G^2}{m_b^2} + \frac{\tilde{\rho}_D^3}{m_b^3}(19 + 8\log\rho) - \frac{r_E^4}{m_b^4}\left(\frac{1292}{45} + \frac{40}{3}\log\rho\right) - \frac{s_B^4}{m_b^4}(8 + 2\log\rho) + \frac{13}{120}\frac{s_{qB}^4}{m_b^4} + \frac{s_E^4}{m_b^4}\left(\frac{63}{5} + 4\log\rho\right) + \frac{r_G^4}{m_b^4}\left(\frac{827}{45} + \frac{22}{3}\log\rho\right), \tag{4.10}$$

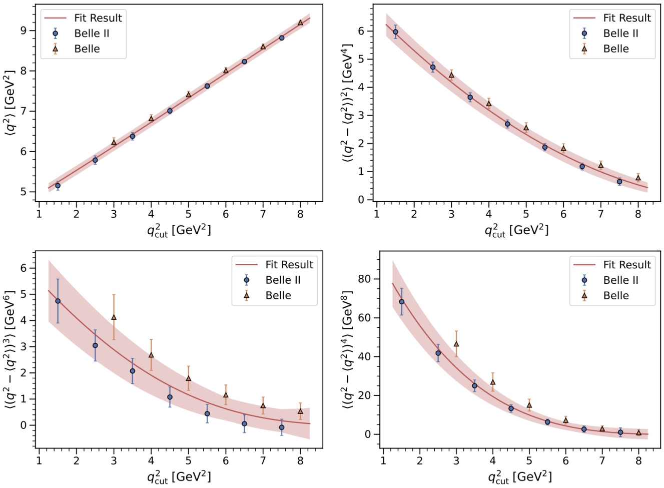
$$\mathcal{Q}_2 = \frac{2}{15}\mu_3 - \frac{16}{15}\frac{\mu_G^2}{m_b^2} + \frac{\tilde{\rho}_D^3}{m_b^3}\left(\frac{358}{15} + 8\log\rho\right) - \frac{r_E^4}{m_b^4}\left(\frac{2888}{45} + \frac{64}{3}\log\rho\right) - \frac{s_B^4}{m_b^4}\left(\frac{259}{15} + 4\log\rho\right) + \frac{s_{qB}^4}{m_b^4}\left(\frac{251}{180} + \frac{1}{3}\log\rho\right) + \frac{s_E^4}{m_b^4}\left(\frac{908}{45} + \frac{16}{3}\log\rho\right) + \frac{r_G^4}{m_b^4}\left(\frac{1373}{45} + \frac{28}{3}\log\rho\right), \tag{4.11}$$

Data on q^2 Moments I

Belle Collaboration [2109.01685, 2105.08001]



Data on q^2 Moments II



2205.10274 (Bernlochner et al.)

$\Longrightarrow$ New V_{cb} Determination

$$R^*(q^2_{\rm cut}) \qquad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_3, \mu_G^2, \tilde\rho_D^3, r_E^4, r_G^4, s_E^4, s_B^4, s_{qB}^4, m_b, m_c$$

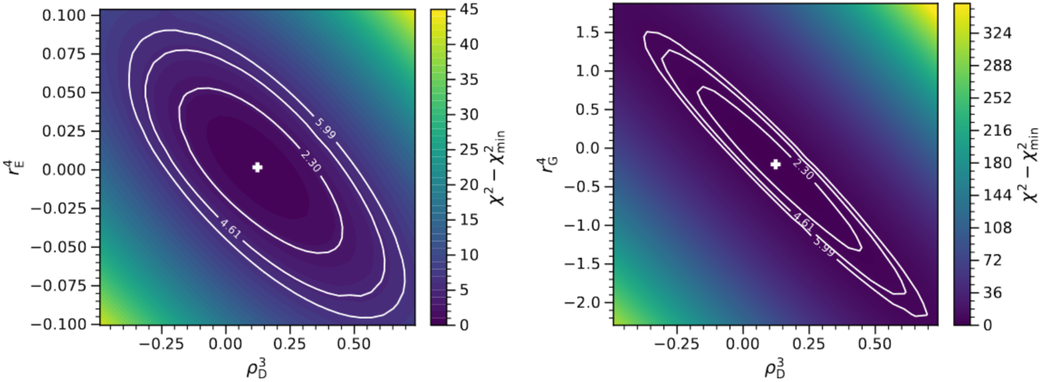
$$\downarrow$$

$$\mathrm{Br}(\bar B \to X_c \ell \bar\nu) \propto \frac{|V_{cb}|^2}{\tau_B}\Big[\Gamma_{\mu_3}\mu_3 + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde\rho_D}\frac{\tilde\rho_D^3}{m_b^3} + \Gamma_{r_E}\frac{r_E^4}{m_b^4} + \Gamma_{r_G}\frac{r_G^4}{m_b^4} + \Gamma_{s_B}\frac{s_B^4}{m_b^4} + \Gamma_{s_E}\frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}}\frac{s_{qB}^4}{m_b^4}\Big]$$

$$\downarrow$$

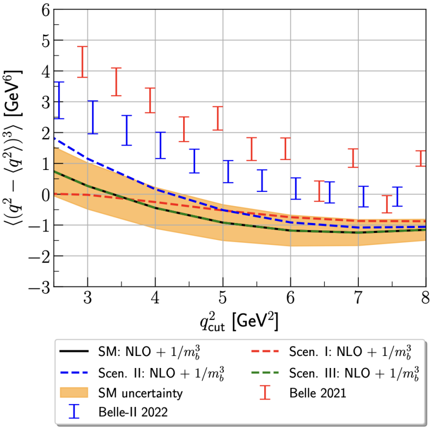
$$V_{cb} = (41.69 \pm 0.63) \cdot 10^{-3}$$

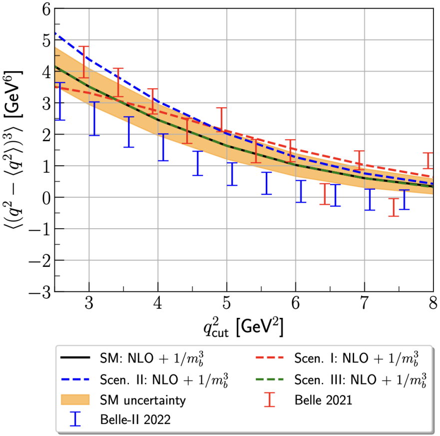
- Agrees with previous determinations
- It includes a data driven determination of the $1/m^4$ HQE Parameters
- $1/m^4$ turns our to be small $\to$ good fot the HQE



Interesting side remark: The value of ρ_D:

- Gambino et al.: $\rho_D = (0.185 \pm 0.031) GeV^3$ (kinetic scheme)
- Bernlochner et al. $\rho_D = (0.03 \pm 0.02) GeV^3$ (kinetic scheme)





Modified Heavy Quark Expansion: $B \to X_u \ell \bar{\nu}$

- Problem: Cuts needed to suppress charmed decays
- Forces us into corners of phase space, where the usual OPE breaks down
- Expansion parameter $\Lambda_{\mathrm{QCD}}/(m_b - 2E_\ell)$
- Instead of HQE Parameters: Shape Functions $f(\omega)$

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle$$

- Universal for all heavy-to-light decays
- Systematics: S_{oft}C_{ollinear}E_{ffective}T_{heory} calculation
  - Several subleading shape functions
  - perturbative QCD corrections

Shape Functions

- Shape function vs. local OPE: Moment Expansion

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

- Perturbative "jetlike" contributions: Convolution

$$S(\omega, \mu) = \int dk\; C_0(\omega - k, \mu) f(k)$$

- Charged Lepton Energy Spectrum (H: hard QCD corrections)

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega\, \Theta(m_b(1-y) - \omega) H(\mu) S(\omega, \mu)$$

Approaches

- Obtaining the Shape functions:
  - From Comparison with $B \to X_s\gamma$
  - From the knowledge of (a few) moments
  - From modeling
- QCD based:
  - BLNP (Bosch, Lange, Neubert, Paz)
  - GGOU (Gambino, Giordano, Ossola, Uraltsev)
  - SIMBA (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)
- QCD inspired:
  - Dressed Gluon Exponentiation (Andersen, Gardi)
  - Analytic Coupling (Aglietti et al.)
- Attempts to avoid the shape functions (Bauer Ligeti, Luke ...)

Theo. uncertainty in $V_{ub,incl}$ is still (7 ... 10) %

In Progress: Update of BLNP

- Systematic Framework: Soft Collinear Efeective Theory (SCET)
- Include the known α_s^2 Corrections
- Include the konwn higher Moments of the shape functions
- Use the kinetic mass scheme, Comparison to $b \to c$
- New, more flexible parametrization of the shape function, including also new constraints

Hope: Resolve the V_{ub} problem

Semi-Tauonic Decays

Inclusive $B \to X_c \tau \bar{\nu}$

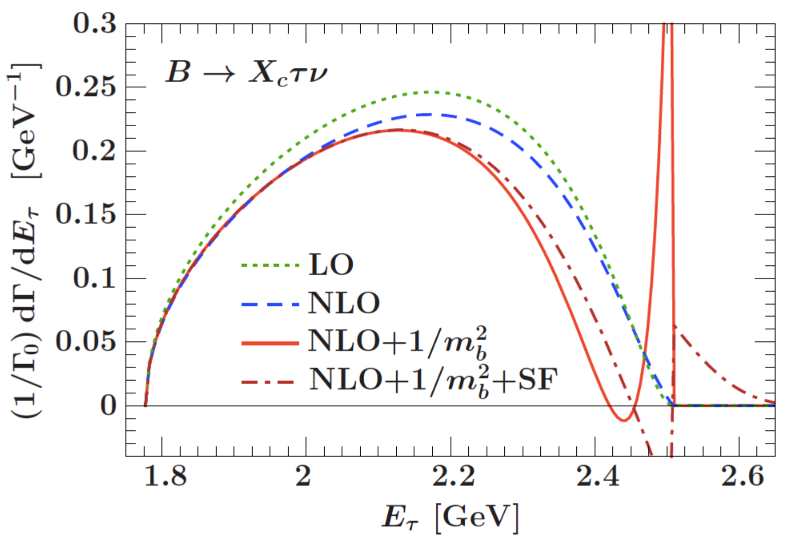
- One can use the HQE to compute $B \to X_c \tau \bar{\nu}$
(LIgeti, Tackmann)

$$\mathrm{Br}(B \to X_c \ell \bar{\nu}) = (2.42 \pm 0.06)\% \quad \text{(includes } \alpha_s \text{ and } 1/m_b^2\text{)}$$

- More precise calculations are under way (Shahriaran et al.)
- Measurement: (b-Admixture from LHC, LEP, Tevatron, SppS)

$$\mathrm{Br}(b \to X_\ell \bar{\nu}) = (2.41 \pm 0.23)\%$$

- seems fairly well under control


$B \to X_c \tau \nu$
LO
NLO
NLO+$1/m_b^2$
NLO+$1/m_b^2$+SF
$(1/\Gamma_0)\, d\Gamma/dE_\tau$ [GeV^{-1}]
E_τ [GeV]
0.3
0.25
0.2
0.15
0.1
0.05
0
1.8
2
2.2
2.4
2.6

Inclusive $B \to X_c \tau \bar{\nu}$

- For $B \to D^{(*)} \tau \bar{\nu}$ we have the general form:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \, |V_{cb}|^2 \, |\vec{p}_{D^{(*)}}| q^2}{96\pi^3 m_B^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times \left[(|H_+|^2 + |H_-|^2 + |H_0|^2) \left(1 + \frac{m_\tau^2}{2q^2}\right) + \frac{3 m_\tau^2}{2q^2} |H_s|^2 \right]$$

- In particular: The rate depends on the form factors proportional to q_μ
- In the HQL: Known in terms of $\xi(w)$
- Additional factor m_τ^2 / m_B^2 in front of these form factots

- Based on this, we get the SM predictions

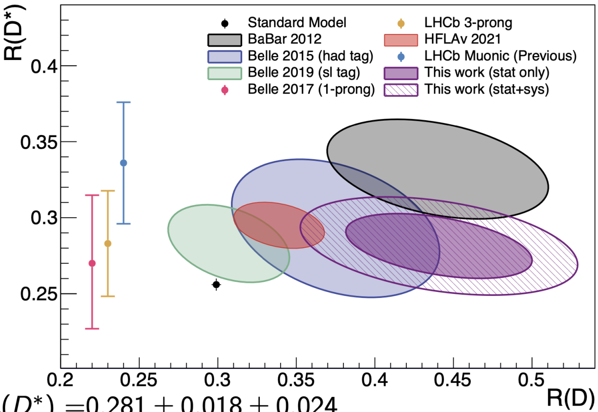
$$\mathrm{Br}(B \to D\tau\bar{\nu}) = (0.66 \pm 0.05)\%$$
$$\mathrm{Br}(B \to D^*\tau\bar{\nu}) = (1.43 \pm 0.05)\%$$

and

$$R(D) = \frac{\Gamma(B \to D\tau\bar{\nu})}{\Gamma(B \to D\ell\bar{\nu})} = 0.297 \pm 0.017$$
$$R(D^*) = \frac{\Gamma(B \to D^*\tau\bar{\nu})}{\Gamma(B \to D^*\ell\bar{\nu})} = 0.252 \pm 0.003$$

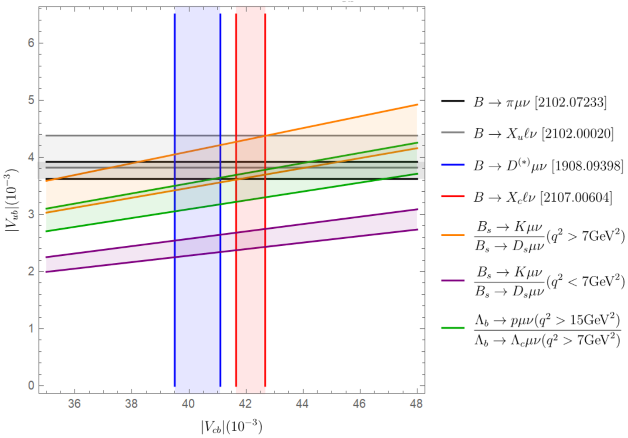
$B \to D\tau\bar{\nu}$ and $B \to D^*\tau\bar{\nu}$
exhaust about 86% of the inclusive rate.

Current status (Talk by Ciezarek Oct 19th, 2022)



- $\mathcal{R}(D^*) = 0.281 \pm 0.018 \pm 0.024$
- $\mathcal{R}(D) = 0.441 \pm 0.060 \pm 0.066$

Some Personal Conclusions



(From K. Vos)

The V_{cb} "Problem"

- Inclusive V_{cb} is in a very good shape
  - $1/m^4$ becomes under control
  - No hint for any problem with the HQE
  - Next step: Investigate possible (small) Violation of Duality
  - (Sub) percent precision seems within reach
- Exclusive V_{cb} from the lattice
  - Lattice calculations exhibit some problems: Different slopes
  - HQE based estimates indicate a larger value, still consistent with lattice
- Continuum methods are close to the second value

The V_{cb} problem will be solved soon!

The V_{ub} Problem

- Exclusive V_{ub}
  - Lattice yields consistent picture of the form factor for $B \to \pi\ell\bar{\nu}$
  - Very consistent with sum rule estimates for $B \to \pi\ell\bar{\nu}$
  - $B \to \rho\ell\bar{\nu}$ is not very precise,
    need the ppion phase space distribution for $B \to \pi\pi\ell\bar{\nu}$
- Inclusive V_{ub}
  - Model dependence through the shape function is hard to estimate
  - BLNP and GGOU are QCD based approaches
  - ... and BLNP needs and update
  - DGE and ADFR are models, now way to estimate the corresponding uncertainty.

More work on the inclusive side needed

Other Great Stuff

There is a lot more Semilptonics:

- Semitauonic Decays
- Semileptonic Decays of Bottom Baryons
- Semileptonic Decays of B_s
- Semileptonic Decays of B_c (includes $b \to c$ and $c \to s$!)

Even once the V_{xb} puzzles are all solved, we should still enjoy Semilep-Tonic