



Introduction to CPV and all that

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Content:

- Symmetries & non-observables
- CP Violation in Standard Model Lagrangien
- Phenomenology of CP Violation and Mixing

Symmetries I

T.D. Lee: "The root to all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; the non-observables."

There are four main type of symmetries:

- Permuation symmetry:
 Bose-Einstain and Fermi-Dirac statistics
- Continous space-time symmetries: translation, rotation, acceleration
- Discrete symmetries:

space inversion, time inversion, charge inversion

unitarity symmetries: gauge invariances:
 U₁(charge), SU₂(isospin), SU₃(color), ...





Symmetries II

Non-observables	Symmetry Transformations	Conservation Laws or Selection Rules
Difference between identical particles	Permutation	BE. or FD. statistics
Absolute spatial position	Space translation $\vec{r} \rightarrow \vec{r} + \vec{\Delta}$	momentum
Absolute time	Time translation $t \rightarrow t + \tau$	energy
Absolute spatial direction	Rotation	angular momentum
Absolute right (or left)	$\vec{r} \rightarrow -\vec{r}$	parity
Absolute sign of electric charge	$e \rightarrow -e$	charge conjugation
Relative phase between states of different charge Q	$\psi \to e^{iQ\theta}\psi$	charge
Relative phase between states of different baryon number B	$\psi \rightarrow e^{N \theta} \psi$	baryon number
Relative phase between states of different lepton number L	$\psi \to e^{t \ell \theta} \psi$	lepton number
Difference between different coherent mixture of p and n states	$\binom{p}{n} \rightarrow U\binom{p}{n}$	isospin

Discrete Symmetries C,P,T

- Parity, P
 - Partiy reflects a system through the origin
 Converts RH coordinate system to LH ones
 - Vectors change sign, but axial vectors remain unchanged

$$ec{x}
ightarrow -ec{x}$$
, $ec{p}
ightarrow -ec{p}$, but $ec{L} = ec{x} imes ec{p}
ightarrow ec{L}$

- Charge Conjugation, C
 - Turns a particle into its anti-particle $e^+ \rightarrow e^-, K^- \rightarrow K^+$
- Time Reversal, T
 - Changes, e.g. the direction of motion of particles: $t \rightarrow -t$







Recall: Dirac Matrices

$$\gamma^{0} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}; \quad \gamma^{i} = \begin{pmatrix} 0 & \sigma_{i} \\ -\sigma_{i} & 0 \end{pmatrix}, \quad i = 1, 2, 3; \quad \gamma^{5} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix};$$

I: 2×2 unit matrix; σ_i : Pauli matrices

Four-component spinors $\psi = \begin{bmatrix} \chi \\ \psi \end{bmatrix}$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix};$$

Projection to left/right handed part: $\frac{1-\gamma^5}{2}\psi = \psi_L$; $\frac{1+\gamma^5}{2}\psi = \psi_R$

Recall: Transformation Properties

				Р	С
		(\vec{x},t)	\rightarrow	$(\vec{-x},t)$	$(ec{x},t)$
5	scalar field:	$\Phi(\vec{x},t)$	\rightarrow	$\vec{\Phi(-x,t)}$	$\Phi^{\dagger}(ec{x},t)$
ps	seudo field:	$P(\vec{x},t)$	\rightarrow	$-P(\vec{-x},t)$	$P^{\dagger}(\vec{x},t)$
	dirac field:	$\psi(ec{x},t)$	\rightarrow	$\gamma_0 \psi(\vec{-x}, t)$	$i\gamma^2\gamma^0\overline{\psi}^T(\vec{x},t)$
١	vector field:	$V_{\mu}(ec{x},t)$	$) \longrightarrow$	$V^{\mu}(-\vec{x},t)$	$-V^{\dagger}_{\mu}(ec{x},t)$
	axial field:	$A_{\mu}(\vec{x}, t$	$) \rightarrow$	$-A^{\mu}(-\vec{x},t)$) $A^{\dagger}_{\mu}(\vec{x},t)$
			Р	С	CP
S:	$\overline{\psi_1}\psi_2$	\rightarrow	$\overline{\psi_1}\psi_2$	$\overline{\psi_2}\psi$	$\overline{\psi_2}\psi_1$
P:	$\overline{\psi_1}\gamma_5\psi_2$	\rightarrow	$-\overline{\psi_1}\gamma_5\chi$	$\psi_2 \qquad \overline{\psi_2}\gamma_5$	$\psi_1 \qquad -\overline{\psi_2}\gamma_5\psi_1$
V:	$\overline{\psi_1}\gamma_\mu\psi_2$	\rightarrow	$\overline{\psi_1}\gamma^\mu\psi$	$-\overline{\psi_2}\gamma_{\mu}$	$_{\mu}\psi_{1}$ $-\overline{\psi_{2}}\gamma^{\mu}\psi_{1}$
A:	$\overline{\psi_1}\gamma_\mu\gamma_5\psi_2$	\rightarrow –	$-\overline{\psi_1}\gamma^\mu\gamma_5$	$_5\psi_2$ $\overline{\psi_2}\gamma_\mu\gamma_\mu$	$\psi_5\psi_1 -\overline{\psi_2}\gamma^\mu\gamma_5\psi_1$
(from "CP Violation/Jarlskog" or "CP Violation/Branco")					

P Violation of Weak Interaction (CC)

Maximal Parity violation of weak interaction (charged current)! (max. violation of symmetrie: transformed process doesn't exist)

$$\begin{pmatrix} V_{e} \\ e^{-} \end{pmatrix}_{L} \begin{pmatrix} V_{\mu} \\ \mu^{-} \end{pmatrix}_{L} \begin{pmatrix} V_{\tau} \\ \tau^{-} \end{pmatrix}_{L}$$

$$e_{R}^{-}, V_{eR} \mu_{R}^{-}, V_{R}^{-}, \tau_{R}^{-}, V_{R}^{-}$$

$$\begin{pmatrix} U \\ \bullet \\ d \end{pmatrix}_{L} \begin{pmatrix} C \\ s \end{pmatrix}_{L} \begin{pmatrix} t \\ b \end{pmatrix}_{L}$$

$$U_{R}, d_{R} C_{R}, s_{R} t_{R}, b_{R}$$

P & C Violation in Weak IA (CC)



P & C Violation in Weak IA (CC)



P & C Violation in Weak IA (CC)



P Violation in NC?



Example:

$$\nu: g_L = +\frac{1}{2} \quad g_R = 0 \qquad e: g_L = -0.27 \quad g_R = +0.23$$

Instead of g_l and g_R the vector and axial couplings often used:

$$g_V = g_L + g_R$$
 $g_A = g_L - g_R$

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CPT Invariance I

Invariance formal: $[CPT, X] = 0 \leftrightarrow (CPT)X(CPT)^{-1} = X$

X (operator of an observable) commute with transformation

however not equiv. to (CPT) = 1

initial state $\pi^{-}(\vec{p_{1}})p(\vec{p_{2}}) \rightarrow K^{0}(\vec{p_{3}})\Lambda(\vec{p_{4}})$ P transformation $\pi^{-}(-\vec{p_{1}})p(-\vec{p_{2}}) \rightarrow K^{0}(-\vec{p_{3}})\Lambda(-\vec{p_{4}})$ C transformation $\pi^{+}(-\vec{p_{1}})\bar{p}(-\vec{p_{2}}) \rightarrow K^{0}(-\vec{p_{3}})\bar{\Lambda}(-\vec{p_{4}})$ T transformation $\bar{K^{0}}(\vec{p_{3}})\bar{\Lambda}(\vec{p_{4}}) \rightarrow \pi^{+}(\vec{p_{1}})\bar{p}(\vec{p_{2}})$

Completely different process, but same matrix element!

Invariance under any transformation T: transformed process has same probability to happen as initial one, or Lagrangien is invariant under T.

CPT Invariance II

Local Field theories always respect:

- Lorentz Invariance
- Symmetry under CPT operation
 - \rightarrow mass of particle = mass of anti-particle
 - \rightarrow total decay rate of particle = total decay rate of anti-particle

(proof Lüders, Pauli, Schwinger)

• Question 1:

Mass diff. between K_L and K_S : $\Delta m = 3.5 \times 10^{-6}$ eV; CPT violation?

- Question 2: Lifetime of $K_s = 0.089$ ns, while lifetime of $K_L = 51.7$ ns; CPT violation?
- Question 3:

B factories measure decay rate $B \to J/\psi K_s$ and $\bar{B} \to J/\psi K_s$ to be clearly not the same. How can it be?

Toy Theory (I)

Consider a spin-1/2(Dirac) particle ("nuclean") interacting with a spin-0 (Sclar) object ("meson").

For simplicity, here real scalar field

$$\begin{split} L &= i \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi - i \bar{\psi} \gamma_{\mu} \partial^{\mu} \psi - m \bar{\psi} \psi & \text{nucleon field} \\ &+ \frac{1}{2} \partial^{\mu} \bar{\phi} \partial_{\mu} \phi - V(\phi)^{2} & \text{meson potential} \\ &+ \bar{\psi} (a + i b \gamma^{5}) \psi \phi + \bar{\psi} (a^{*} - i b^{*} \gamma^{5}) \psi \phi & \text{nuclean-meson IA} \end{split}$$

What are the symmetries under C, P, CP? Can a,b be any complexe number?

Toy Theory (II)

Consider a spin-1/2(Dirac) particle ("nuclean") interacting with a spin-0 (Sclar) object ("meson").

For simplicity, here real scalar field

$$\begin{split} L &= i \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi - i \bar{\psi} \gamma_{\mu} \partial^{\mu} \psi - m \bar{\psi} \psi & \text{nucleon field} \\ &+ \frac{1}{2} \partial^{\mu} \bar{\phi} \partial_{\mu} \phi - V(\phi)^2 & \text{meson potential} \\ &+ \bar{\psi} (a + i b \gamma^5) \psi \phi + \bar{\psi} (a^* - i b^* \gamma^5) \psi \phi & \text{nuclean-meson IA} \end{split}$$

vector field, scalar, pseudo-scalar

Toy Theory III

$$\begin{split} L &= i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi - m\bar{\psi}\psi \\ &+ \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - V(\phi)^{2} \\ &+ \bar{\psi}(a+ib\gamma^{5})\psi\phi + \bar{\psi}(a^{*}-ib^{*}\gamma^{5})\psi\phi \end{split}$$
$$\begin{split} L &= i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi - i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi \\ &+ \frac{1}{2}\partial^{\mu}\bar{\phi}\partial_{\mu}\phi - V(\phi)^{2} \qquad P t \\ &+ \bar{\psi}(a-ib\gamma^{5})\psi\phi + \bar{\psi}(a^{*}+ib^{*}\gamma^{5})\psi\phi \end{split}$$
$$\begin{split} L &= -i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi - m\bar{\psi}\psi \\ &+ \frac{1}{2}\partial^{\mu}\bar{\phi}\partial_{\mu}\phi - V(\phi)^{2} \qquad C t \\ &+ \bar{\psi}(a+ib\gamma^{5})\psi\phi + \bar{\psi}(a^{*}-ib^{*}\gamma^{5})\psi\phi \end{split}$$

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C transformation

$$\begin{split} L &= -i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi \\ &+ \frac{1}{2}\partial^{\mu}\bar{\phi}\partial_{\mu}\phi - V(\phi)^{2} \\ &+ \bar{\psi}(a + ib\gamma^{5})\psi\phi + \bar{\psi}(a^{*} - ib^{*}\gamma^{5})\psi\phi \end{split}$$

CP transformation

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Local Gauge Invariance

= Lagrangian must be invariant under local gauge transformations

Theory of massless Fermions:
$$L = i\overline{\psi} (\gamma^{\mu} \partial_{\mu}) \psi$$

"global" U(1) gauge transformation:

"global" U(1) gauge transformation:
$$\psi(X) \rightarrow \psi'(X) = e^{i\alpha}\psi(X)$$

"local" U(1) gauge transformation: $\psi(X) \rightarrow \psi'(X) = e^{i\alpha(X)}\psi(X)$

Is the Lagrangian invariant?

$$\psi(x) \to e^{i\alpha(x)}\psi(x) \quad ; \quad \overline{\psi}(x) \to e^{-i\alpha(x)}\overline{\psi}(x)$$
$$\partial_{\mu}\psi(x) \to e^{i\alpha(x)}\partial_{\mu}\psi(x) \quad + \quad ie^{i\alpha(x)}\psi(x)\partial_{\mu}\alpha(x)$$

Then:

$$i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi \rightarrow i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi$$

$$\overline{\psi}\gamma^{\mu}\psi\partial_{\mu}\alpha(x)$$
 No

ot variant!

Gauge Field

=> Introduce the covariant derivative:

$$D_{\mu} \equiv \partial_{\mu} - i e A_{\mu}$$

and <u>demand</u> that A_{μ} transforms as:

$$A_{\mu} \to A_{\mu}' = A_{\mu} + \frac{1}{e} \partial_{\mu} \alpha \left(x \right)$$

$$L \rightarrow L' = L$$

Conclusion:

- Introduce charged fermion field (electron)
- Demand invariance under local gauge transformations (U(1))
- The price to pay is that a gauge field A_{μ} and the IA with the field must be introduced at the same time.

Standard Model Lagrangian

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$

- $L_{Kinetic}$: Introduce the massless fermion fields Require local gauge invariance \rightarrow gauge bosons \Rightarrow CP conserving
- L_{Higgs} : Introuce Higgs potential with $\langle \phi \rangle \neq 0$ Spontaneous sym. breaking $\rightarrow W^+$, W^- & Z^0 get masses \Rightarrow CP conserving
- L_{Yukawa} : Ad hoc interaction between Higgs field & fermions \Rightarrow CP violating with a single phase
- $L_{Yukawa} \rightarrow L_{mass}$:

fermion weak eigenstates - mass matrix non-diagonal \Rightarrow CPV fermion mass eigenstates - mass matrix diagonal \Rightarrow no CPV

• $L_{Kinetic}$ for mass eigenst.: CKM matrix CPV w. single phase



 $L_{Kinetic}$: Fermions + gauge bosons + interactions Introduce fermion fields; demand local gauge invariance Start with the Dirac Lagrangian: $L = i \overline{\psi} (\partial^{\mu} \gamma_{\mu}) \psi$ Replace: $\partial^{\mu} \rightarrow D^{\mu} = \partial^{\mu} + ig_s G^{\mu}_a L_a + ig W^{\mu}_b T_b + ig B^{\mu} Y$ Fields: G_a^{μ} 8 gluons W^{μ}_{μ} : weak bosons W_1 , W_2 , W_3 B^{μ} : hypercharge bosons Generators: L_a : Gel-Mann matrices $\frac{\lambda}{2}$ (3×3) SU(3)_c T_h : Pauli matrices $\frac{\sigma}{2}$ (2×2) SU(2)_L Y: Hypercharge $U(1)_Y$

For the remainder we only consider electroweak $SU(2)_L \times U(1)_Y$



$$q = quarks, leptons; \begin{pmatrix} q_{jL} \\ q'_{jL} \end{pmatrix}, q_{jR}, q'_{jR} \text{ with } j = 1,2,3$$

$$L_{Kinetic} = \sum_{j=1}^{N} \overline{[(q,q')_{j,L}i\gamma^{\mu}(\partial_{\mu} - ig_{1}\frac{\vec{\sigma}}{2}\vec{W}_{\mu} - ig_{2}\frac{1}{6}\vec{B}_{\mu})} \begin{pmatrix} q_{j,L} \\ q'_{j,L} \end{pmatrix}$$

$$\overline{q_{jR}i\gamma^{\mu}(\partial_{\mu} - ig_{1}\frac{2}{3}\vec{B}_{\mu})g_{jR}} + \overline{q'_{jR}i\gamma^{\mu}(\partial_{\mu} - ig_{1}\frac{-1}{3}\vec{B}_{\mu})q'_{jR}}] + h.c.$$

Lagrangian violates P and C, but conserves CP

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vector & axial vector: $i\bar{\psi}_L\gamma^{\mu}\psi_L = i\bar{\psi}\gamma^{\mu}\frac{1-\gamma^5}{2}\psi \rightarrow \mathsf{P} \rightarrow i\bar{\psi}\gamma_{\mu}\frac{1+\gamma^5}{2}\psi$ $i\bar{\psi}_L\gamma^{\mu}\psi_L = i\bar{\psi}\gamma^{\mu}\frac{1-\gamma^5}{2}\psi \to \mathbf{C} \to i\bar{\psi}\gamma^{\mu}\frac{-1-\gamma^5}{2}\psi = -i\bar{\psi}\gamma^{\mu}\frac{1+\gamma^5}{2}\psi$

vector feld:

$$\begin{split} & i\vec{\sigma}\vec{W} \to \mathsf{P} \to i\vec{\sigma}\vec{W} \\ & i\vec{\sigma}\vec{W} \to \mathsf{C} \to -i(\vec{\sigma}\vec{W})^{\dagger} \end{split}$$

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L_{Higgs} & Symmetrie Breaking

$$L_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_H; V_H = rac{1}{2} \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2$$

 $\phi^{\dagger}\phi$: real scalar field \rightarrow conserves C, P and consequently CP



Spontaneous Symmetry Breaking:

The Higgs field adopts a non-zero vacuum expectation value



Add ad-hoc IA between ϕ and fermions in gauge invariante way: only quarks right now

$$L_{Yukawa} = \sum_{j,k=1}^{N} \left(Y_{jk} \overline{(q,q')_{jL}} \begin{pmatrix} \phi^{0*} \\ -\phi^{-} \end{pmatrix} q_{kR} + Y_{jk}^{'} \overline{(q,q')_{jL}} \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} q_{kR}^{'} + h.c.
ight)$$

Spontaneous symmetrie breaking:

$$\phi_0 \to \phi_0 + v; \qquad \phi = \begin{pmatrix} 0\\ \phi_0 + v \end{pmatrix}$$

 \rightarrow 3 Higgs fields eaten up \rightarrow W,Z boson gets massive

/

 \rightarrow Higgs boson appears

$$L_{Yukawa} = \sum_{j,k=1}^{N} \left(m_{jk} \overline{q_{jL}} q_{kR} + m'_{jk} \overline{q'_{jL}} q'_{kR} + h.c. \right) \left(1 + \frac{\phi^0}{v} \right)$$

 $m_{jk}=-rac{v}{\sqrt{2}}Y_{jk}, \quad m_{jk}'=-rac{v}{\sqrt{2}}Y_{jk}'$

no physical quark fields (no mass eigenstates)



$$L_{Yukawa} = \sum_{j,k=1}^{N} \left(m_{jk} \overline{q_{jL}} q_{kR} + m'_{jk} \overline{q'_{jL}} q'_{kR} + h.c. \right) \left(1 + \frac{\phi^0}{v} \right)$$

All terms occur in pairs (+ h.c.) such as:

 $Y_{ij}\bar{\psi}_{Li}\phi\psi_{Rj} + Y_{ij}^*\bar{\psi}_{Rj}\phi^{\dagger}\psi_{Li}$

 \Downarrow CP transformation

 $Y_{ij}\bar{\psi}_{Rj}\phi^{\dagger}\psi_{Li} + Y_{ij}^{*}\bar{\psi}_{Li}\phi\psi_{Rj}$

If $V_{ij} \neq V_{ij}^*$, L_{Yukawa} violates CP formally: CPV $\Leftrightarrow Im(det[YY^{\dagger}, Y'Y'^{\dagger}]) \neq 0$

(related to add. degrees of freedom, quark phases)

Physical Quark Fields

convention

Diagonalize mass matrices:

(matrix theory: possible w/ help of 2 unitary matices)

$$U_L m U_R^{+} = D \equiv Diag.(m_u, m_c, m_t)$$

 $U'_{I}m'U'^{+}_{R} = D' \equiv Diag.(m_d, m_s, m_b)$

 $U_{1}U_{1}^{+}=1$

Substituting into L(f,H) one obtains for u-type quarks:

$$\overline{q}_{jL}m_{jk}q_{kR} = \overline{q}_{L}mq_{R} = \overline{q}_{L}U_{L}^{+}U_{L}mU_{R}^{+}U_{R}q_{R}$$

$$= \overline{U_{L}q_{L}}DU_{R}q_{R} = \overline{U_{L}q_{L}}\begin{pmatrix} m_{u} & & \\ & m_{c} & \\ & & m_{t} \end{pmatrix}U_{R}q_{R}$$

$$q_{L}^{phys} = U_{L}q_{L}$$

$$g_{L}^{'phys} = U_{L}'q_{L}$$
Similar relations also for right-handed quarks

 $q_L^{(p)(y)} = U_L^{\prime} q_L^{\prime}$

analog for d-type quarks

$$L_{Yukawa} \rightarrow L_{mass}$$

$$L_{mass} = (1 - \frac{\phi^0}{v})(m_u u^{phys} \overline{u}^{phys} + m_c s^{phys} \overline{s}^{phys} + m_t t^{phys} \overline{t}^{phys} + m_d d^{phys} \overline{d}^{phys} + m_s c^{phys} \overline{c}^{phys} + m_b b^{phys} \overline{b}^{phys} + h.c.)$$

Conserves C and P seperately, thus as well CP

however go back to $L_{Kinetic}$ and write it with mass eigenstates ...

Charged Current IA

Rewrite CC part of $L_{Kinetic}$ with respect to mass eigenstates

$$X_{C} = [W_{\mu}^{1} - iW_{\mu}^{2}]\overline{q_{L}}\gamma^{\mu}q_{L}' + h.c.$$
$$= [W_{\mu}^{1} - iW_{\mu}^{2}]\overline{q_{L}^{phys}}\gamma^{\mu}U_{L}U_{L}'^{\dagger}q_{L}'^{phys} + h.c.$$
$$= [W_{\mu}^{1} - iW_{\mu}^{2}]\overline{q_{L}^{phys}}\gamma^{\mu}Vq_{L}'^{phys} + h.c. = [W_{\mu}^{1} - iW_{\mu}^{2}]J_{c}'' + h.c.$$

$$V \equiv U_L U_L^{\prime +} \text{ and } J_C^{\mu} \equiv \overline{(u, c, t)}_L \gamma^{\mu} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

CP Violation & Mixing Matrix

CC is maximally P and C violating; CP conservation requires V to be real

 $X_{C} = [W_{\mu}^{1} - iW_{\mu}^{2}]\overline{u_{j}}\gamma^{\mu}V_{jk}(1-\gamma_{5})d_{k} + [W_{\mu}^{1} + iW_{\mu}^{2}]\overline{d_{k}}\gamma^{\mu}V_{jk}^{*}(1-\gamma_{5})u_{j}$ $\Downarrow \text{ CP Transformation}$

 $[W^{1}_{\mu} + iW^{2}_{\mu}]\overline{d_{k}}\gamma^{\mu}V_{jk}(1-\gamma_{5})u_{j} + [W^{1}_{\mu} - iW^{2}_{\mu}]\overline{u_{j}}\gamma^{\mu}V^{*}_{jk}(1-\gamma_{5})d_{k}$

Same CP Violation as in Yukawa term (directly inherited).



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CKM Matrix I

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$
flavour CKM matrix mass

18 parameters (9 complex elements)-5 relative quark phases (unobservable)-9 unitarity conditions

= 4 independent parameters 3 Euler angles and 1 Phase

4 fundamental Standard Model Parameters (out of ${\sim}28$)

CKM Matrix II

Lagrangian insensitive to phases of left-handed fields: possible redefinition:

 $u_{L} \to e^{i\phi(u)}u_{L} \quad c_{L} \to e^{i\phi(c)}c_{L} \quad t_{L} \to e^{i\phi(t)}t_{L}$ $d_{L} \to e^{i\phi(d)}d_{L} \quad s_{L} \to e^{i\phi(s)}s_{L} \quad b_{L} \to e^{i\phi(b)}b_{L}$

 $\phi(q)$: real numbers

$$V = \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

5 unobservable phase differences.

Standard Model & CPV

- CP is explicitly broken
- There is a single source (phase) of CP violation
- CPV appears only in charge current interaction of quarks
 → flavour changing interactions
- CPV direct consequence of spontaneous symmetry breaking.
 Addresses the question of origin of matter, beyond the search for the Higgs boson

CKM Matrix III



Diagonal elements of CKM matrix are close to one. Only small of diagonal contributions. Mixing between quark families is "CKM suppressed".

Unitarity Triangle I

Wolfenstein Parameterization: λ , A, ρ , η ; ($\lambda \approx 0.22$)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Only very small complexe contributions, up to third order in λ (\sim 0.5%) only in V_{ub} and V_{td}

Unitarity Triangle I

Unitary CKM matrix: $VV^{\dagger} = 1 \rightarrow 6$ "triangle" relations:





$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

$$V_{td}V_{ud}^{*} + V_{ts}V_{us}^{*} + V_{tb}V_{ub}^{*} = 0$$
Important for **B**_d and **B**_s decays

Remaining 4 relations lead to degenerated triangles: same area (J/2) but very different sides.

Unitarity Triangle II



Unitarity Triangle III



Current status of knowledge on "the" CKM triangle. Sofar all measurements consistent with each other.

Phenomenology of Mixing I



Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix}B^{0}\\\bar{B^{0}}\end{pmatrix} = H\begin{pmatrix}B^{0}\\\bar{B^{0}}\end{pmatrix} = \begin{pmatrix}H_{11} & H_{21}\\H_{12} & H_{22}\end{pmatrix}\begin{pmatrix}B^{0}\\\bar{B^{0}}\end{pmatrix}$$
$$= \begin{pmatrix}M - \frac{i}{2}\Gamma\end{pmatrix}\begin{pmatrix}B^{0}\\\bar{B^{0}}\end{pmatrix} = \begin{pmatrix}m_{11} - \frac{i}{2}\Gamma_{11} & m_{21} - \frac{i}{2}\Gamma_{21}\\m_{12} - \frac{i}{2}\Gamma_{12} & m_{22} - \frac{i}{2}\Gamma_{22}\end{pmatrix}\begin{pmatrix}B^{0}\\\bar{B^{0}}\end{pmatrix}$$

CPT theorem:

$$m_{11} = m_{22} = m(B_0) = m(\bar{B}^0)$$

$$\Gamma_{11} = \Gamma_{22} = \Gamma$$

$$= \frac{1}{\tau(B^0)} = \frac{1}{\tau(\bar{B}^0)}$$

off-diagonal elements \Rightarrow mixing

 $\square *$

 M, Γ hermetic:

$$m_{12} = m_{21}; \Gamma_{12} = \Gamma_{21}$$

 $m_{12} = \Delta m; \Gamma_{12} = \Delta \Gamma$

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Phenomenology of Mixing II

$$H = \begin{pmatrix} m_{11} - \frac{i}{2}\Gamma_{11} & \frac{\Delta m}{2} - \frac{i}{2}\Delta\Gamma \\ \frac{\Delta m}{2} - \frac{i}{2}\Delta\Gamma & m_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$







For K⁰ important, for B⁰ negligible



Phenomenology of Mixing III

	K^0/\overline{K}^0	D^0/\overline{D}^0	$B^0/\overline{B}{}^0$	B_s/\overline{B}_s
au [ps]	$89.4 \pm 0.1;$ 51700 ± 400	$0.413 \pm .003$	1.548 ± 0.021	1.49 ± 0.06
$\Gamma [s^{-1}] y = \frac{\Delta \Gamma}{2\Gamma}$	$5.61 \cdot 10^9$ -0.9966	$\begin{array}{c} 2.4 \cdot 10^{12} \\ y < 0.06 \end{array}$	$\begin{array}{c} (6.41 \pm 0.16) \cdot 10^{11} \\ y \lesssim 0.01^* \end{array}$	$(6.7 \pm 0.3) \cdot 10^{11} - (0.01 \dots 0.10)^*$
$\Delta m [s^{-1}]$ $\Delta m [eV]$ $x = \frac{\Delta m}{\Gamma}$	$\begin{array}{c} (5.300\pm0.012)\cdot10^9\\ (3.49\pm0.01)\cdot10^{-6}\\ 0.945\pm0.002\end{array}$	$< 7 \cdot 10^{10}$ $< 5 \cdot 10^{-6}$ < 0.03	$(4.89 \pm 0.09) \cdot 10^{11}$ $(3.2 \pm 0.1) \cdot 10^{-4}$ 0.76 ± 0.02	$> 15 \cdot 10^{12}$ $> 1.0 \cdot 10^{-2}$ $21 \dots 40^{*}$

kaons: mixing in decay and mixing in oscillationD mesons: very slow mixing (discovered 2007 at Babar)B mesons: mixing in oscillation (discovered 2006 at Tevatron)

Phenomenology of Mixing IV

Diagonalizing of $(M - \frac{i}{2}\Gamma) \to \text{mass eigen states:}$ $|B_L >= p|B^0 > +q|\bar{B^0} >, |B_L(t) >= |B_L > e^{-\frac{\Gamma_L}{2}t}e^{-im_L t}$ $|B_H >= p|B^0 > -q|\bar{B^0} >, |B_H(t) >= |B_H > e^{-\frac{\Gamma_H}{2}t}e^{-im_H t}$

 $|\boldsymbol{p}|^2+|\boldsymbol{q}|^2=1$ complex coefficients

Flavour eigenstates:

$$|B^{0}\rangle = \frac{1}{2p}(|B_{L}\rangle + |B_{H}\rangle)$$

$$|\bar{B^{0}}\rangle = \frac{1}{2q}(|B_{L}\rangle - |B_{H}\rangle)$$

$$m_{H,L} = m \pm Re\sqrt{H_{12}H_{21}}$$

$$\Gamma_{H,L} = \Gamma \mp 2Im\sqrt{H_{12}H_{21}}$$

$$\Delta m = m_H - m_L = 2Re\sqrt{H_{12}H_{21}}$$

$$\Delta \Gamma = \Gamma_H - \Gamma_L = -4Im\sqrt{H_{12}H_{21}}$$

Phenomenology of Mixing V

$$\begin{aligned} & \mathsf{CPT \ conversation!} \\ & P(B^0 \to B^0) = P(\bar{B^0} \to \bar{B^0}) = \\ & \frac{1}{4} \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos(\Delta m t) \right) \\ & P(B^0 \to \bar{B^0}) = \frac{1}{4} |\frac{q}{p}|^2 \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos\Delta m t \right) \\ & P(\bar{B^0} \to B^0) = \frac{1}{4} |\frac{p}{q}|^2 \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos\Delta m t \right) \\ & \mathsf{CP \ violation \ in \ mixing:} \\ & P(B^0 \to \bar{B^0}) \neq P(\bar{B^0} \to B^0) \ \Rightarrow |\frac{q}{p}| \neq 1 \end{aligned}$$



Mixing @ Babar & CDF



D Meson Mixing



 $x^{'2} = (-0.22 \pm 0.30 \pm 0.20) \times 10^{-3}$ $y^{'} = (9.7 \pm 4.4 \pm 2.9) \times 10^{-3}$

CP Transformation & Weak Interaction

Quarks

---- CP -----



$$\begin{pmatrix} \overline{d}' \\ \overline{s}' \\ \overline{b}' \end{pmatrix} = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} \overline{d} \\ \overline{s} \\ \overline{b} \end{pmatrix}$$



CP Violation

CP violation: $|\mathcal{A}(B \to f)|^2 \neq |\mathcal{A}(\bar{B} \to \bar{f})|^2$

Within weak interaction, moving from particle to antiparticle, system amplitudes are complex conjugated.

No CP violation if:

- There is only one amplitude contributing to the decay: $|\mathcal{A}|^2 = |\mathcal{A}^*|^2$
- The sum of two amplitudes, where both are complex conjugated when moving from particle to antiparticle system: $|\mathcal{A}_1 + \mathcal{A}_2|^2 = (\mathcal{A}_1 + \mathcal{A}_2)(\mathcal{A}_1^* + \mathcal{A}_2^*) = |\mathcal{A}_1^* + \mathcal{A}_2^*|^2$

For CP violation one needs two complex amplitudes, where one of them is complex conjugated and one not when moving from particle to antiparticle system.

CP Violation



 $|\mathcal{A}|^{2} = |\mathcal{A}|^{2} = |\mathcal{A}|^{2} = A_{1}^{2} + A_{2}^{2} + 2A_{1}A_{2}\cos(\Delta\phi + \Delta\delta) \qquad A_{1}^{2} + A_{2}^{2} + 2A_{1}A_{2}\cos(-\Delta\phi + \Delta\delta)$

 A_1 and A_2 need to have different weak phases ϕ and different CP invariant (e.g. strong) phases δ .

CP Violation

3 Types of CP violation:

1) CP violation in mixing (not present in B system)

2) CP violation in decay (sometimes called "direct" CPV):Different decay amplitudes contributingto the same finals state

 CP violation in interference:
 Same final state can be reached directly via decay and as well through mixing and then decay.

CP Violation in B Mixing



2 dominant diagrams with same phase; t dominates in loop (GIM mechanism)

GIM Mechanism

GIM: Glashow, Iliopolus, Maiani (1970)



Equal quark masses, no mixing possible: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

top not dominant, because it is so heavy, however due to $m_u \sim m_c \neq m_t$

Historically this resulted in the prediction of the charm quark!

Model independent: CP violation in mixing $< O(\frac{\Delta\Gamma}{\Delta m})$

	B_d	B_s
$\Delta m = m_H$ - m_L	$0.5~{ m ps}^{-1}$	17.8 ps^{-1}
$\Delta \Gamma / \Gamma$ = (Γ_L - Γ_H)/ Γ	$\mathcal{O}(0.01)$	<i>O</i> (0.1)
$\tau = 1/\Gamma$	1.5 ps	1.5 ps

Model independent: CP violation in mixing $< O(\frac{\Delta\Gamma}{\Delta m})$

	B_d	B_s
$\Delta m = m_H$ - m_L	$0.5~{ m ps}^{-1}$	17.8 ps^{-1}
$\Delta\Gamma/\Gamma$ = (Γ_L - Γ_H)/ Γ	$\mathcal{O}(0.01)$	$\mathcal{O}(0.1)$
$\tau = 1/\Gamma$	1.5 ps	1.5 ps

$$\frac{\Delta\Gamma}{\Delta m} = \frac{\Delta\Gamma}{\Gamma} \frac{\Gamma}{\Delta m} = \frac{\Delta\Gamma}{\Gamma} \frac{1}{\tau * \Delta m}$$
$$B_d : \mathcal{O}(0.01) \frac{1}{1.5ps * 0.5ps^{-1}} \sim \mathcal{O}(0.01)$$
$$B_s : \mathcal{O}(0.1) \frac{1}{1.5ps * 18ps^{-1}} \sim \mathcal{O}(0.01)$$

CP violation in $B_{d/s}$ Mixing is negligible!

CP Violation in Decay



CP Asymmetrie:

$$|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2 = 2|A_1||A_2|[\cos(\arg(V_{tb}^*V_{ts}) + \Delta\delta) - \cos(\arg(V_{tb}^*V_{ts}) - \Delta\delta)]$$

CP Violation in Decay



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CP Violation in Interference

Same final state through decay & mixing + decay



$$\mathcal{A}_{1} = \mathcal{A}_{mix}(B^{0} \to B^{0}) * \mathcal{A}_{decay}(B^{0} \to J/\Psi K_{s})$$

$$= \cos(\frac{\Delta m t}{2}) * A * e^{i\omega}$$

$$\mathcal{A}_{2} = \mathcal{A}_{mix}(B^{0} \to \bar{B^{0}}) * \mathcal{A}_{decay}(\bar{B^{0}} \to J/\Psi K_{s})$$

$$= i\sin(\frac{\Delta m t}{2}) * e^{+i\phi} * A * e^{-i\omega}$$

 $\Delta \phi = \phi - 2\omega$ (assume no CP violation in mixing and in decay) $\Delta \delta = \pi/2 \Leftarrow$ mixing introduces second phase difference

$$B_d
ightarrow J/\psi K_s$$



 $\begin{aligned} \mathsf{CP} & |J/\psi K_s \rangle = \xi |J/\psi K_s \rangle = -1 |J/\psi K_s \rangle \\ \phi_{mix} &= \arg((V_{td}V_{tb}^*)^2) = 2\beta \\ \omega &= \arg((V_{cb}V_{cs}^*)(V_{us}V_{ud}^*)) = 0 \end{aligned}$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

 $CP|J/\psi K_s>$

 B_d : $J^P = 0^{-1}$ (Pseudoskalar) J/ψ : : $J^{CP} = 1^{-1-1}$ (Vector) K_s : : $J^{CP} = 0^{-1-1}$ (Pseudoskalar)



 $CP|J/\psi K_s >$

 B_d : $J^P = 0^{-1}$ (Pseudoskalar) J/ψ : : $J^{CP} = 1^{-1-1}$ (Vector) K_s : : $J^{CP} = 0^{-1-1}$ (Pseudoskalar)



Drehimpulserhaltung:

$$\mathbf{0} = \mathbf{J} \left(J/\psi \phi \right) = |\vec{S} + \vec{L}|; \rightarrow \mathbf{L} = \mathbf{1}$$

 $P(J/\psi\phi) = P(J/\psi) * P(\phi) * (-1)^{L}$ $CP(J/\psi\phi) = CP(J/\psi) * CP(\phi) * (-1)^{L}$ = -1;

 \rightarrow CP odd Endzustand (ω = -1)

$$B_d
ightarrow J/\psi K_s$$



 $\mathcal{A}(t) = \frac{\Gamma(B \to J/\psi K_s)(t) - \Gamma(B \to J/\psi K_s)(t)}{\Gamma(\bar{B} \to J/\psi K_s)(t) + \Gamma(B \to J/\psi K_s)(t)}$

At B Factories, correlated states \rightarrow at $t = t_0$ Flavour of signal B determined by Flavour of tagging B.

 $B_d o J/\psi K_s$



 $B_d \to J/\psi K_s$



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 $B_d \to J/\psi K_s$



 $\mathcal{A}(t) = \sin(2\beta)\sin(\Delta m_d t)$

Babar: $sin(2\beta) = 0.722 \pm 0.040 \pm 0.023$ Belle: $sin(2\beta) = 0.652 \pm 0.039 \pm 0.020$

Why is measured "raw asymmetry" smaller then $\sin(2\beta)$? Which quantities determine the resolution?

 \rightarrow See Uli's talk

Esher's View on CPV



⇔ P transformation

⇔ C transformation

Hotel Lindenhof

Following people stay in Hotel Lindehof:

single room: Gudrun Hiller, Christoph Ilgner, Michael Schmelling double room: Osvaldo Aqunes, Markward Britsch, Andreas Crivellin, Bjoern Duling, Jenny Girrbach, Dmitry Popov, Stefan Schacht, Dominik Scherer, David Straub, Danny van Dyk, Susanne Westhoff,