

(FEB. 2009

BARYOGENESIS

THE ORIGIN OF MATTER
IN OUR UNIVERSE

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COSMOLOGY/
ASTROPHYSICS



ELEMENTARY
PARTICLE PHYSICS

WEINBERG
"FIRST THREE MIN." ✓

NEW OBSERVATIONS

- CMB (WMAP..)
- LARGE REDSHIFT GALAXIES SUPERNOVAE
- GRAVITATIONAL LENSING DEFECTS

THEORETICAL EXPLANATIONS

INFLATION \leadsto FLUCTUATIONS

DARK MATTER / ENERGY

STRUCTURE FORMATION

STILL NOT VERY SPECIFIC HINTS FOR ELEM. PARTICLE PHYSICS

- BARYONASYMMETRY

$$\eta = \frac{N_B - (N_{\bar{B}})}{N_\gamma} = (6.1 \pm 0.4 \text{ } ^{+0.4}_{-0.3}) 10^{-10} \text{ FROM WMAP}$$

IN AGREEMENT WITH PRIMORDIAL NUCLEOSYNTHESIS

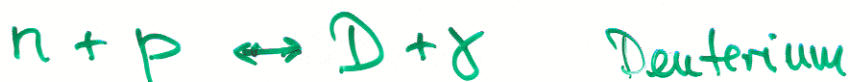
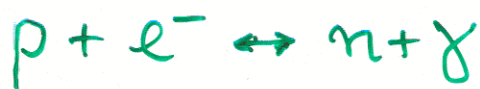
STRONGER CONNECTION TO ELEM. P. PHYSICS

(BEYOND THE SM NEEDED!?)

ESTIMATE OF Ω_B

- STAR MATTER (LUMINOUS)
GAS CLOUDS

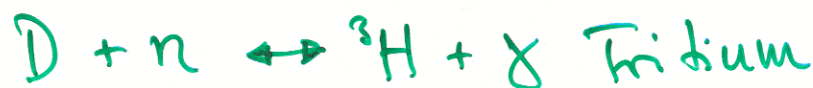
- EARLY NUCLEOSYNTHESIS
OF LIGHT ELEMENTS



Deuterium



Helium



Tritium

⋮

Lithium

$$(T > 10^6 \text{ eV})$$

$$\sim 10^{10} \text{ K}$$

$$t > 1 \text{ sec}$$

MORE RECENT

- POSITION AND HEIGHT
OF ACUSTIC PEAKS
IN DISTRIBUTION OF ΔT -
FLUCTUATIONS OF THE
COSMIC MICROWAVE background
(WMAP-Data)

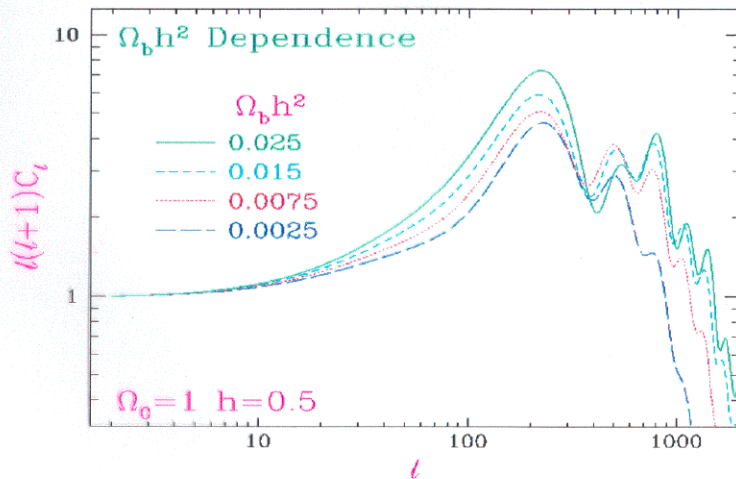
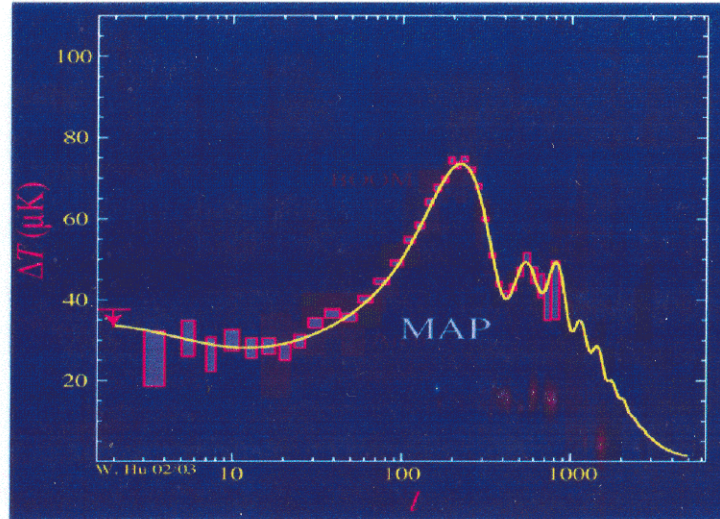
$$\Omega_B \sim 0.04 !$$

$$(\Omega_{\text{lum}} \leq 0.01)$$

Ω_B INFLUENCES SOUND VELOCITY
AT RECOMBINATION TIME !

Baryonic matter and cmb

• 3 •



***baryons:** increase compression (odd) peaks, decrease rarefaction peaks*

INFLATION

- SOLVES CAUSALITY PROBLEM IN BIG BANG TH.
- $\Omega = 1$ ($g = g_c$)
FLAT UNIVERSE
- CREATES FLUCTUATIONS LEADING TO STRUCTURE FORMATION

NO PARTICLES LEFT AFTER EXPONENTIAL GROWTH

HAVE TO CREATE BARYON ASYMMETRY

$$n_B \sim \Delta n_B = n_B - n_{\bar{B}} \quad \text{SMALL} \quad \frac{n_{\bar{B}}}{n_\gamma} \lesssim 10^{-18}$$

AFTER PAIR ANNIHILATION

SAKHAROV NECESSARY CRITERIA

- B-VIOLATION ✓
- C, CP - VIOLATION

→ KOLB WOLFRAM...

In the absence of preference for matter/antimatter
B nonconserving reactions will produce B and \bar{B}
at the same rate

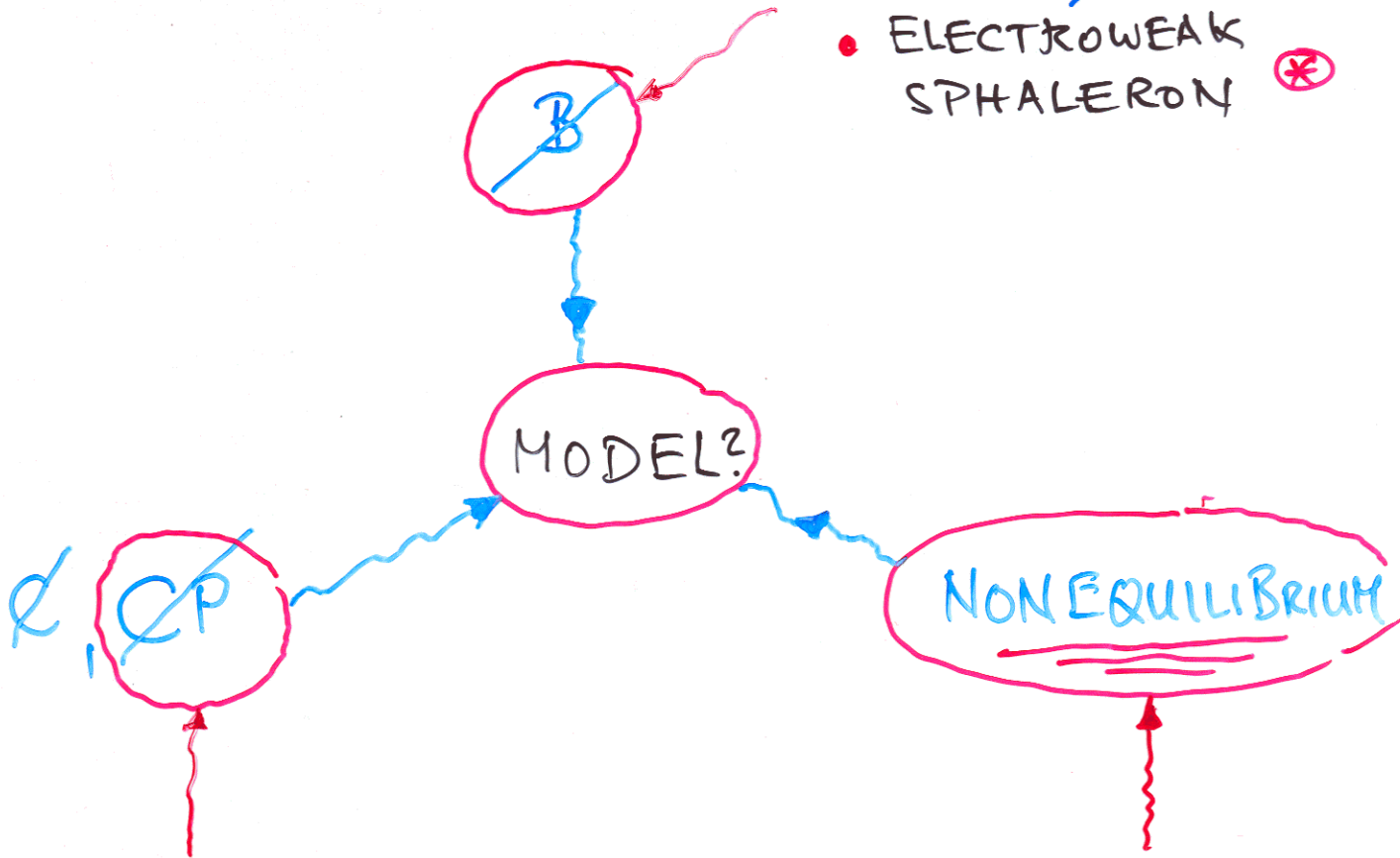
- NONEQUILIBRIUM

Chemical potential for nonconserved q.n. vanishes
 $\mu_B = \mu_{\bar{B}}$ (CPT) → same thermal distribution

• BARYO GENESIS

SAKHAROV '67

- GUT, MAJORANA NEUTR.
- ELECTROWEAK SPHALERON *



- CKM - MATRIX *
- PHASES IN NONSTANDARD TH
- SPONTANEOUS BREAKING

- EXPANDING UNIVERSE
- OUT OF EQUIL. DECAY
- PHASE TRANSITION *

* POSSIBLE IN SM?

SHAPOSHNIKOV

PROGRAM

SOME IMPORTANT MODELS

- ELECTROWEAK BARYOGENESIS (SM, MSSM, NMSSM or MSSM)
- LEPTOGENESIS
- AFFLECK-DINE BARYOGENESIS
- COHERENT BARYOGENESIS
- COLD BARYOGENESIS ALMOST THE SM

always needed

TECHNICAL POINTS NEEDED

- ~~STANDARD~~ MODEL
- SPHALERON TRANSITION
- ~~short~~ THERMAL HISTORY OF UNIVERSE
- PHASE TRANSITION (EWK.)
- ~~short~~ SUPERSYMMETRY
- TRANSPORT EQS. (~~short~~ BOLTZMANN...)
- MAJORANA NEUTRINOS

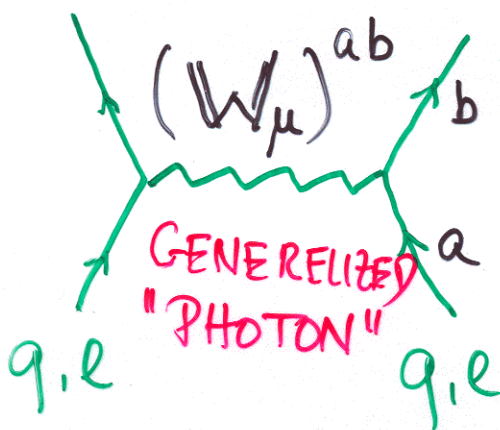
- ELECTROWEAK THEORY
INSIDE THE STANDARD MODEL
OF ELEMENTARY PARTICLE PH.

THREE GENERATIONS OF QUARKS AND LEPTONS

$$3 \times \begin{pmatrix} \nu \\ e^- \end{pmatrix}_{\text{LEFT HANDED}}, \quad \begin{pmatrix} u \\ d \end{pmatrix}_L^{a=1,2,3 \text{ (COLOR)}}, \quad e_R^-, u_R^a, d_R^a$$

TABLE
→

WEAK (ELWK) FORCES MEDIATED
BY WEAK GAUGE BOSONS COUPLING
TO L.H. QUARKS + LEPTONS



$$\underline{a, b = 1, 2}$$

→ MIXING

$$\textcircled{SU(2)}_{\text{WEAK}} \times U(1)_Y \times SU(3)_{\text{COLOR}}$$

$$W_\mu = \sum_{i=1}^3 W_\mu^i \tau_i; \quad \mathcal{L} = \frac{1}{2} \text{tr } F_{\mu\nu} F^{\mu\nu}$$

The Fundamental Fermions

(a) Leptons

Lepton	Symbol	Charge (e)	Mass (GeV/c^2)
Electron	e^-	-1	$5.1099906(15) \times 10^{-4}$
e-Neutrino	ν_e	0	$< 1.8 \times 10^{-8}$
Muon	μ^-	-1	$0.10565839(6)$
μ -Neutrino	ν_μ	0	$< 2.5 \times 10^{-4}$
Tau	τ^-	-1	$1.7841(32)$
τ -Neutrino	ν_τ	0	$< 3.5 \times 10^{-2}$

(b) Quarks

Quark flavor	Symbol	Charge (e)	Mass (GeV/c^2)
Down	d	$-\frac{1}{3}$	0.008
Up	u	$\frac{2}{3}$	0.004
Strange	s	$-\frac{1}{3}$	0.15
Charm	c	$\frac{2}{3}$	1.2
Bottom	b	$-\frac{1}{3}$	4.7
Top	t	$\frac{2}{3}$	≥ 40

The masses are given in the usual particle-physics units $1 \text{ GeV}/c^2 = 1.782662 \times 10^{-27} \text{ kg}$

The Fundamental Bosons

Name	Symbol	Spin (\hbar)	Mass (GeV/c^2)	Charge (e)
Graviton	G	2	0	0
Photon	γ	1	0	0
Charged weak bosons	W^\pm	1	$81.0(1.3)$	± 1
Neutral weak boson	Z	1	$92.4(1.8)$	0
Gluons	g_1, \dots, g_8	1	0	0
Higgs	H	0	?	0

$$F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g_{\text{WEAK}} [W_\mu, W_\nu]$$

LIKE IN E-DYN. BUT WITH SELF COUPLING



- ! MASSIVE W-BOSONS (FIELD / PARTICLES)
($\sim 80 \text{ GEV}$) \rightarrow "WEAK" INTERACTION

COUPLING TO A DOUBLET OF COMPLEX SCALAR "HIGGS" FIELDS $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
WITH A (CLASSICAL!) (AMPLITUDE) 2 :

$V < \phi^+ \phi >$
GAUGE COUPL. g_w^2

$< \phi^+ \phi >$

\rightarrow LATER!

- PHASE TRANSITIONS

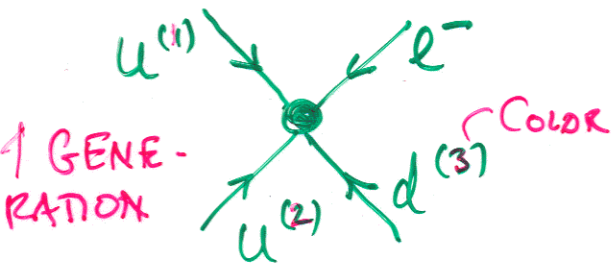
\leadsto THERMODYNAMICS, STATISTICAL MECHANICS

GINZBURG-LANDAU THEORY

... HUANG
SCHWABER

• SPHALERON TRANSITION IN EWK TH

$SU(2)_{\text{WEAK}}$ HAS TOPOLOGICAL NONTRIVIAL GAUGE FIELD CONFIGURATIONS (WINDING NUMBERS, CHERN-SIMONS NUMBER...) INDUCING TUNNELING BETWEEN QUARK- AND LEPTON BOUBLETS, VIOLATING $B+L$

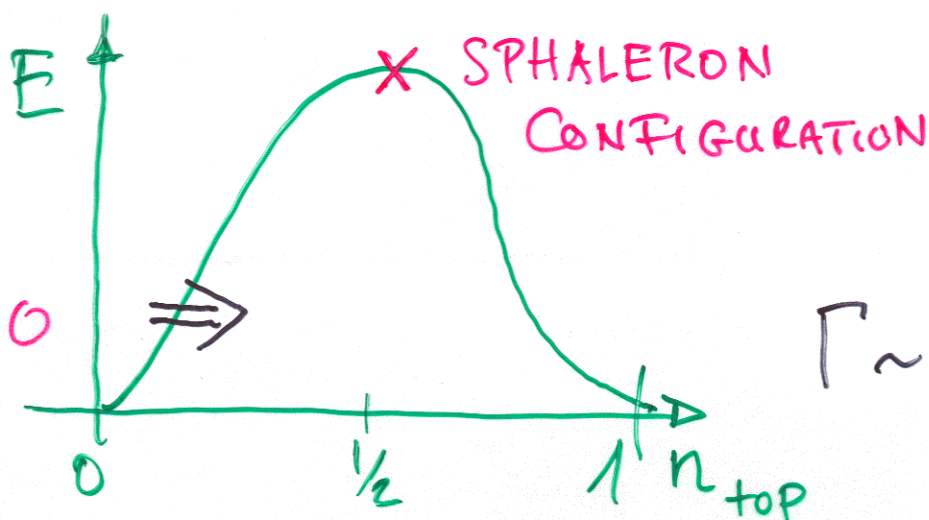


INSTANTON
INDUCED
AT $T=0$

$$e^{-\frac{8\pi^2}{g_w^2}}$$

Instanton action

ALSO : THERMAL TRANSITION VIOLATES $B+L$



$$\Gamma \sim T^4 e^{-\frac{M_w(T)}{\alpha_w kT}}$$

$\frac{\phi(T)}{T}$

↑
UNSUPPRESSED
IN HOT PHASE
WITH $\phi(T)=0$

SOME ARGUMENTS FOR B+L VIOLATION

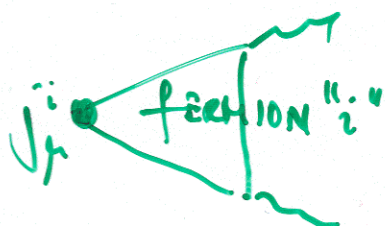
• QUANTUM ANOMALY

$$\int d^4x \partial_\mu (\overline{\psi}_L^i \gamma^\mu \psi_L^i) = \frac{1}{64\pi^2} \int d^4x F_{\mu\nu}^A \tilde{F}^{\mu\nu A}$$

U(1) CURRENT
OF SU(2) DOUBLETS

$\text{Gauss} = Q^i(+\infty) - Q^i(-\infty)$ CHANGE IN TOPOLOG-
QUANTUM NUMBER

$n = 1, 2, \dots$
instanton



$$(\partial_\mu \mathbb{1} + ig A_\mu) \psi_\lambda = \lambda \psi_\lambda \quad \text{DIRAC EIGENFUNCTIONS}$$

$\# \lambda = 0$ ZERO MODES = $\#$ TOPOL. QUANTUM
NUMBER OF A_μ

$$\int [d\psi] e^{-\int L(\psi)}$$

GRASSMANN

$$\psi = \sum_{\lambda \neq 0} c_\lambda \psi_\lambda + c_0 \psi_0$$

GRASSMANN!

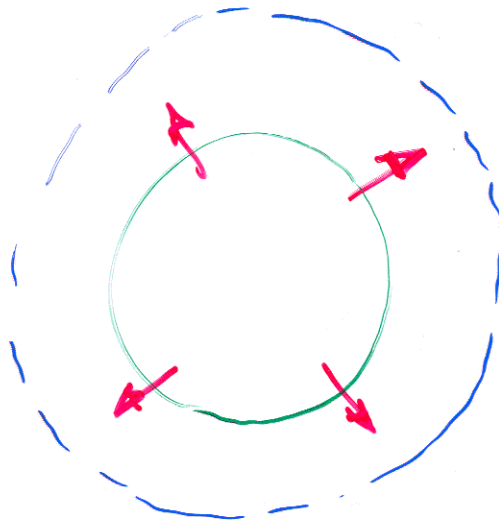
$$= \prod_{\lambda \neq 0} \int dc_\lambda \int dc_0 e^{\text{"}c_0 \text{ INDEP.}"}$$

NEED FURTHER c_0

• THE EARLY UNIVERSE

HOW TO REACH
? THE TEMPERATURE
OF THE ELECTROW.P.T.

- IT IS HOMOGENEOUS
- IT COOLS DOWN IN EXPANDING
- IT WAS VERY HOT!



T_γ TODAY $\sim 2.7^\circ\text{K}$

- A SHORT STORY OF THE (VERY) EARLY UNIVERSE AND BARYOGENESIS

• EINSTEIN EQ. IN SYMM. UNIVERSE (Robertson-Walker)

• RADIATION DOMINANCE

$$H^2 = \left(\frac{\dot{R}(t)}{R} \right)^2 = \frac{\kappa}{8} \rho_{\text{RAD}} T^4$$

$$\rho_{\text{RAD}} R^4 = C \quad \text{STEFAN-BOLTZMANN}$$

$\sim \text{"RT-LAW"}$

$$R(t) = \left(\frac{4\kappa C}{3} \right)^{1/4} t^{1/2}$$

$$RT \sim \text{const.}$$

$$T \sim t^{-1/2}$$

$$1 \text{ GeV} \sim 10^{13} \text{ } ^\circ\text{K}$$

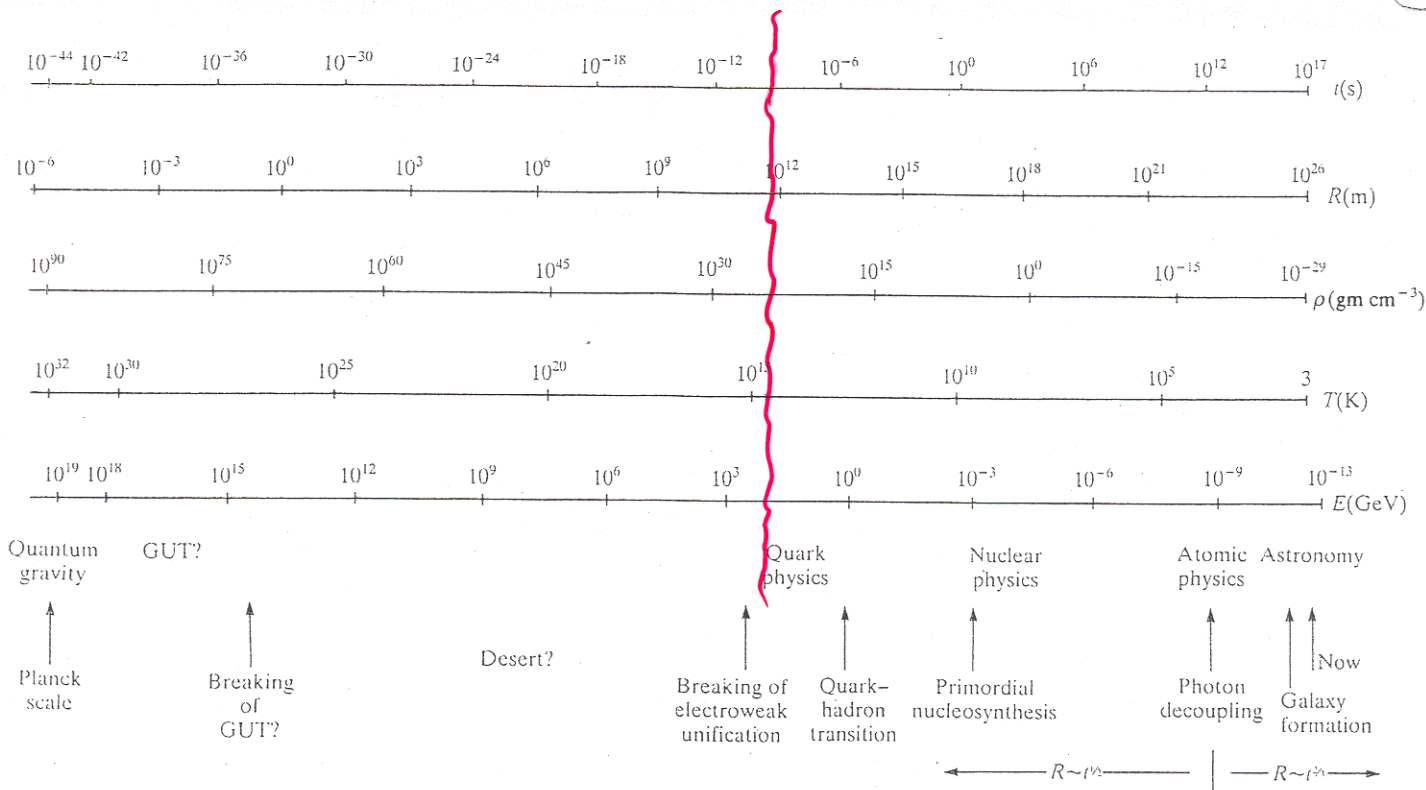
$$\begin{array}{ccc} \text{100 keV} & \xrightarrow{\times 10^6} & \text{100 GeV} \\ \text{"FIRST THREE MINUTES" sec} & & 10^{-12} \text{ sec} \\ \text{(WEINBERG)} & & \end{array}$$

STILL
LABORATORY
ENERGY

$$R \sim 10^{13} \text{ cm}$$

$$\Omega = \rho / \rho_c$$

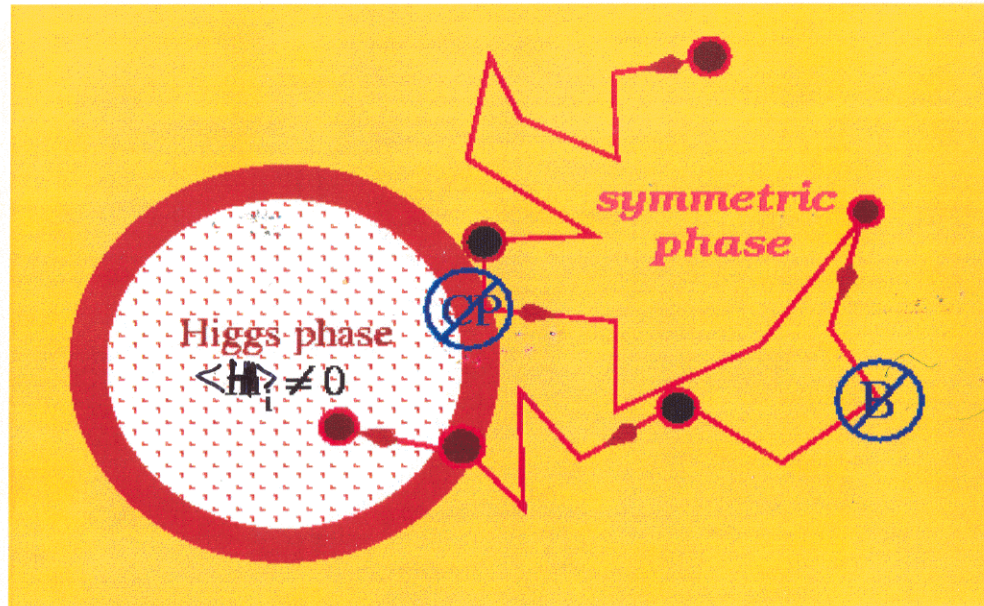
$$\rho_c = \frac{3H^2}{8\pi G}$$



The "history" of the universe from the Planck time to the present, showing how the size of the presently observable universe R , the average density ρ , the temperature T , and the energy per particle kT , have varied with time t according to the hot big bang model. Some of the major "events" and the dominant type of physics in each epoch are indicated.

Electroweak baryogenesis at a strong 1st order transition

CHARGE TRANSPORT



- expanding bubbles of higgs phase
- CP violation on bubble walls \Rightarrow CREATE CHIRAL ASYM.
- B violation in symmetric phase (SPHALERON)

NEED • STRONG B -VIOLATION
IN HOT ("UNBROKEN") PHASE

• FREEZEOUT OF B -VIOLATION
IN LOW TEMPERATURE ("HIGGS") PHASE
($T_{\text{SPH}} < H$)

\Rightarrow STRONG FIRST ORDER P.T.

! IT TURNED OUT THAT IN THE
• SM THERE IS NO STRONG 1. ORDER
P.T.

LATER

BUT : POSSIBLE IN VARIANTS OF
THE SM:

"BEYOND
THE SM"

MSSM MINIMAL SUPERSYMMETRIC
SM

NMSSM NEXT TO ...

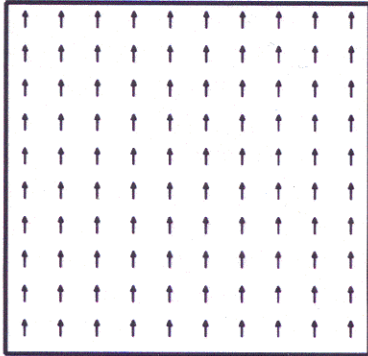
n MSSM NEARLY ...

2-HIGGS MODELS

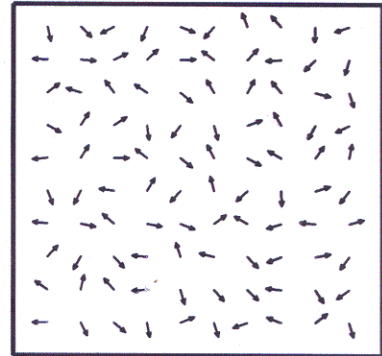
COSMOLOGY MIGHT REQUIRE TO MODIFY
THE SM ANYWAY!

Symmetry Restoration

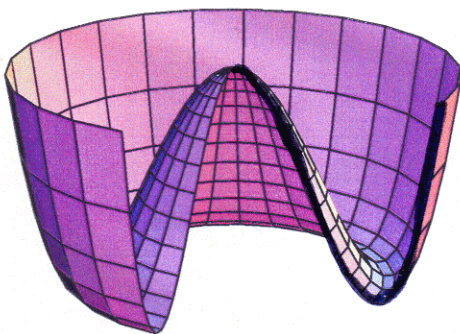
Low Temperature



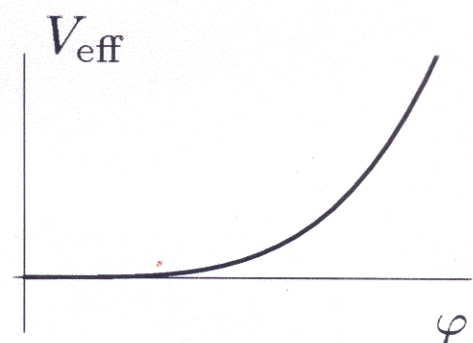
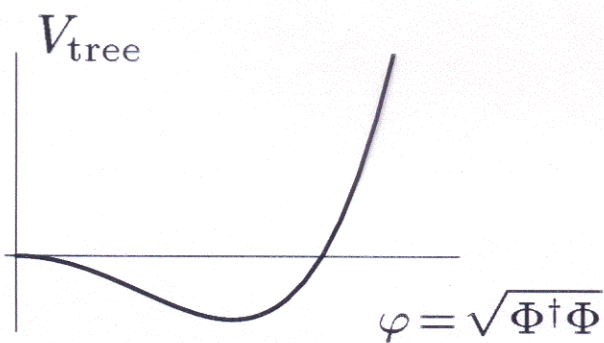
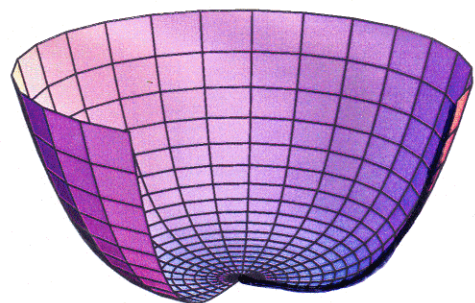
High Temperature



Broken Symmetry

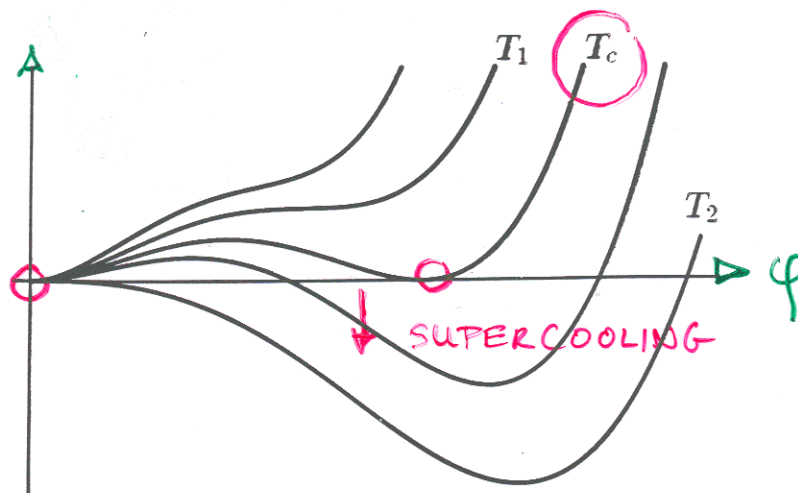


Restored Symmetry



First Versus Second Order

first order:

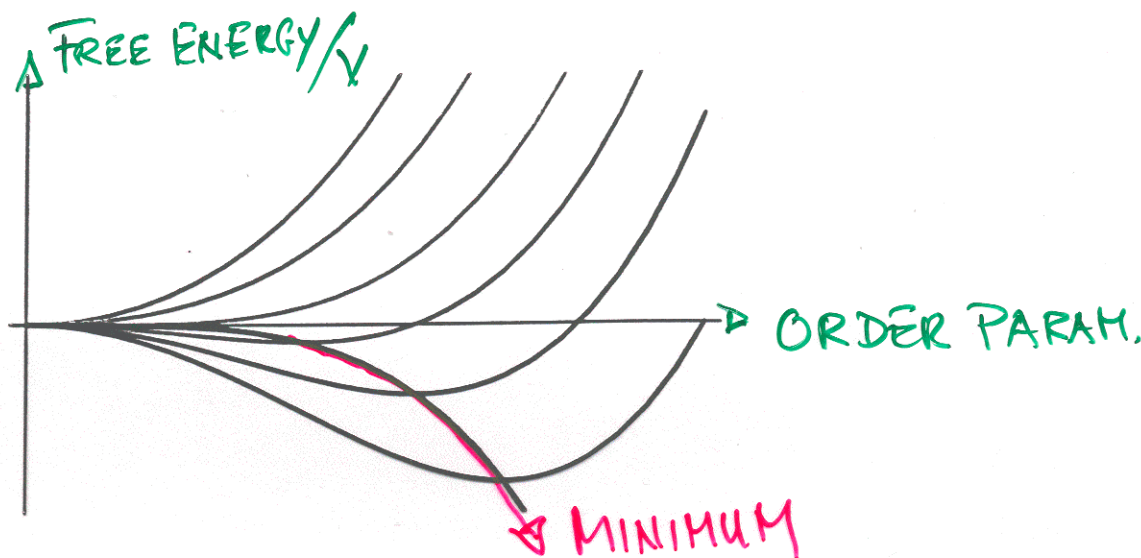


"EFFECTIVE POTENTIAL"

The field value at the global minimum jumps at the critical temperature (bubble nucleation).

There are deviations from the thermodynamic equilibrium.

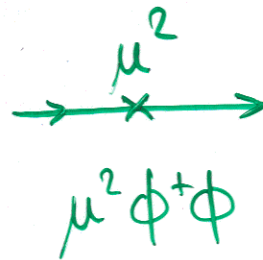
second order:



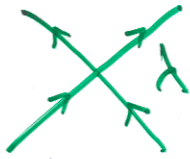
The field value at the global minimum departs continuously from 0 at the critical temperature.

• THE EFFECTIVE HIGGS POTENTIAL AT $T \neq 0$

"TREE" COUPLING



$$\mu^2 \phi^+ \phi$$



$$\frac{\lambda}{4} (\phi^+ \phi)^2$$



$$g_w^2 A_\mu^2 \phi^+ \phi$$

$$m_{\text{GAUGE B.}}^2 = \frac{1}{4} g_w^2 \langle \phi^+ \phi \rangle$$

$$\phi = \underbrace{\langle \phi \rangle}_{\text{CLASSICAL}} + \phi_Q$$

SIMPLE : CALCULATE GRANDCANONICAL POT. OF GAUGE BOSONS (+ HIGGSES...)

$$Z_3 = \sum_{N=0}^{\infty} \sum_{\{n_{\vec{p}}\}} e^{-\beta (E\{n_{\vec{p}}\} - \mu N)}$$

$\{n_{\vec{p}}\}, \sum_{\vec{p}} n_{\vec{p}} = N$

$\downarrow 0$

$$\beta = \frac{1}{kT}$$

$$E\{n_{\vec{p}}\} = \sum_{\vec{p}} n_{\vec{p}} (\epsilon_{\vec{p}} = (m_G^2(\phi) + \vec{p}^2)^{1/2})$$

$$Z_3 = \prod_{\vec{p}} \sum_{n_{\vec{p}}=0}^{\infty} e^{-\beta (\epsilon_{\vec{p}} - \mu) n_{\vec{p}}} = \prod_{\vec{p}} \frac{1}{1 - e^{-\beta (\epsilon_{\vec{p}} - \mu)}}$$

"QUANTUM BOSON GAS"

$$\mu=0$$

$$\mathcal{Z} = -\frac{1}{\beta} \ln \mathcal{Z}_3 = V_1(\phi) =$$

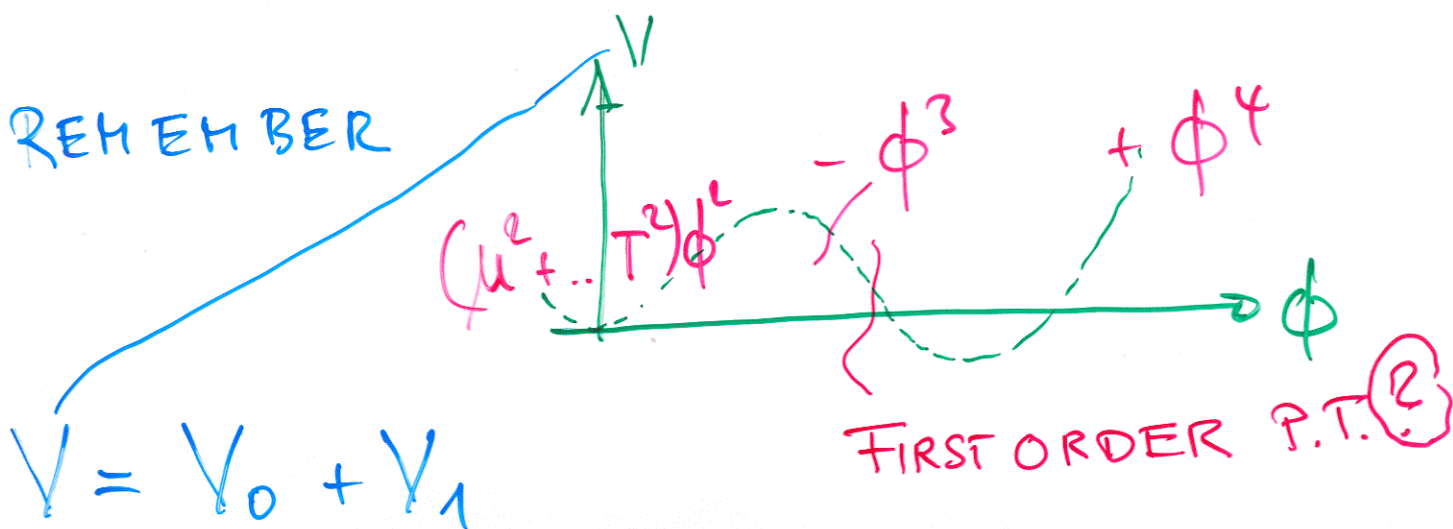
$$= \frac{1}{\beta} \int \frac{d^3 p}{(2\pi)^3} \ln \left(1 - e^{-\beta (m_G^2(\phi) + \vec{p}^2)^{1/2}} \right)$$

$$= \underbrace{\dots T^4}_{\text{STEFAN-BOLTZMANN}} + \dots T^2 m_G^2(\phi) - \dots T (m_G^2(\phi))^{3/2}$$

$$- \dots m_G^4 \ln \frac{m_G^2}{T^2} + \dots$$

$$\sim g_w^2 \phi^2 T^2 !$$

$$\text{" } -\phi^3 \text{-TERM"}$$



SIMILAR: "DEBYE" MASS $\sim g^2 T^2$

OF LONGITUDINAL GAUGE BOS.

NOT OF TRANSVERSAL ("A₀")

MORE CLEAN: THERMO FIELD THEORY

DISCUSS INFRARED-BEHAVIOR WITH MASSLESS TRANSV. GB $\phi \rightarrow 0$

COMPARE

$$Z = \text{tr } \rho = \sum_n \langle n | e^{-\beta H} | n \rangle$$

WITH QUANTUM MECH. TRANSITION $i \rightarrow j$

$$\langle j | e^{iHt/\hbar} | i \rangle \xrightarrow{\text{late}} \langle j | e^{iHt/\hbar} | i \rangle$$

$$\beta \approx -it/\hbar$$

$$\left. \begin{array}{l} X(0) = X_i \\ X(t) = X_j \end{array} \right\} \Rightarrow X(0) = X(\beta)$$

PERIODIC

"EUCLIDEAN TIME" it .

Quantum Field Th. \leadsto ThermoFieldTh.

$$\phi(x_4, \vec{x}) = \phi(x_4 + \beta, \vec{x})$$

?
 Euclid.

FOURIER-TRANSFORM

$$\int \frac{dp_4}{2\pi} \rightarrow \frac{1}{\beta} \sum_n$$

$$p_4 \beta = 2\pi n$$

FEYNMAN-PROPAGATOR

$$\frac{1}{\vec{p}^2 - p_0^2 + m^2} \rightarrow \frac{1}{(2\pi n T)^2 + \vec{p}^2 + m^2}$$

$$(n = 0, \pm 1, \dots)$$

MATSUBARA FREQ.

FOR BOSONS

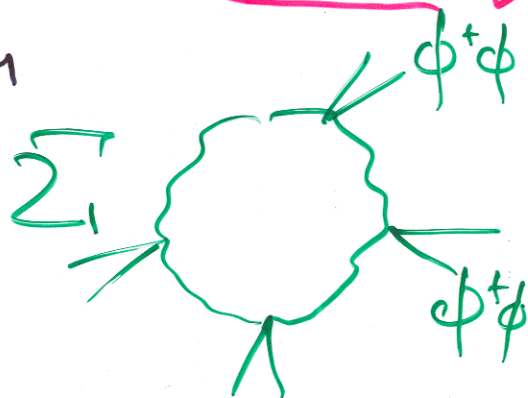
EFFECTIVE POTENTIAL

$$T \sum_{n=-\infty}^{+\infty} \int \frac{d^3 p}{(2\pi)^3} \ln \left((2\pi n T)^2 + p^2 + m^2 \right)$$

$$= (T=0 \text{ piece}) + \frac{1}{\beta} \int \frac{d^3 p}{(2\pi)^3} \ln \left(1 - e^{-\beta(m^2 + p^2)^{1/2}} \right)$$

See above!

$n=0$ GIVES " ϕ^3 "-TERM
(3 DIM.)



FOR $\langle \phi^\dagger \phi \rangle \rightarrow 0$

(GAUGE BOSON
CONDENSATE
IMPORTANT ?!)

HAVE INFRARED PROBLEMS
FOR MASSLESS GAUGE BOSONS

$n=0$:

$$\frac{1}{\vec{p}^2 + \dots \underline{\underline{g^2 T^2}}}$$

AVOIDS IR -
DIVERGENCE



"HARD THERMAL
LOOPS"

}
FOR LONGITUDINAL
GAUGE FIELDS

$n \neq 0$ MODES (MASSIVE $2\pi nT$!)

INFLUENCE THE EFF. ACTION OF $n=0$

\Rightarrow "INTEGRATE OUT" $n \neq 0$

OBTAIN EFFECTIVE 3-DIMENSIONAL TH.

NAIV: $\int_0^\beta dt_E \int d^3x \rightarrow \frac{1}{T} \int d^3x$

HAVE TO PERFORM 1/2-LOOP PERTURBATION
THEORY O.K.! TO OBTAIN "DIMENSIONALLY REDUCED"
THEORY \Rightarrow AGAIN GAUGE THEORY WITH HIGGS
FIELD, NOW 3D - "TRUNCATED"

\leadsto FIG.

MASS SCALES

$$\begin{array}{l} M_H, T \\ \left\{ \begin{array}{l} \text{MATSUBARA} \\ \text{OTHER MASSIVE} \\ \text{STATES} \end{array} \right. \end{array} \Rightarrow g_w T \xRightarrow{\text{DEBYE}} \underbrace{g_w^2 T}_{\text{3D THEORY}}$$

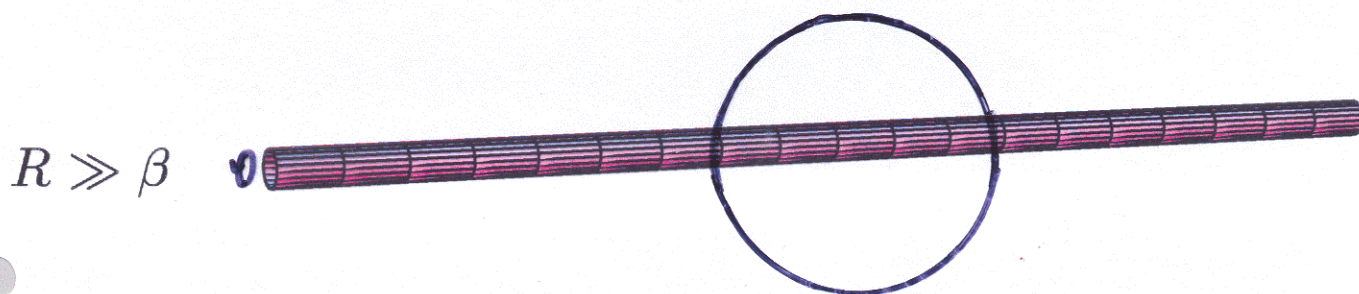
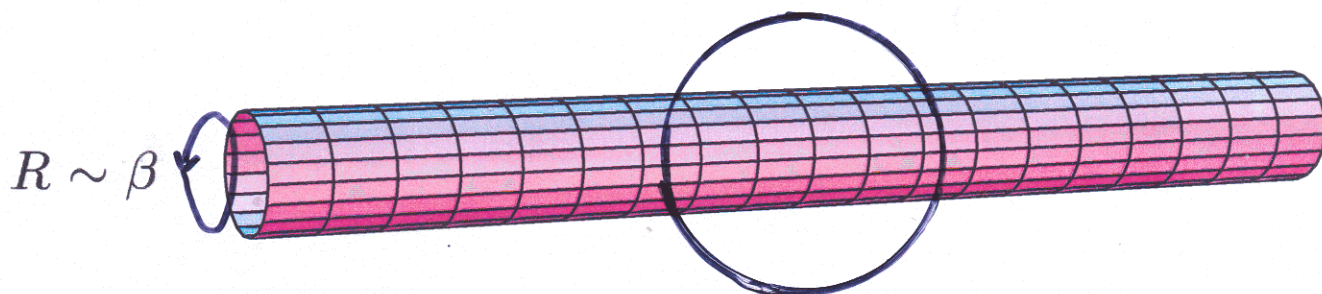
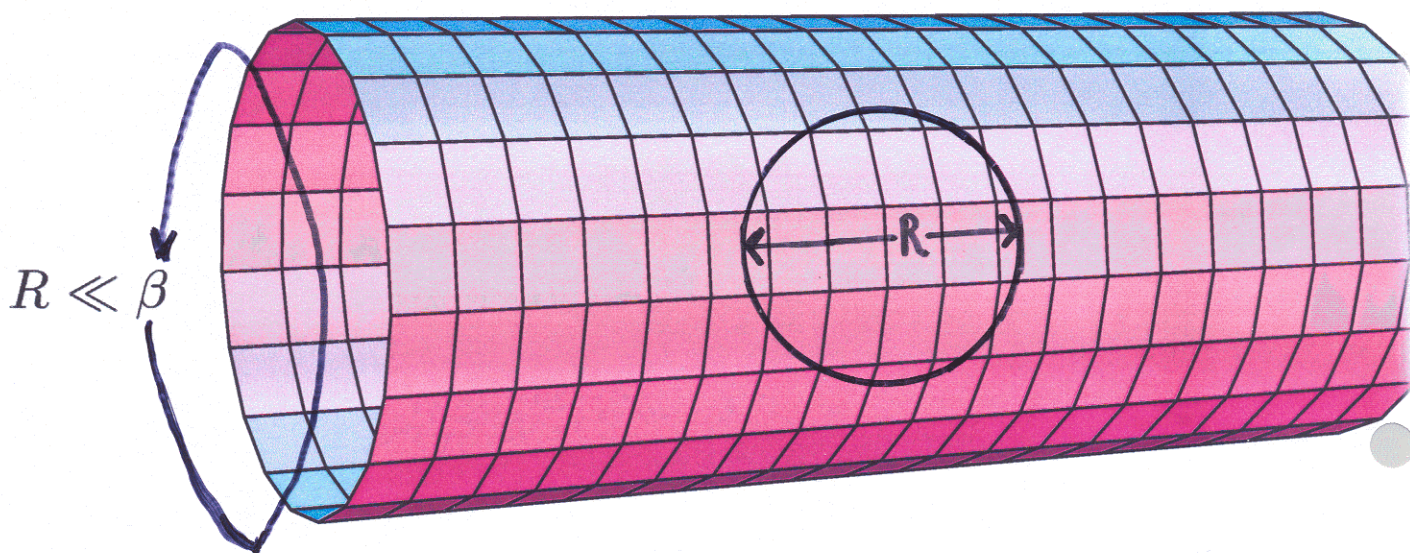
CAN BE STRONGLY

IR SENSITIVE (TRANSVERSAL
GAUGE BOSONS!)

\leadsto LATTICE GAUGE TH. CALCUL.

$$\beta = \frac{1}{T}$$

Dimensional Reduction

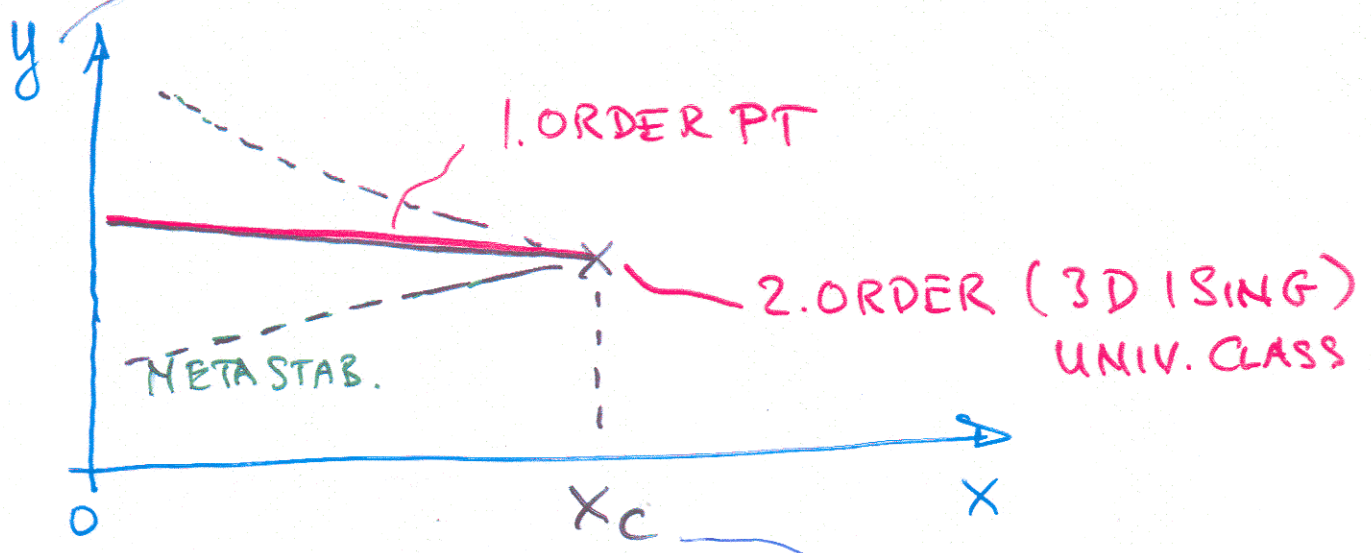


$$\mathcal{L} = \frac{1}{4} W_{ij}^a W_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2 + \dots$$

$$g_3^2 = g_4^2 T_{+}$$

$$x = \lambda_3 / g_3^2, \quad y = m_3^2 (g_3^2) / g_3^4$$

$\left\{ \begin{array}{l} \text{HIGGS-MASS DEP.} \\ \sim (T - T_c^*) \end{array} \right.$



"CROSS-OVER" at $m_H \gtrsim m_W$
 NO PHASE TRANSITION

ELECTROWEAK BARYOGENESIS

SM -

CKM - ~~CP~~

VERY SMALL

(BUT !! SEE LATER)

NO PHASE TRANSITION

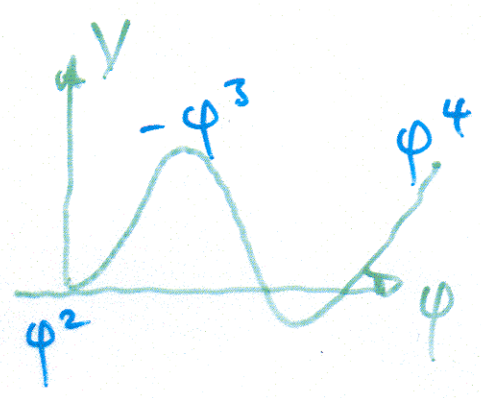
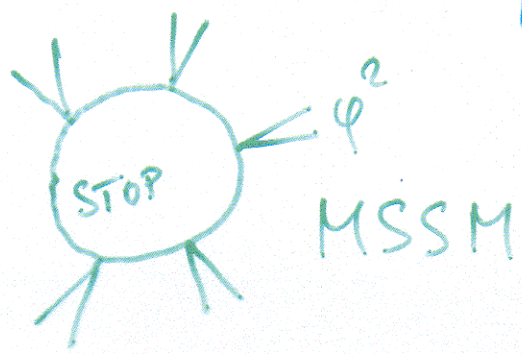
Kajantie
Laine
Rummukainen

FOR $m_h \gtrsim m_w$ (\leadsto "CROSSOVER")



SUPER SYMMETRIC VARIANTS

INCREASE " φ^3 " TERM



BODEKER
LAINE
JOHN
SCH.

HUBER
SCH.

— NMSSM
nMSSM

GET STRONG 1. ORDER
PHASE TRANSITION

SUSY

"MINIMAL" MODEL

("MSSM") ^(*) (8)

$H \rightarrow H_1, H_2, \tilde{H}_1, \tilde{H}_2$ HIGGSINOS

$q \rightarrow q, sq$ SQUARKS

$l \rightarrow l, sl$ SLERTONS

$g \rightarrow g, \tilde{g}$ GAUGINOS

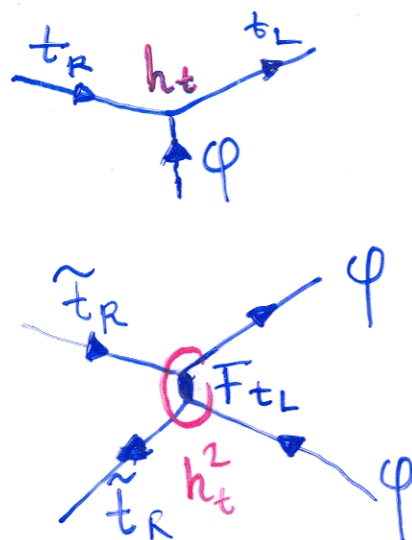
STANDARD MODEL COUPLINGS + //

SOFT SUSY-BREAKING PARAM.

IN LOOP:

$$m_{\tilde{t}_R}^2 = m_u^2 + m_{top}^2 + (m_{THERMAL}^2)$$

$\underbrace{\hspace{10em}}_{\substack{\text{SUSY} \\ \text{BREAKING} \\ \text{MASS}}} \quad \underbrace{\hspace{10em}}_{\substack{\sim h_t^2 \\ \sim \varphi^2 \\ \sim T^2}}$



MSSM WITH STRONG FIRST ORDER PT
 $\left(\frac{V(T_c)}{T_c} \gtrsim 1\right)$

- $m_h \lesssim \dots 110 \text{ GeV} \dots \rightarrow \tan \beta$

- experim. $m_h \gtrsim 108 \text{ GeV}$ (m_{A_0} depend.)
 $((^{CFR\gamma} = 114 \text{ GeV} ??))$

- $160 \text{ GeV} \lesssim m_{\text{stop}_R} \lesssim m_{\text{top}}$ (Conservativ. ...)
 avoid stop_R-Condensate

- experim. $m_{\text{stop}_R} \gtrsim 100 \text{ GeV}$ ($m_{\text{Neutralino}}$ depend.)

LOWERING THE EXPER. HIGGS-MASS BOUND
 WITH STRONG CP-VIOLATION

PILAFSIS
 WAGNER

MODEL CAN BE RULED OUT BY EXPER. IN
 THE NEAR FUTURE - OR CONFIRMED!

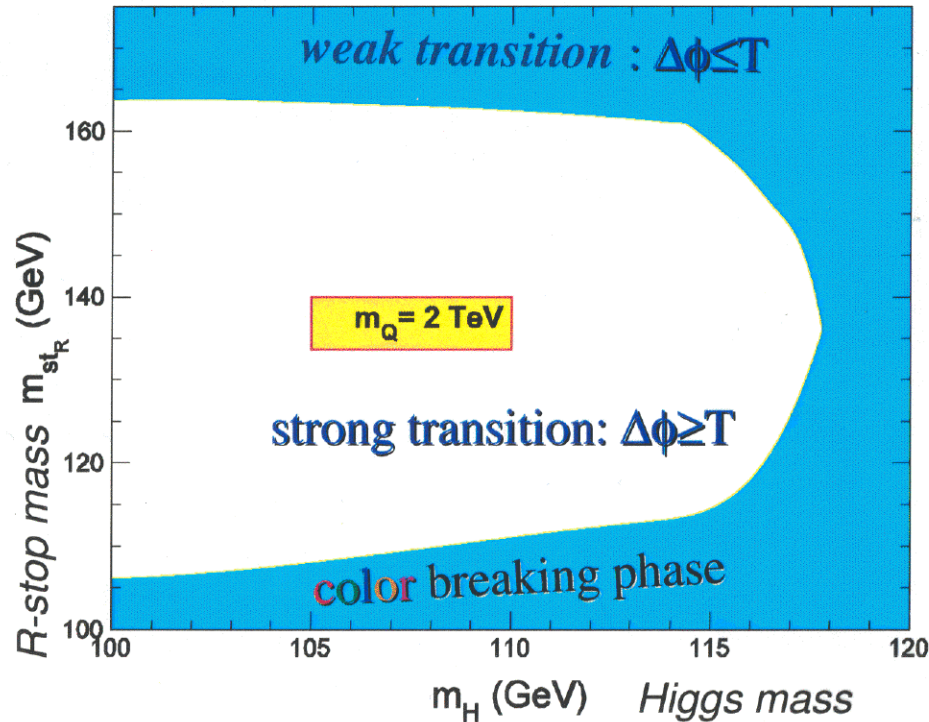
NMSSM STRONG FIRST ORDER PT.
 EVEN FOR $m_h \sim 120 \text{ GeV}$!
 REDUCTION OF EXP. m_h -BOUND
 BY H-SINGLET MIXING
 LARGE PARAMETER SPACE

S. HUBER
 M.G. SCH.

Strong first order transition in MSSM

allowed “triangle” for MSSM:

Carena, Quiros, Seco, Wagner, 2000



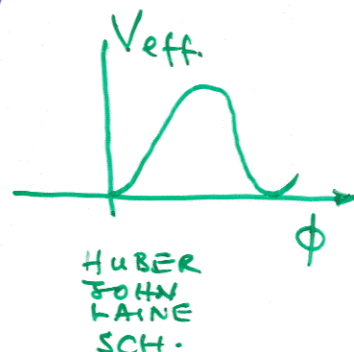
ELECTROWEAK BARYGENESIS -

A CONCRETE PROCEDURE!

- CRITICAL BUBBLE (1. ORDER P.T. !)

- MULTIDIMENSIONAL IN FIELDS
(HIGGS)

- TRANSITION PROBABILITY (LANGER FORMAL.)
 $\sim e^{-S_{\text{eff}}}$



- SUPERCOOLING
NUCLEATION TEMPERATURE ("1 BUBBLE / UNIVERSE")

- STATIONARY EXPANSION OF BUBBLE

HIGGS : SYMM. PHASE
→ V_w

$V_w = ?$, WALL-PROFILE
DEFLAGATION

BUBBLE PRESSURE = - FRICTION

→ $V_w, \delta f_{\text{NONEQUIL.}}$

MOORE
PROKOPEC
JOHN, SCH.

- DIFFUSION IN PRESENCE
OF MOVING BUBBLE WALL \Leftarrow QUANTUM
BOLTZMANN EQ
- WITH CP-VIOLATING WALL OR (MSSM)

EXPLICIT ~~CP~~ INTER
ACTION

GENERATES CHIRAL ASYMMETRY

$$n_{q_L} - n_{\bar{q}_L}$$

- ~~B~~ BY "HOT" SPHALERON OF ELWK. THEORY
IN FRONT OF BUBBLE WALL

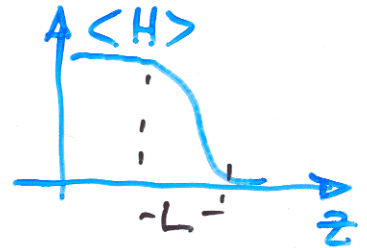
TRANSPORT EQS.

Prob. in phase space

$$\bullet \left(\frac{\partial}{\partial t} + \underbrace{\frac{d\vec{X}}{dt}}_{\vec{V}} \cdot \vec{\nabla}_X + \underbrace{\frac{d\vec{P}}{dt}}_{\text{FORCE}} \cdot \vec{\nabla}_P \right) f(\vec{X}, \vec{P}, t) = \text{COLLISION TERM}$$

BOLTZMANN EQ.

CP-VIOLATING FORCE IN BUBBLE WALL.
WE HAVE "THICK WALL"



$$\bullet L \gg \frac{1}{T}$$

→ EXPECT

WKB APPROXIMATION O.K.
("QUASICLASSICAL")

• CP-VIOLATION IS \hbar -EFFECT (QUANTUM EFFECT)

→ "QUANTUM BOLTZMANN EQS."
BAYM-KADANOFF EQS.

CONSIDER CORRELATORS (GREEN'S FUNCTIONS..)

$$\langle \phi(\vec{x}', t') \phi(\vec{x}, t) \rangle \quad \Omega \leftarrow \text{ENSEMBLE}$$

$$\vec{X} = \frac{\vec{x} + \vec{x}'}{2}, \quad \Delta \vec{x} = \vec{x}' - \vec{x} \xrightarrow{FT} \vec{P} \rightsquigarrow f(\vec{X}, T, \vec{P}, E)$$

$$T = \frac{t + t'}{2}, \quad \Delta t = t' - t \xrightarrow{\text{WIGNER TRANSFORM}} E$$

• "SIMPLE" WITH CP-VIOLATING DIRAC MASS

$$m = |m| e^{i\theta}, \quad m = m(\underline{z}) \rightarrow |m|(z), \theta(z)$$

$$F_z = - \frac{|m|^2}{\dots} + s \left(\frac{|m|^2 \theta'}{\dots} \right)'$$

CP viol.
 $\hbar \frac{d}{dx} \theta$

• MSSM

$$M = \begin{pmatrix} m_2 & g H_2 \\ g H_1 & \mu \end{pmatrix}$$

CP-VIOLATING
PHASES

MASS-MATRIX

IN CHARGED -

HIGGSINO SYSTEM

$$\psi_R = \begin{pmatrix} \tilde{W}_R^+ \\ \tilde{h}_{1,R}^+ \end{pmatrix}, \psi_L = \begin{pmatrix} \tilde{W}_L^+ \\ \tilde{h}_{2,L}^+ \end{pmatrix}$$

NOTE

QUANTUM TRANSPORT EQS. NOW

ALSO USED IN LEPTOGENESIS

* MSSM

T. KONSTANTIN
T. PROKOPEC
H. G. SCH.
H. SECO

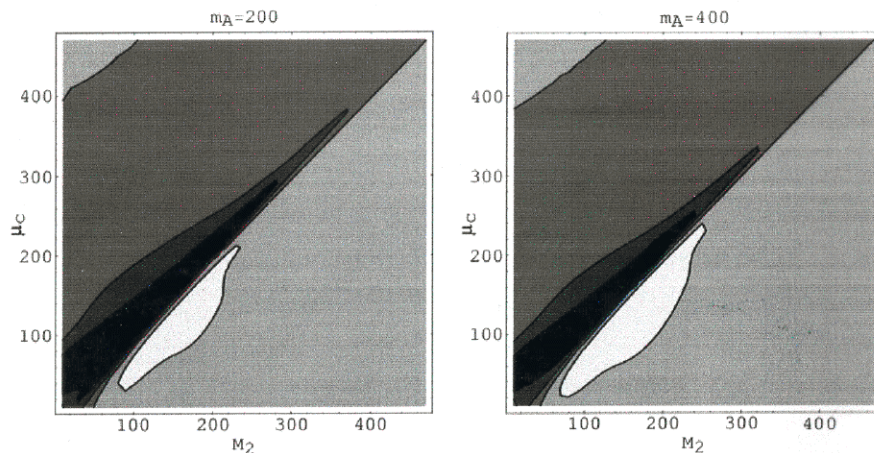


FIG. 5: The baryon-to-entropy ratio $\eta_{10} = 10^{10} \times \eta$ in the (M_2, μ_c) parameter space from $(0 \text{ GeV}, 0 \text{ GeV})$ to $(400 \text{ GeV}, 400 \text{ GeV})$. For the left plot the value $m_A = 200 \text{ GeV}$ is used, for the right plot $m_A = 400 \text{ GeV}$. The black region denotes $\eta_{10} > 1$, where baryogenesis is viable. The other four regions are bordered by the values of η_{10} , $\{-0.5, 0, 0.5, 1\}$, beginning with the lightest color.

MAXIMAL CP-VIOLATION

! RESTRICTIONS BY exp. n/e - ELECTRIC DIPOLE
~ CP-VIOL. PHASE < 0.1

$$|d_e| \leq 1.6 \cdot 10^{-27} \text{ ecm}$$

Regnier et al PRL 88
071805, 2002

LIMITS

ELECTRIC DIPOLE MOMENT FROM MSSM

•16•

The current measurement bound of the electron electric dipole moment (EDM)

$$|d_e| \quad 1.6 \times 10^{-27} \text{ ecm}$$

Regan et al, Phys. Rev. Lett. 88:071805, 2002

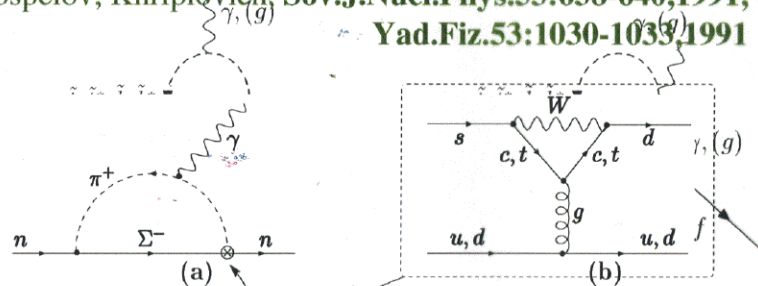
The standard model (MSM) value for eEDM (4 loop)

$$d_e^{\text{CKM}} \quad 1 \times 10^{-38} \text{ ecm}$$

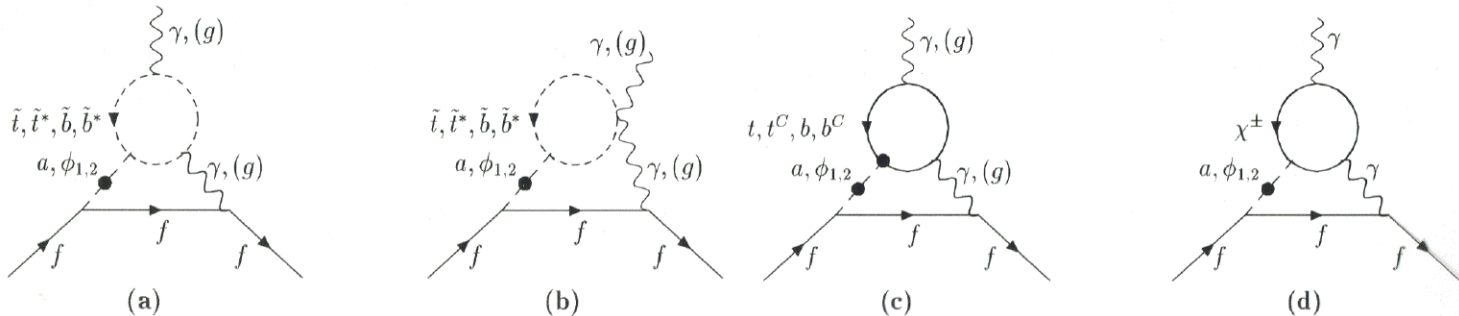
Pospelov, Khriplovich, Sov.J.Nucl.Phys.53:638-640,1991,
Yad.Fiz.53:1030-1033,1991

The standard model (MSM) value
for neutron EDM (2 loop penguin)

$$d_n^{\text{CKM}} \sim 1 \times 10^{-32} \text{ ecm}$$



The MSSM 2 loop Higgs contribution for electron EDM



MSSM

'Superpotential'

$$W = y_{\dots}^{(e)} H_1 \in L \cdot \bar{e} \cdot + y_{\dots}^{(d)} H_1 \in Q \cdot \bar{d} \cdot \\ + y_{\dots}^{(u)} H_2 \in Q \cdot \bar{u} \cdot + \mu H_1 \in H_2$$

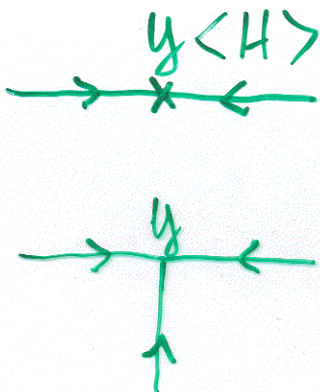
NMSSM

$$W = W_{\text{MSSM}} + \lambda S H_1 \in H_2 - m^2 S + \frac{k}{3!} S^3 \\ (+ \mu H_1 \in H_2)$$

nMSSM

$$W = W_{\text{MSSM}} + \lambda S H_1 \in H_2 - m^2 S$$

+ SUSY BREAKING TERMS (SOFT)

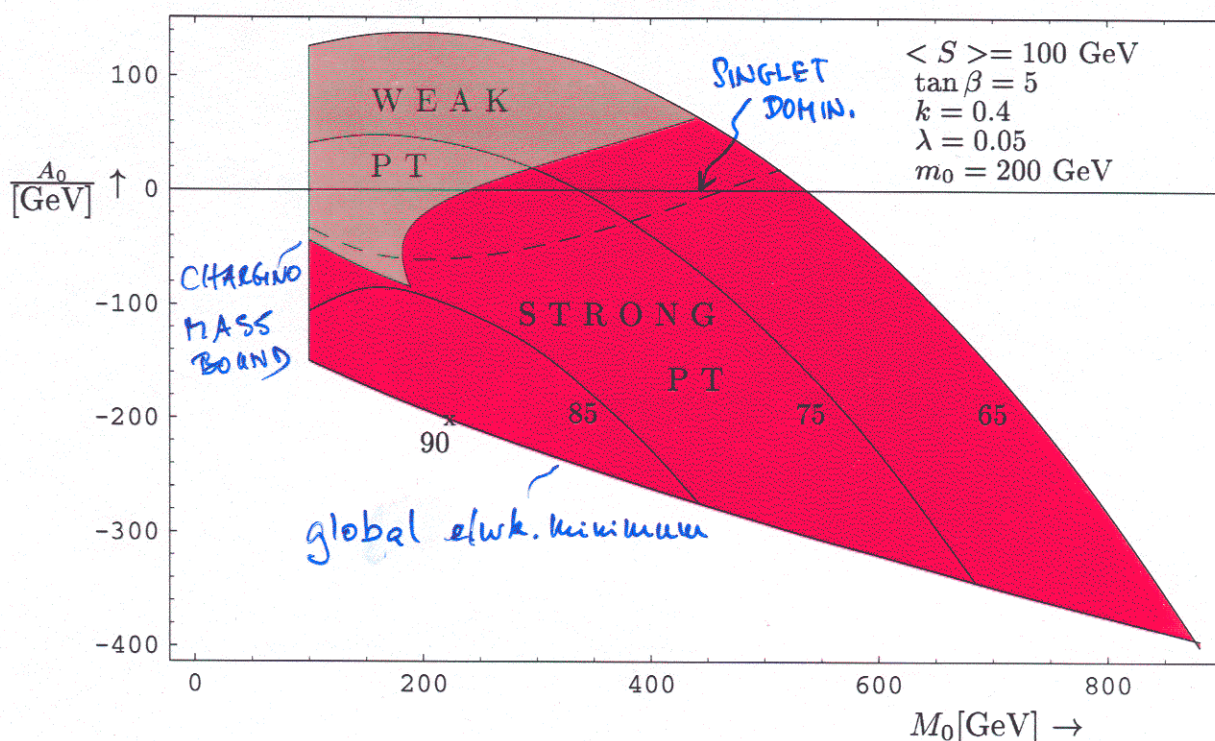


$$H_1 = (H_1^0, H_1^-)$$

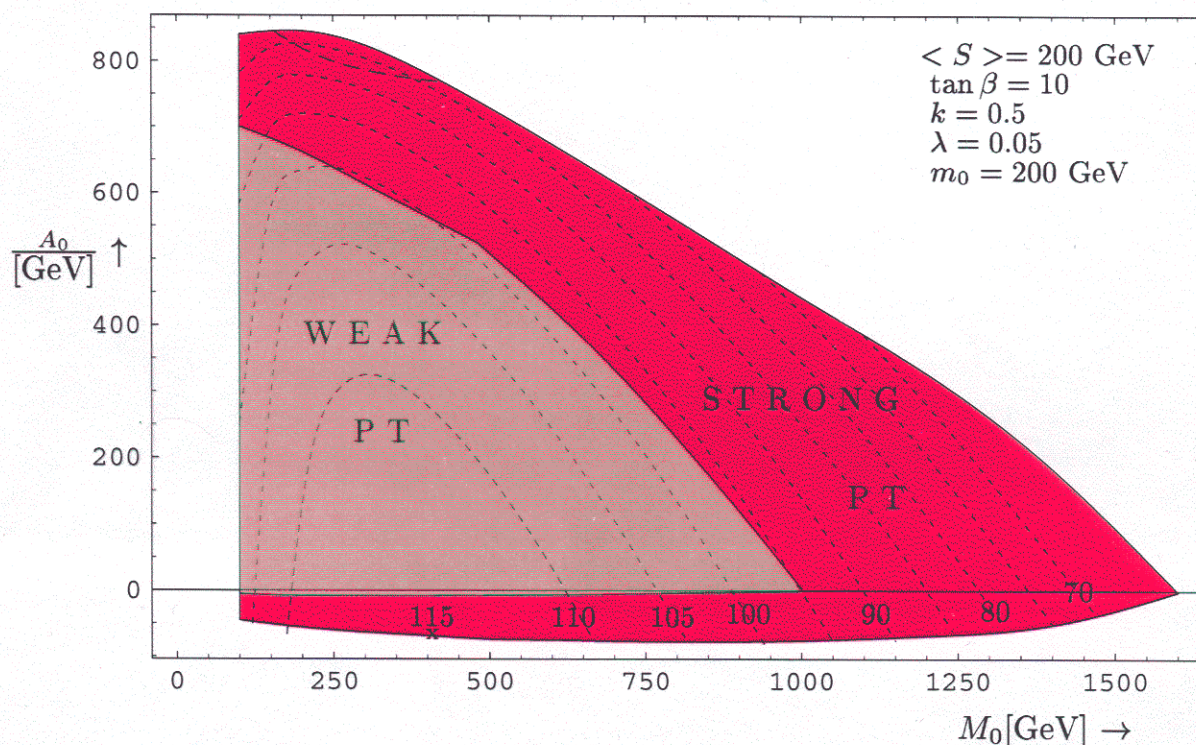
$$H_2 = (H_2^+, H_2^0)$$

NMSSM/RESULTS for strength of the PT:

S. Huber
H.G. Soff.



$\langle S \rangle = 100 \text{ GeV}$



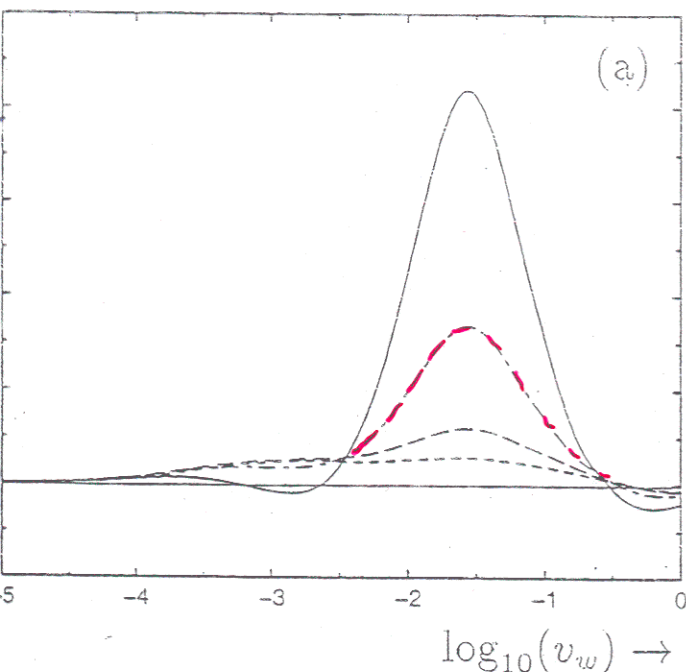
$\langle S \rangle = 200 \text{ GeV}$

$$M_H^{\max} = 115 \text{ GeV} \text{ and } \frac{v_c}{v_e} > 1$$

$$\eta^* = \eta_B / \eta_{B \text{ observed}}$$

NHSSM

S. HUBER
H. POH.



↑
 η^*

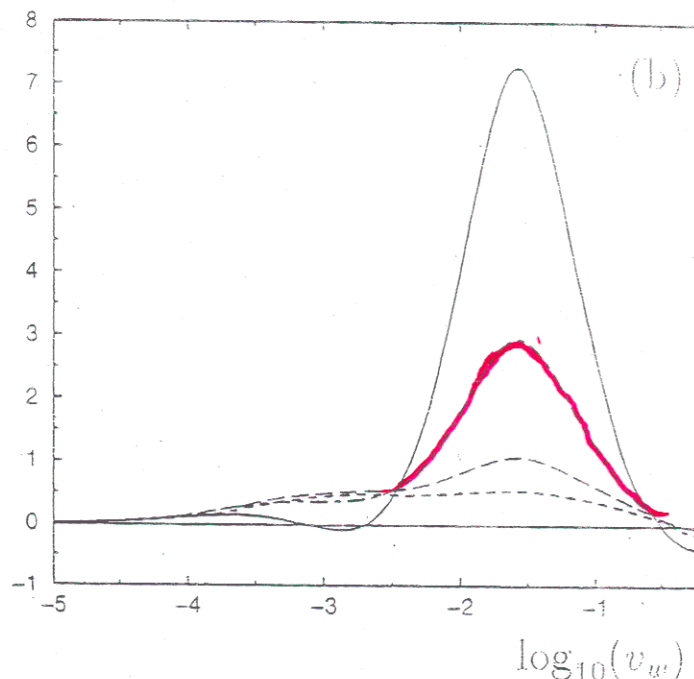
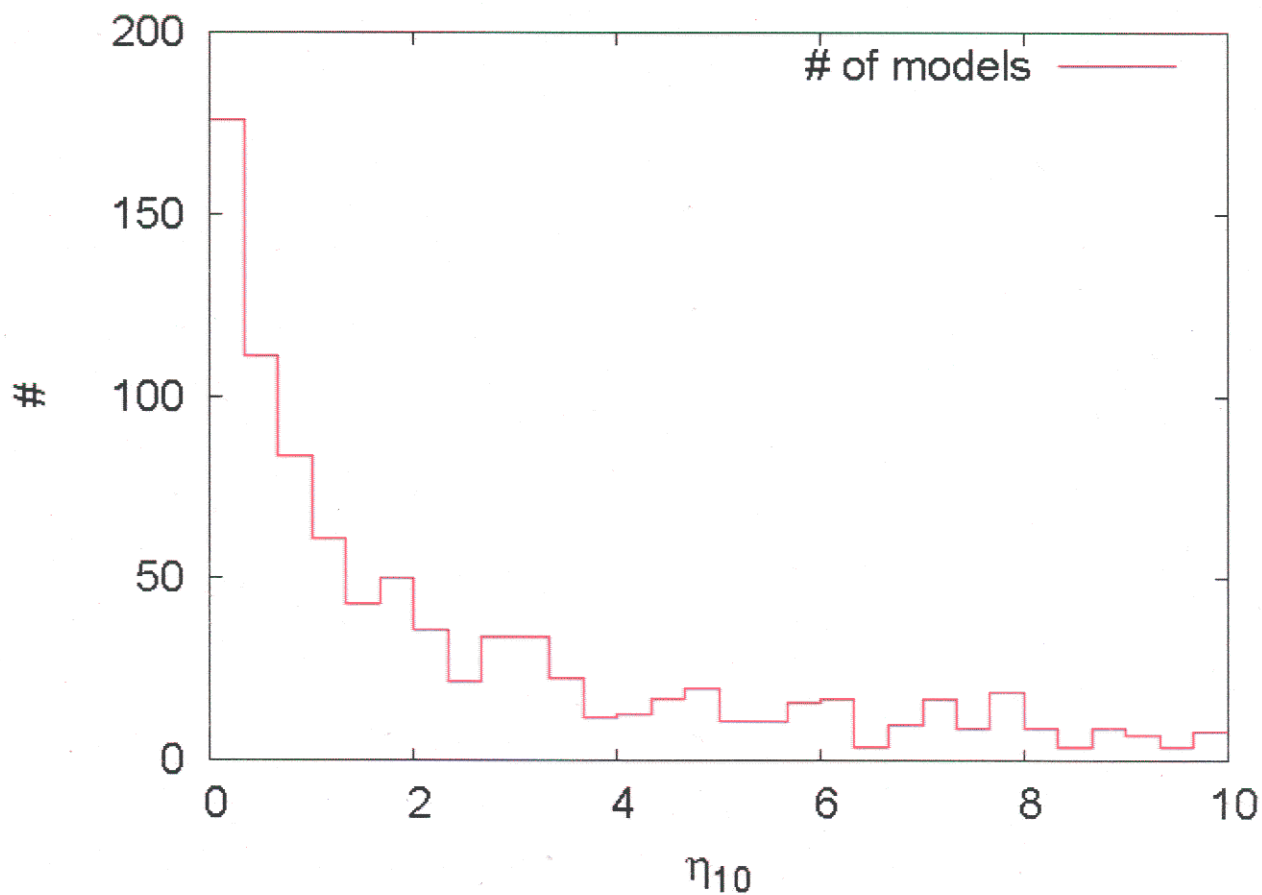


Figure 10: The chargino contribution to the baryon asymmetry in units of 2×10^{-11} as a function of the wall velocity for different values of the wall thickness L , $10/T$, $5/T$, $3/T$ (from below). We use the squark spectrum C and the explicit CP-violation considered in the context of fig. 6. (b): The same quantities for the transitionally CP-violating bubble wall of fig. 7 and the squark spectrum D.

$$\eta_B = \frac{n_B}{n_\gamma} = \frac{135}{g^*} \frac{\Gamma_{\text{SPHALETON}}}{2\pi^2 v_w T} \int_0^\infty d\bar{z} \mu_{B_L}(\bar{z})$$

IN FRONT OF WALL

(η^* IN UNITS OF 2×10^{-11} !)



Produced baryon asymmetry in random nMSSM models.

HUBER '06
KONSTANTIN
PROKOPEC
SCHMIDT

$\eta_{10} \approx 1$ exp.

• MAJORANA NEUTRINOS N

(preparing for leptogen.)

$$\begin{array}{c} N_L \quad N_L \\ \rightarrow \quad \leftarrow \\ M \end{array}$$

$$\mathcal{L} = \dots \frac{1}{2} \bar{N}^c M N$$

"Weyl nota."

* MAJORANA MASS VIOLATES LEPTON NUMBER!

$$\begin{array}{c} m = y \langle H \rangle \\ \rightarrow \quad \leftarrow \\ e_L^- \quad e_L^+ \end{array}$$

$$\mathcal{L} = \dots \bar{e}_R m e_L$$

DIRAC MASS (IN DIRAC NOTATION)

data

$$\Delta m_{21}^2 = m_{sol}^2 = (7.9 \pm 0.3) 10^{-5} \text{ eV}^2$$

$$\Delta m_{32}^2 = m_{atm}^2 = (2.6 \pm 0.2) 10^{-3} \text{ eV}^2$$

DIRAC SPINOR $\begin{pmatrix} \chi_{1L} \\ \chi_{2L}^c \end{pmatrix}$

MAJORANA SPINOR $\begin{pmatrix} \chi_{1L} \\ \chi_{1L}^c \end{pmatrix}$

SEESAW MECHANISM

$$(\nu_L \quad N) \begin{pmatrix} 0 & \overbrace{m}^{y \langle H \rangle} \\ m^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

$$\Rightarrow \text{diagonal.} \quad m_\nu \approx \frac{m^2}{M}$$

$$m_N \sim M$$

• INCLUDING FLAVOUR

$$y_i^{(l)} (H \in L_i) \bar{e}_i$$

$$+ \lambda_{ik} (H^* L_i) N_k$$

$$+ \frac{1}{2} M_i N_i N_i$$

$$H = \begin{pmatrix} H^- \\ H^0 \end{pmatrix}$$

(WEYL NOT.)

[AFTER (PARTIAL) DIAGONALIZATION]

ASSUME: " M_1 " IN THERMAL EQUILIBRIUM
($M_{2,3} \gg M_1$)

* OUT OF EQUILIBRIUM DECAY $\tau_{\text{DECAY}} < H$

HAVE 3 CP-PHASES (ONE OF CKM TYP)

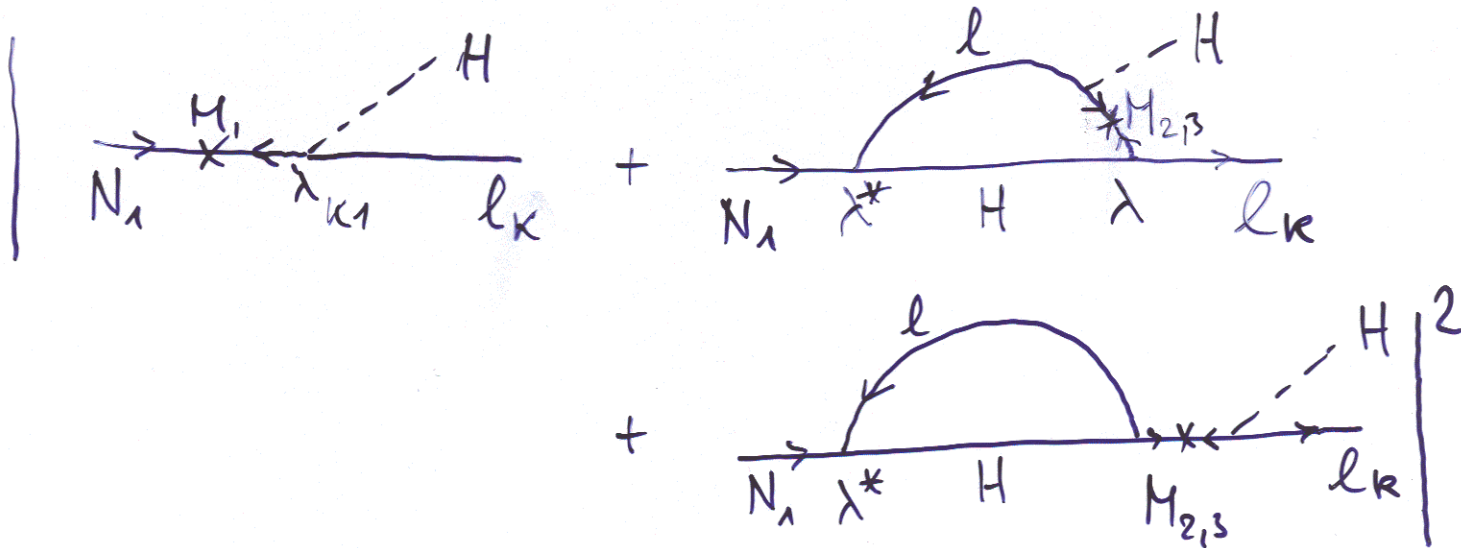
* \Rightarrow CP VIOLATING DECAY

$\tau_{\text{REHEAT}} \gtrsim M_1/5$ (DANGEROUS FOR SUSY
"GRAVITINO PROBLEM")

$$\epsilon_k = \frac{\Gamma(N_1 \rightarrow H + l_k) - \Gamma(N_1 \rightarrow \bar{H} + \bar{l}_k)}{\dots + \dots}$$

W. BUCHMÜLLER
? "DARK MATTER"

NEED TREE/ONE LOOP INTERFERENCE



$$\epsilon_k = \frac{3 M_1}{16 \pi v_H^2 [\lambda^\dagger \lambda]_{11}} \text{Im} \{ [\lambda]_{k1} [m^\dagger \lambda]_{1k} \}$$

RELATED TO NEUTRINO MASS

$$\Gamma_i = \frac{1}{8\pi} (\lambda^\dagger \lambda)_{ii} M_i$$

SPHALERON

LEPTON ASYMMETRY \rightarrow BARYON ASYMMETRY

$M_1 > 10^9 \text{ GeV}$ NEEDED

WASH OUT FOR $\tilde{m}_1 = \frac{v_H^2 (\lambda^\dagger \lambda)_{11}}{M_1} \gtrsim m^\dagger = 10^{-3} \text{ eV}$

BARYON ASYM. THEN IS INDEPENDENT
OF INITIAL N_1 ABUNDANCE AND
BARYON ASSYM.

ALSO: NON THERMAL LEPTOGENESIS

BOLTZMANN EQS. (SIMPLE CASE!)

$$\left[\begin{aligned} \frac{d N_{N_1}}{dz} &= - (D + S) (N_{N_1} - N_{N_1}^{\text{EQUIL.}}) \\ \frac{d N_{B-L}}{dz} &= - \epsilon_1 D (N_{N_1} - N_{N_1}^{\text{EQUIL.}}) - W N_{B-L} \end{aligned} \right]$$

$$z = M_1/T, \quad D = \Gamma_D/Hz, \quad S = \Gamma_S/Hz$$

$$W = \Gamma_W/Hz$$

INCREASING $z \sim$ DECREASING T - INCREASING TIME

$$B = \frac{8 \overset{\text{3 FAM.}}{\textcircled{N_f}} + 4}{22 N_f + 1} (B-L)$$

IN EQUILIBRIUM, SINGLE CHEMICAL POTENTIAL
LEFT

$$T \gg v_H$$

AFFLECK - DINE BARYOGENESIS / LEPTOGENESIS

MSSM \rightarrow SCALAR SUPERPARTNERS
OF LEPTONS AND QUARKS

\rightarrow SCALAR FIELD ϕ_i POTENTIAL

$$V = \underbrace{\sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2}_{\text{F-TERM}} + \underbrace{\frac{1}{2} g^2 \left(\sum_i \phi_i^* T^a \phi_i \right)}_{\text{D-TERM}}$$

+ SUSY BREAKING

e.g. $W = \dots y_i^{(e)} L_i \in H_1 \bar{L}_i$

$\Rightarrow |y_i^{(e)}|^2 \underbrace{|L_i \in H_1|}_{\text{"}\phi\text{"}}$

" ϕ " CARRIES LEPTON NUMBER

FLAT DIRECTION WITHOUT EXPANSION OF
UNIVERSE AND SUSY - BREAKING

" ϕ - CONDENSATE" ROLLING DOWN POTENTIAL

$$\ddot{\phi}(t) + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$\left(H = \frac{1}{2t} \right.$$

IF RAD. DOM.)

START $V = m^2 |\phi|^2$

(i) $H \gg m$ STRONG DAMPING

(ii) $H \ll m$ ϕ -OSCILLATION OF $\text{RE}[\partial_m \phi]$
 ϕ -SHRINKING IN TIME

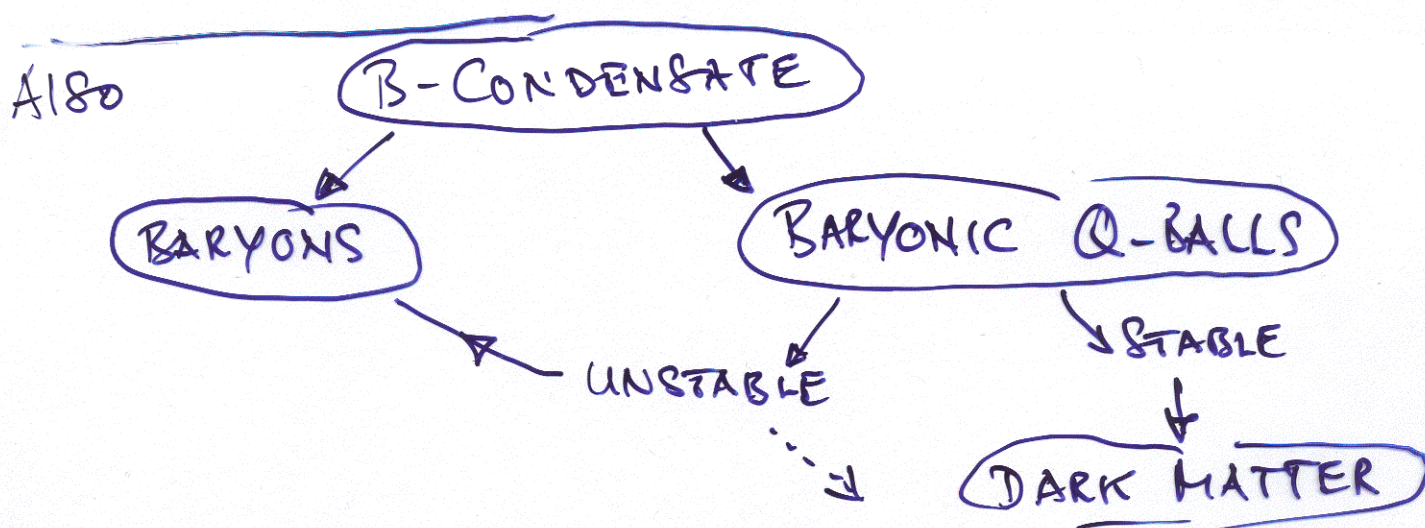
(iii) INTERACTION TERMS $\overbrace{CP\text{-VIOLATING}}$
 $|\phi|^4 + b \phi^3 \phi^* + c \phi^4 + \text{c.c.}$

GIVE PERTURBATIONS, CP-VIOLATING EFFECT (VIOL. OF "ANGULAR MOMENTUM")

FOR A LIMITED TIME CREATES LEPTON NUMBER $\sim i \phi^* \partial_0 \phi$

(iv) CONDENSATE (HERE Lept.) DESTABILIZES (BARYONIC IN OTHER TERMS)
 DECAY INTO LEPTONS WITH ASYMM.

→ SPHALERON BARYON ASYMMETRY



• COHERENT BARYOGENESIS

B. GARBRECHT
T. PROKOPEC
H.B. SCH.

- SCALAR FIELD CONDENSATE INDUCES TIME DEPENDENT MASS (MATRIX) IN COSMOLOGY ← WITHOUT B/L CHARGE!
- COHERENT PARTICLE PRODUCTION BY NON ADIABATICAL TIME DEPENDENCE OF SCALAR COND.
- CERTAIN CHARGE NUMBERS TRANSFORMED TO B-L
- ~~CP~~ OF MASS MATRIX \Rightarrow ASYMMETRY \Rightarrow BARYON ASYM.

FRAMEWORK
AGAIN!

CONSIDER "QUANTUM BOLTZMANN EQS."

(SCHWINGER-KELDISH CTP...)

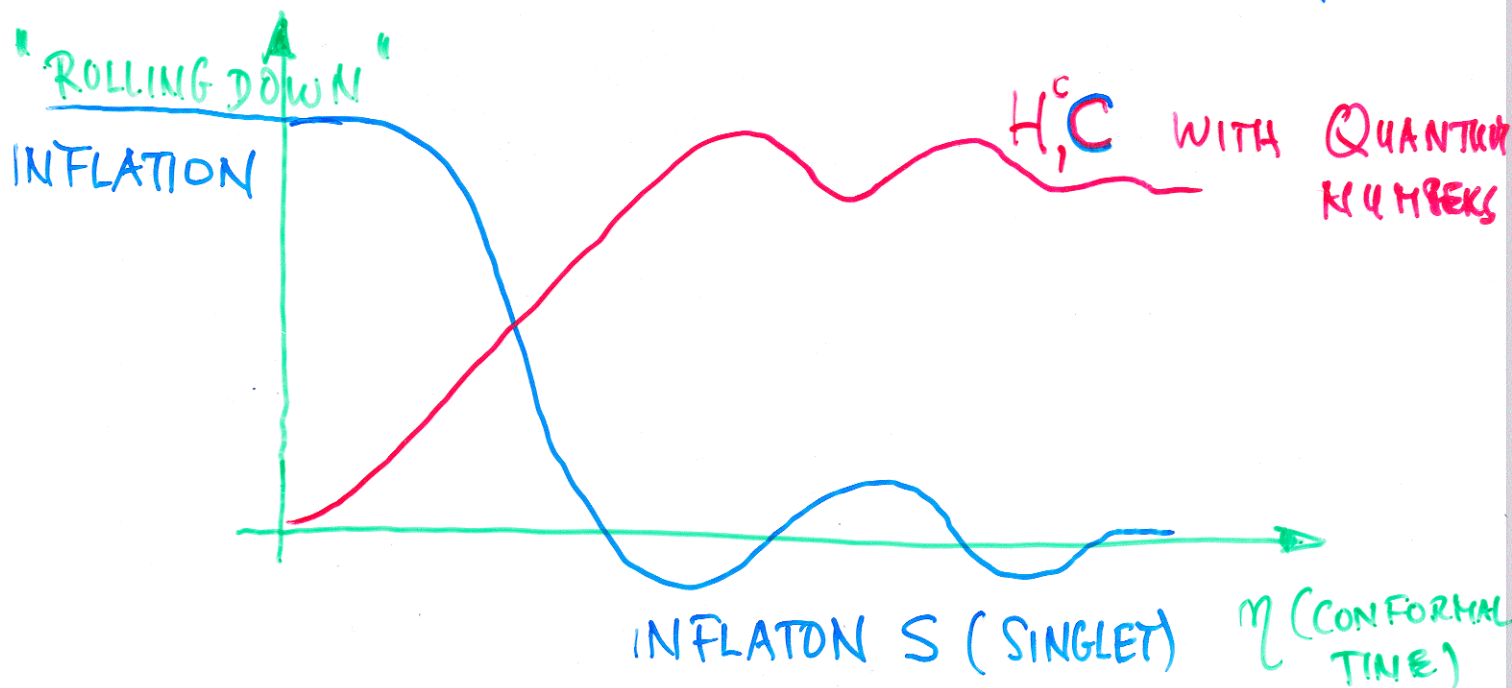
\Rightarrow MATRIX-EQS.

FOR FERMIONS / BOSONS

$$\boxed{Z \leftrightarrow t}$$

• APPLICATION: HYBRID INFLATION (SUPER SYM.)

→ FIG.



GUT - WATERFALL

EXAMPLES:

- PATI - SALAM : $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$
 \downarrow "COLOR" \downarrow
 $SU(3)_C \times U(1) \quad U(1)$
 $\underbrace{\hspace{10em}}_{U(1)_Y}$
- $SO(10) \rightarrow SM$

$$W_{\text{SUPERPOT.}} \supset K S (\bar{H}^c H^c - \mu^2) + \dots \quad \text{CP-VIOL. COUPLINGS}$$

$$H^c = (\bar{4}, 1, 2)$$

PATI-SALAM

$$C = [16]$$

SO(10)

HYBRID INFLATION

1

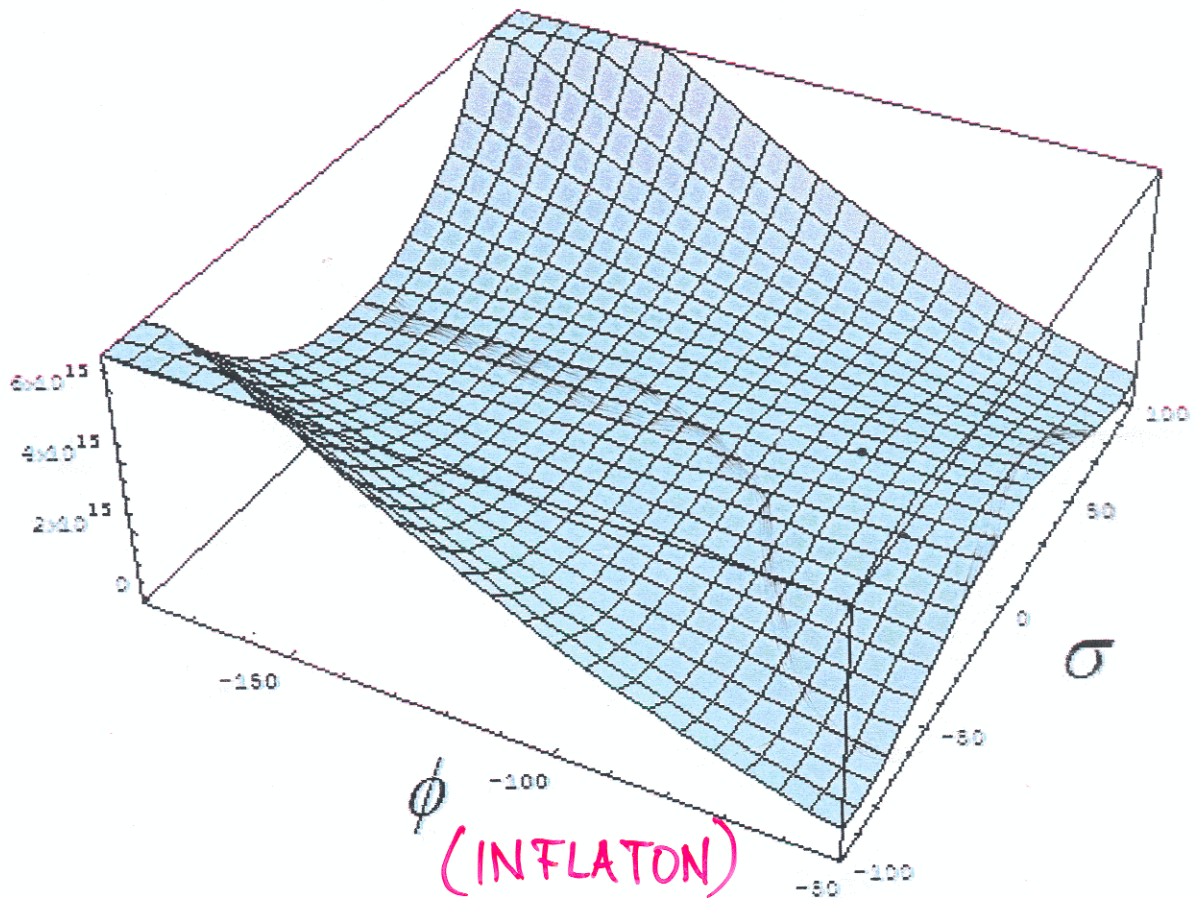
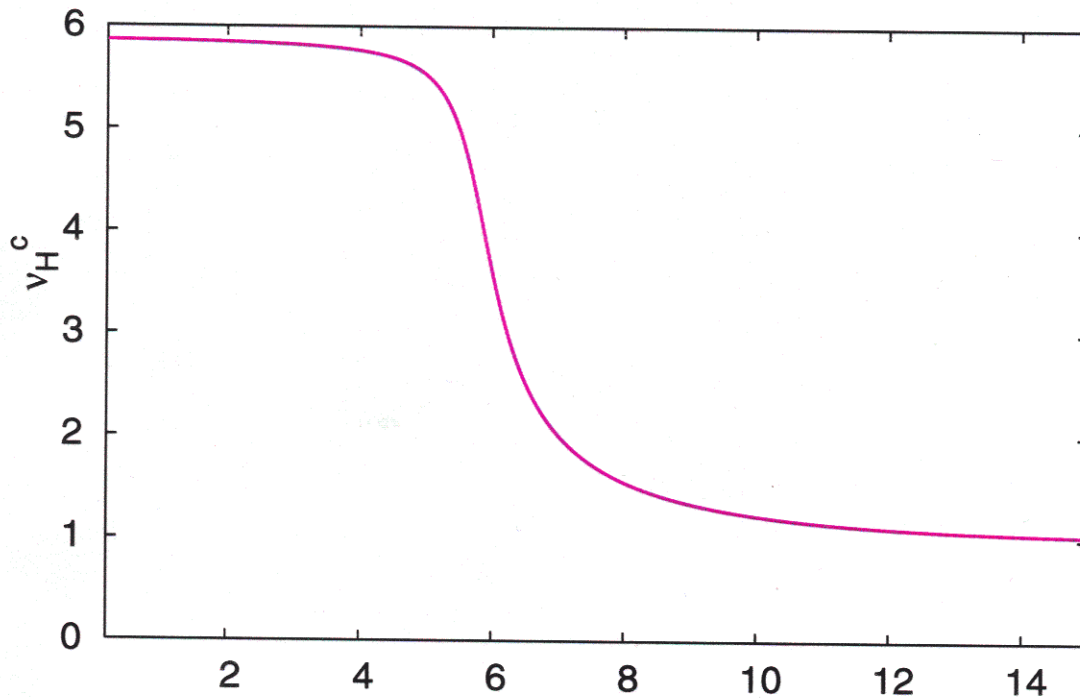


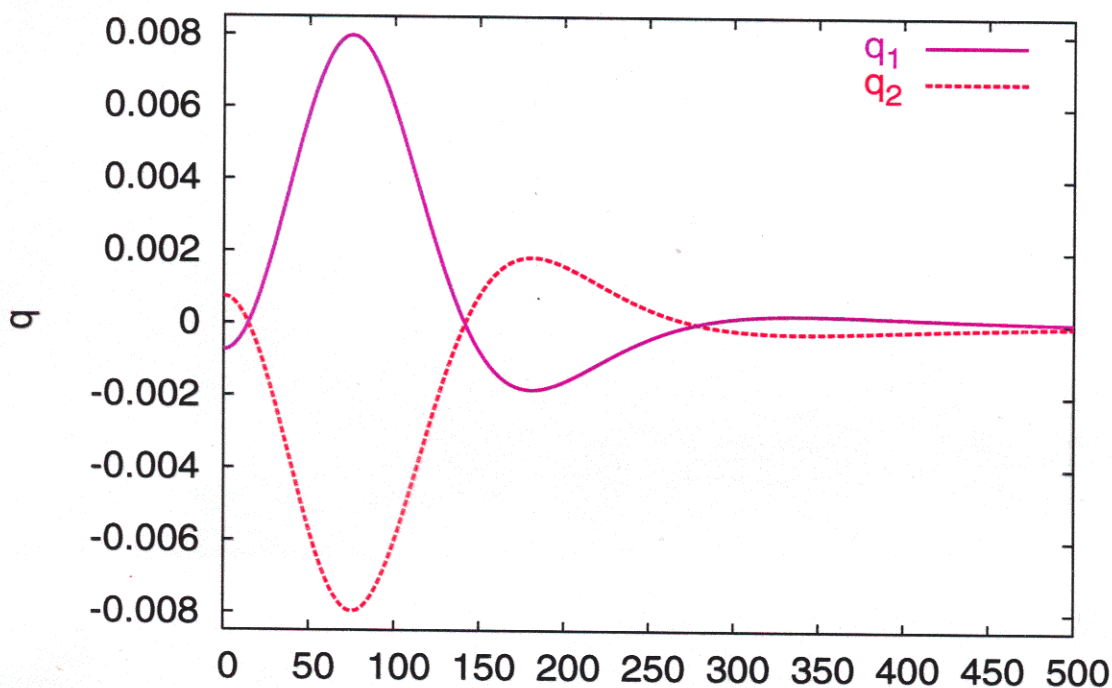
Figure 1: Hybrid Potential, using $m_{pl} = 10^9$, $\lambda = 10^4$, $g = 8 \cdot 10^3$, $m = 1.5 \cdot 10^{-5} m_{pl}$, and $M = 10^{-3} \cdot m_{pl}$.

ADISORN ADULPRAMTHAI

COHERENT BARYOGENESIS IN HYBRID INFLATION



Evolution of the field ν_H^c (η conformal time).



The produced charges of the Dirac fermions χ_{1j} , χ_{2j} , summed over both helicities.

$\kappa = 0.007$	$\mu = 2.0 \times 10^{16} \text{ GeV}$	$\zeta = 0.12i$	$M_S = 50\mu$
$\beta = 1$		$\xi = 0.12$	$\Gamma = 0.1\mu$

REALISTIC
PARAMETERS
OF SENOGUZE
SCHAFFER-INFLATION

• COUPLINGS

(i) $\gamma \underbrace{F^c \bar{H}^c F^c \bar{H}^c}_{\text{SINGLET}} / M_S$

→ MAJORANA MASS FOR ν

$\chi_1 \Rightarrow d_{\bar{H}}^c \xrightarrow{\langle \nu_{\bar{H}}^c \rangle} d + \nu_{\text{MAJORANA}}$

SCALAR FERMION

NO ~~CP~~ DECAY
REQUIRED AS
IN LEPTOGENESIS!
(WITH TREE-ONE LOOP
INTERFERENCE)

(ii) $\delta \underbrace{F^c \bar{H}^c F^c \bar{H}^c}_{\epsilon''} / M_S$

$\chi_2 \Rightarrow \bar{d}_H^c \xrightarrow{\langle \nu_H^c \rangle} \bar{u} \bar{d}$

$\leadsto B-L = -\frac{2}{3} Q_2 + \frac{1}{3} Q_1 = Q_1$

AFTER SPHALERON PROCESSES

$B = \frac{10}{31} (B-L)$

ESTIMATE

VACUUM ENERGY $\rho = \left[k^2 \frac{M_S^2}{4\beta} - k\mu^2 \right]^2 \sim \pi^2 g^* T_R^4 / 30$

$S < 2\pi^2 g^* T_R^3 / 45$

$B/n_\gamma > 10^{-10}$ EASILY

DETAILED REHEATING
CALCULATION

- NONTHERMAL LEPTOGENESIS (IN SAME MODEL)

$\langle \nu_H^c \rangle \rightarrow$ MAJORANA NEUTRINO MASS AFTER PREHEATING
 LIGHTEST MASS $M_1 = 3.9 \times 10^{10} \text{ GeV}$
 COMPARE $T_R = 2.7 \cdot 10^9 \text{ GeV}$ } **NONTHERMAL!**

MAXIMAL MIXING AND CP VIOLATION VIA
 1-LOOP INTERFERENCE

$$\frac{n_L}{s} \leq 3 \cdot 10^{-10} \frac{T_R}{m_{\nu_H}^c} \left(\frac{M_1}{10^6 \text{ GeV}} \right) \left(\frac{m_{\nu 3}}{0.04 \text{ eV}} \right) \approx 8 \times 10^{-11}$$

$$\Rightarrow \frac{n_B}{s} \lesssim 3 \times 10^{-11} \quad \text{SHALER!} \quad (\text{SENOGU\AA} / \text{SHAFI})$$

COLD ELECTROWEAK BARYOGENESIS

ALMOST THE SM

- $\mu^2 |H|^2 \Rightarrow -\mu^2 |H|^2$

$$\langle H \rangle = 0 \Rightarrow \langle H \rangle \neq 0$$

IN LOW SCALE (TeV) HYBRID INFLATION

→ REHEATING TEMP. SHALL \ll ELWK SCALE
(100 GeV)

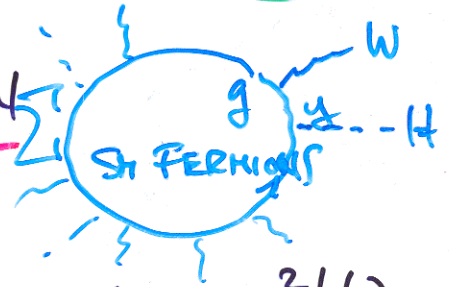
- CP VIOLATING TERM MUCH BIGGER THAN "JARLSKOG DETERMINANT" IN THE EFFECTIVE ACTION !!

INTEGRATE OUT QUARKS

2. ORDER IN DERIVATIVE EXPANSION
(→ "WORLD LINE METHOD")

INCLUDING CKM - CP - VIOLATION

A. HERNANDEZ
T. KONSTANTIN
H.G. SCH.



- DO CLASSICAL EVOLUTION WITH $\mu^2(t)$ IN EFFECTIVE THEORY INCLUDING CP-VIOLATING TERM ON LATTICE.
MEASURE BARYON NUMBER VIOLATION INSPECTING

$$N_{CS}(t) - N_{CS}(0) = \frac{1}{16\pi^2} \int dt \int d^3x \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

AND "HIGGS - WINDING" N_W

STATIONARY CASE (NO DERIVATIVES OF FIELDS)

⇒ "JARLSKOG DETERMINANT" RULES ~~CP~~

$$\Delta_{CP} = \boxed{J} \prod \frac{\Delta \tilde{m}_u^2}{v^2} \prod \frac{\Delta \tilde{m}_d^2}{v^2} \approx \underline{\underline{10^{-19}}}$$

BLWK SCALE DIAGONALIZED

$$C = U^\dagger D \quad \text{CKM MATRIX}$$

$$\boxed{J} = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin \delta \approx (3.0 \pm 0.3) \times \underline{\underline{10^{-5}}}$$

$$S_{CP} = \frac{1}{8(4\pi)^2} \frac{3}{16} \boxed{J} K^{CP} \epsilon^{\mu\nu\lambda\sigma} \int d^4x \left(\sum_\mu W_{\nu\lambda}^+ W_{\sigma}^- \times \right. \\ \left. \times (W_\sigma^+ W_\lambda^- + W_\lambda^+ W_\sigma^-) + c.c. \right)$$

$\sim \tilde{m}_c^2$

$$K^{CP} \approx 9.87 \quad \text{IN "BROKEN" PHASE}$$

$$\boxed{\frac{K^{CP}}{\tilde{m}_c^2}} \approx \frac{32}{9\tilde{m}_c^2 (\tilde{m}_c^2 - \tilde{m}_s^2)^3 (\tilde{m}_c^2 - \tilde{m}_t^2)^3 (\tilde{m}_s^2 - \tilde{m}_t^2)^2} \times$$

$$\left(\tilde{m}_s^6 \tilde{m}_t^6 (\tilde{m}_s^2 - \tilde{m}_t^2)^2 + 3\tilde{m}_c^{14} (\tilde{m}_s^2 + \tilde{m}_t^2) \right.$$

$$- 5\tilde{m}_c^2 \tilde{m}_s^4 \tilde{m}_t^4 (\tilde{m}_s^2 - \tilde{m}_t^2)^2 (\tilde{m}_s^2 + \tilde{m}_t^2) - 12\tilde{m}_c^{12} (\tilde{m}_s^4 + \tilde{m}_t^4)$$

$$+ \tilde{m}_c^4 \tilde{m}_s^2 \tilde{m}_t^2 (\tilde{m}_s^2 - \tilde{m}_t^2)^2 (13\tilde{m}_s^4 + 28\tilde{m}_s^2 \tilde{m}_t^2 + 13\tilde{m}_t^4) + 18\tilde{m}_c^{10} (\tilde{m}_s^6 + \tilde{m}_t^6)$$

$$+ \tilde{m}_c^8 (-12\tilde{m}_s^8 + 37\tilde{m}_s^6 \tilde{m}_t^2 - 74\tilde{m}_s^4 \tilde{m}_t^4 + 37\tilde{m}_s^2 \tilde{m}_t^6 - 12\tilde{m}_t^8)$$

$$\left. + \tilde{m}_c^6 (3\tilde{m}_s^{10} - 41\tilde{m}_s^8 \tilde{m}_t^2 + 41\tilde{m}_s^6 \tilde{m}_t^4 + 41\tilde{m}_s^4 \tilde{m}_t^6 - 41\tilde{m}_s^2 \tilde{m}_t^8 + 3\tilde{m}_t^{10}) \right)$$

$$- \frac{64\tilde{m}_c^4 \tilde{m}_s^2 \tilde{m}_t^2 (\tilde{m}_c^2 - \tilde{m}_t^2) (\tilde{m}_c^2 - 3\tilde{m}_s^2 + 2\tilde{m}_t^2) \log \left[\frac{\tilde{m}_s^2}{\tilde{m}_c^2} \right]}{3(\tilde{m}_c^2 - \tilde{m}_s^2)^4 (\tilde{m}_s^2 - \tilde{m}_t^2)^3}$$

$$+ \frac{64\tilde{m}_c^4 \tilde{m}_s^2 (\tilde{m}_c^2 - \tilde{m}_s^2) \tilde{m}_t^2 (\tilde{m}_c^2 + 2\tilde{m}_s^2 - 3\tilde{m}_t^2) \log \left[\frac{\tilde{m}_t^2}{\tilde{m}_c^2} \right]}{3(\tilde{m}_c^2 - \tilde{m}_t^2)^4 (\tilde{m}_s^2 - \tilde{m}_t^2)^3}$$

ALREADY SIMPLIFIED!

$$\tilde{m}_b^2 = \tilde{m}_c^2$$

COMPUTER ALGEBRA

"NONPERTURBATIVE"
IN \tilde{m}' 's

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