

Rare B Decays and all that ...

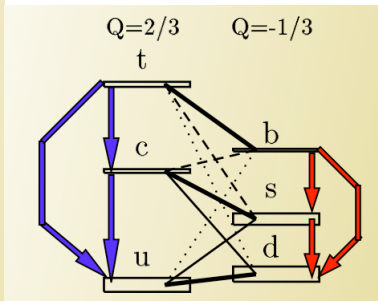
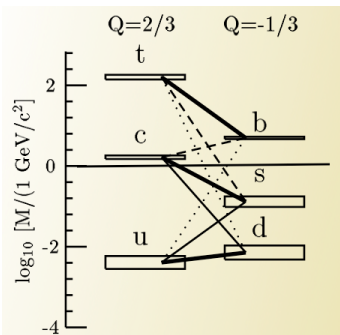
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Nekarzimmern 2008

Preliminary Remarks

- **Rare Decays:** Transitions are suppressed by either
 - **Small (combinations of) CKM Matrix elements** or
 - **Loop factors $1/(16\pi^2)$** or
 - **both**



Outline of the event:

- Part 1: Rare decays in the Standard Model (TM)
- Part 2: Effective Hamiltonian (TF)

Rare B Decays

Introduction and Standard Model

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Outline Part 1

- 1 Quarks in the SM: $SU(2)_L \times U(1)_Y$
 - Symmetries and Quantum Numbers
 - Quark Mixing and CKM Matrix
- 2 Peculiarities of Flavour in the Standard Model
 - Peculiarities of SM CP / Flavour
- 3 Rare Decays: Small CKM Elements
- 4 Rare Decays: Loop Suppressed FCNC Decays

Gauge Structure of the Standard Model

I assume a few things to be known:

- The Standard Model is a gauge theory based on $SU(3)_{QCD} \otimes SU(2)_{Weak} \otimes U(1)_{Hypercharge}$
- Eight gluons, three weak gauge bosons, one photon
- Matter (quarks and leptons):
Multiplets of the gauge group \rightarrow Quantum numbers
- Spontaneous Symmetry Breaking:
Introduction of scalar fields
- Massless Goldstone Modes:
Higgs Mechanism:
 $\phi \rightarrow$ longitudinal modes of gauge bosons: $\phi \sim \partial_\mu W^\mu$

Matter Fields: Quarks

- Left Handed Quarks:

$SU(3)_C$ Triplets, $SU(2)_L$ Doublets

$$Q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad Q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$SU(2)_L$ will be gauged

- Right Handed Quarks:

$SU(3)_C$ Triplets, $SU(2)_R$ Doublets

$$q_1 = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad q_2 = \begin{pmatrix} c_R \\ s_R \end{pmatrix} \quad q_3 = \begin{pmatrix} t_R \\ b_R \end{pmatrix}$$

$SU(2)_R$ introduced “artificially”

Quantum Numbers

- Hypercharge

$$Y = T_{3,R} + \frac{1}{2}(B - L)$$

- Charge

$$q = T_{3,L} + Y = T_{3,L} + T_{3,R} + \frac{1}{2}(B - L)$$

Higgs Fields: Standard Model

- Single $SU(2)$ Doublet: Two Complex Fields

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

- Charge Conjugate Field is also an $SU(2)$ Doublet

$$\tilde{\Phi} = (i\tau_2)\Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- = -\phi_+^* \end{pmatrix}$$

- It is useful to gather these into a 2×2 matrix

$$H = \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_- & \phi_0 \end{pmatrix}$$

- Transformation Properties: $L \in SU(2)_L$:

$$\Phi \rightarrow L\Phi \quad \tilde{\Phi} \rightarrow L\tilde{\Phi}$$

- Transformation Properties: $R \in SU(2)_R$:

$$\begin{pmatrix} \phi_0 \\ \phi_- \end{pmatrix} \rightarrow R \begin{pmatrix} \phi_0 \\ \phi_- \end{pmatrix} \quad \begin{pmatrix} \phi_+ \\ -\phi_0^* \end{pmatrix} \rightarrow R \begin{pmatrix} \phi_+ \\ -\phi_0^* \end{pmatrix}$$

- In total:

$$H \rightarrow LHR^\dagger \quad (\text{remember } Q \rightarrow LQ \quad q \rightarrow Rq)$$

- Hypercharges

$$Y\Phi = -\frac{1}{6} \quad Y\tilde{\Phi} = \frac{1}{6} \quad YH = -\frac{1}{2}$$

Gauge Interactions

- $SU(3)_{color}$ is gauged (not relevant for us now)
- $SU(2)_L$ is gauged **Three W_a^μ Bosons**
- Hypercharge is gauged **One B^μ Boson**
- Recipe: Replace the ordinary derivative in the kinetic terms by the covariant one

$$\partial^\mu \rightarrow D^\mu = \partial^\mu - igT_{L,a}W_a^\mu - iYB^\mu$$

+QCD interactions

- Weinberg rotation between W_3^μ and $B^\mu \dots$
- I assume you have heard the rest of the story ...
- **This is not relevant for the phenomenon of masses and mixing !**

Structure of the Standard Model

- Start out from an $SU(2)_L \times SU(2)_R$ symmetric case:
- Kinetic Term for Quarks and Higgs (i : Generation)

$$\mathcal{L}_{kin} = \sum_i [\bar{Q}_i \not{\partial} Q_i + \bar{q}_i \not{\partial} q_i] + \frac{1}{2} \text{Tr} [(\partial_\mu H)^\dagger (\partial^\mu H)]$$

- Potential for the Higgs field

$$V = V(H) = V(\text{Tr} [H^\dagger H])$$

- Interaction between Quarks and Higgs

$$\mathcal{L}_I = - \sum_{ij} y_{ij} \bar{Q}_i H q_j + \text{h.c.}$$

- y_{ij} can be made diagonal: Any Matrix y can be diagonalized by a **Bi-Unitary Transformation**:

$$y = U^\dagger y_{diag} W$$

- Thus

$$\mathcal{L}_l = - \sum_{ijk} \bar{Q}_i (U^\dagger)_{ik} y_k W_{kj} H q_j + \text{h.c.}$$

- Rotation of Q_i and q_j :

$$Q' = UQ \quad q' = Wq$$

- This has no effect on the kinetic term:
 $y_{ij} = y_i \delta_{ij}$ is the general case!

$$\mathcal{L}_l = - \sum_i y_i \bar{Q}_i H q_i + \text{h.c.}$$

Spontaneous Symmetry Breaking

- The Higgs Potential is (Renormalizability):

$$V = \kappa (\text{Tr} [H^\dagger H]) + \lambda (\text{Tr} [H^\dagger H])^2$$

- For $\kappa < 0$ we have SSB:

H acquires a Vacuum Expectation Value (VEV)

$$\text{Tr} [\langle H^\dagger \rangle \langle H \rangle] = -\frac{\kappa}{2\lambda} > 0$$

- Choice of the VEV

$$\langle \text{Re} \phi_0 \rangle = v \text{ or } \langle H \rangle = v \mathbf{1}_{2 \times 2}$$

- Three massless fields: ϕ_+ , ϕ_- , $\text{Im}\phi_0$:
Goldstone Bosons
- $\phi_0 \rightarrow v + \phi'_0$: One massive field
- Higgs Mechanism: **The massless scalars become the longitudinal modes of the massive vector bosons:**
 - * $\phi_{\pm} \sim \partial^{\mu} W_{\mu}^{\pm}$
 - * $\text{Im}\phi_0 \sim \partial^{\mu} Z_{\mu}$
- ϕ'_0 : Physical Higgs Boson

- The Quarks become massive:

$$\mathcal{L}_I = - \sum_i y_i v \bar{Q}_i q_i + \text{h.c.} + \dots$$

- We have $\bar{Q}_1 q_1 = \bar{u}_L u_R + \bar{d}_L d_R$ etc.
- Thus

$$\mathcal{L}_{mass} = -m_u(\bar{u}u + \bar{d}d) - m_c(\bar{c}c + \bar{s}s) - m_t(\bar{t}t + \bar{b}b)$$

- This is not (yet) what we want ...
- We still have too much symmetry!

Custodial $SU(2)$

- Symmetry of the Higgs Sector in the Standard Model:

$$SU(2)_L \otimes SU(2)_R \xrightarrow{SSB} SU(2)_{L+R} = SU(2)_C$$

- Note that we cannot have explicit breaking of $SU(2)_R$ in the Higgs sector:

$$\text{Tr} [H\tau_i H^\dagger] = 0$$

- $SU(2)_C$: Custodial Symmetry!
→ Extra Symmetry in the Higgs sector !
- This is more than needed: Only $U(1)_Y$ is needed
- $U(1)_Y$ will be related to the τ_3 direction of $SU(2)_R$

- Consequences of $SU(2)_C$:
 - Relation between charged and neutral currents:
 ρ parameter
 - Masses of W^\pm and of Z^0 are equal
 - Up- and Down-type quark masses are equal in each family
 - No mixing occurs among the families
- $SU(2)_C$ is broken by:
 - Yukawa Couplings
 - Gauging only the Hypercharge

$$Y = T_3^{(R)} + \frac{1}{2}(B - L)$$

Breaking $SU(2)_C$: Yukawa Couplings

- Explicit breaking of $SU(2)_C$ by Yukawa Couplings:

$$\mathcal{L}'_I = - \sum_{ij} y'_{ij} \bar{Q}_i H(2T_{3,R}) q_j + \text{h.c.}$$

- Effect of this term:
 - Introduces a splitting between up- and down quark masses
 - Introduces mixing between different families
 - Affects the ρ parameter
- Total Yukawa Coupling term:

$$\mathcal{L}_I + \mathcal{L}'_I = - \sum_{ij} \bar{Q}_i H(y_i \delta_{ij} + 2T_{3,R} y'_{ij}) q_j + \text{h.c.}$$

Quark Mass Matrices

- Use the projections

$$P_{\pm} = \frac{1}{2} \pm T_{3,R} \quad \left(\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \text{ or } \left(\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right)$$

- Up quark Yukawa couplings:

$$\mathcal{L}_{mass}^u = - \sum_{ij} \bar{Q}_i H (y_i \delta_{ij} + y'_{ij}) P_+ q_j + \text{h.c.}$$

- Down quark Yukawa couplings:

$$\mathcal{L}_{mass}^d = - \sum_{ij} \bar{Q}_i H (y_i \delta_{ij} - y'_{ij}) P_- q_j + \text{h.c.}$$

- \rightarrow mass terms, **once $\text{Re}\phi_0 \rightarrow v$**

- More compact notation

$$\mathcal{U}_{L/R} = \begin{bmatrix} u_{L/R} \\ c_{L/R} \\ t_{L/R} \end{bmatrix} \quad \mathcal{D}_{L/R} = \begin{bmatrix} d_{L/R} \\ s_{L/R} \\ b_{L/R} \end{bmatrix}$$

- Mass Term for Up-type quarks

$$\mathcal{L}_{mass}^u = -v \bar{u}_L Y^u u_R + \text{h.c.}$$

with $Y^u = (y + y')$

- Mass Term for down-type quarks

$$\mathcal{L}_{mass}^d = -v \bar{d}_L Y^d d_R + \text{h.c.}$$

with $Y^d = (y - y')$

- **Mass matrices:**

$$\mathcal{M}^u = v Y^u \quad \mathcal{M}^d = v Y^d$$

- In general non-diagonal: Diagonalization by a bi-unitary transformation:

$$\mathcal{M} = U^\dagger \mathcal{M}_{diag} W$$

- New basis for the quark fields

$$\mathcal{L}_{mass}^u = -\bar{U}_L U^{u,\dagger} \mathcal{M}_{diag}^u W^u U_R + \text{h.c.}$$

and

$$\mathcal{L}_{mass}^d = -\bar{D}_L U^{d,\dagger} \mathcal{M}_{diag}^d W^d D_R + \text{h.c.}$$

Quark Mixing: The CKM Matrix

- Effect of the basis transformation:
 - Mass matrices become diagonal
 - Interaction with $\text{Re } \phi_0$ (= Physical Higgs Boson) becomes diagonal !
 - Interaction with $\text{Im } \phi_0$ (= Z_0) becomes diagonal !

$$\mathcal{L}_{\text{Re } \phi_0} = -\text{Re } \phi_0 [\mathcal{U}_L Y^u \mathcal{U}_R + \mathcal{D}_L Y^d \mathcal{D}_R]$$

$$\mathcal{L}_{\text{Im } \phi_0} = -\text{Im } \phi_0 [\mathcal{U}_L Y^u \mathcal{U}_R - \mathcal{D}_L Y^d \mathcal{D}_R]$$

- **NO FLAVOUR CHANGING NEUTRAL CURRENTS**
(at tree level in the Standard Model)
- \rightarrow GIM Mechanism

- Effect on the charged current ONLY:
Interaction with ϕ_- :

$$\begin{aligned} & \sum_{ij} \bar{Q}_i (y_i \delta_{ij} + y'_{ij}) \phi_- \tau_- P_+ q_j + \text{h.c.} \\ &= \mathcal{D}_L Y^u \mathcal{U}_R \phi_- + \text{h.c.} \\ &= \bar{\mathcal{D}}_L U^{d,\dagger} (U^d U^{u,\dagger}) Y_{diag}^u W^u \mathcal{U}_R \phi_- + \text{h.c.} \end{aligned}$$

- In the charged currents flavour mixing occurs!
- Parametrized through the
Cabbibo-Kobayashi-Maskawa Matrix:

$$V_{CKM} = U^d U^{u,\dagger}$$

Properties of the CKM Matrix

- V_{CKM} is unitary (by our construction)
- Number of parameters for n families
 - Unitary $n \times n$ matrix: n^2 real parameters
 - Freedom to rephase the $2n$ quark fields:
 $2n - 1$ relative phases
- $n^2 - 2n + 1 = (n - 1)^2$ real parameters
 - * $(n - 1)(n - 2)/2$ are phases
 - * $n(n - 1)/2$ are angles
- Phases are sources of CP violation
- $n = 2$: One angle, no phase \rightarrow no CP violation
- $n = 3$: Three angles, one phase
- $n = 4$: Six angles, three phases

CKM Basics

- Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- Single phase δ :
$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}.$$

- PDG CKM Parametrization:

$$V_{\text{CKM}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

- Large Phases in $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$ and $V_{td} = |V_{td}| e^{i\beta}$

CKM Unitarity Relations

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Off diagonal zeros of $V_{CKM}^\dagger V_{CKM} = 1 = V_{CKM} V_{CKM}^\dagger$

- $V_{CKM}^\dagger V_{CKM} = 1 : \begin{cases} V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0 \\ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0 \\ V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^* = 0 \end{cases}$

- $V_{CKM} V_{CKM}^\dagger = 1 : \begin{cases} V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \\ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \\ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \end{cases}$

Wolfenstein Parametrization of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & \lambda^2 A \\ \lambda^3 A(1 - \rho - i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

- Expansion in $\lambda \approx 0.22$ up to λ^3
- A, ρ, η of order unity

Unitarity Triangle(s)

- The unitarity relations:
Sum of three complex numbers = 0
- Triangles in the complex plane
- Only two out of the six unitarity relations involve terms of the same order in λ :

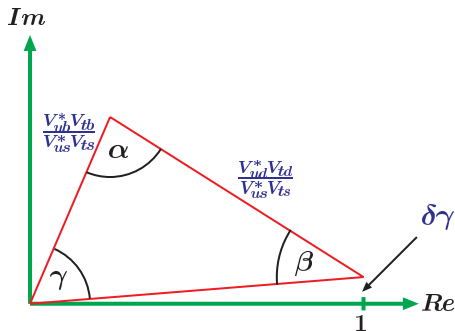
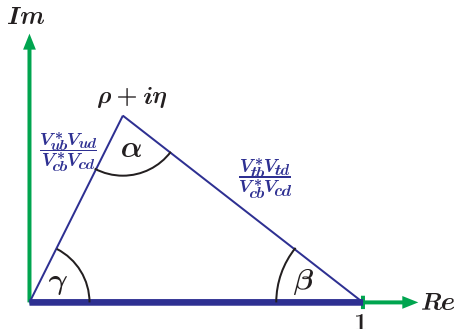
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

- Both correspond to

$$A\lambda^3(\rho + i\eta - 1 + 1 - \rho - i\eta) = 0$$

- This is **THE unitarity triangle** ...



- Definition of the CKM angles α , β and γ
- To leading order Wolfenstein:

$$V_{ub} = |V_{ub}|e^{-i\gamma} \quad V_{tb} = |V_{tb}|e^{-i\beta}$$

all other CKM matrix elements are real.

- $\delta\gamma$ is order λ^5

- Area of the Triangle(s): Measure of CP Violation
- Invariant measure of CP violation:

$$\text{Im}\Delta = \text{Im} V_{ud} V_{td}^* V_{tb} V_{ub}^* = c_{12} s_{12} c_{13}^2 s_{13} s_{23} c_{23} \sin \delta_{13}$$

- Maximal possible value $\delta_{\max} = \frac{1}{6\sqrt{3}} \sim 0.1$
- CP Violation is a small effect:
Measured value $\delta_{\text{exp}} \sim 0.0001$
- CP Violation vanishes in case of degeneracies: (Jarlskog)

$$\begin{aligned} J &= \text{Det}([M_u, M_d]) \\ &= 2i \text{Im}\Delta (m_u - m_c)(m_u - m_t)(m_c - m_t) \\ &\quad \times (m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned}$$

Peculiarities of SM Flavour Mixing

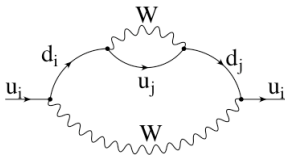
- Hierarchical structure of the CKM matrix
- Quark Mass spectrum is widely spread
 $m_u \sim 10 \text{ MeV}$ to $m_t \sim 170 \text{ GeV}$
- PMNS Matrix for lepton flavour mixing is not hierarchical
- Only the charged lepton masses are hierarchical
 $m_e \sim 0.5 \text{ MeV}$ to $m_\tau \sim 1772 \text{ MeV}$
- Up-type leptons \sim Neutrinos have very small masses
- (Enormous) Suppression of Flavour Changing Neutral Currents:
 $b \rightarrow s, c \rightarrow u, \tau \rightarrow \mu, \mu \rightarrow e, \nu_2 \rightarrow \nu_1$

Peculiarities of SM CP Violation

- Strong CP remains mysterious
- Flavour diagonal CP Violation is well hidden:

e.g electric dipole moment of the neutron:

At least three loops (Shabalin)



$$\begin{aligned}
 d_e &\sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \text{Im}\Delta \mu^3 \\
 &\sim 10^{-32} e \text{ cm} \quad \text{with } \mu \sim 0.3 \text{ GeV} \\
 d_{\text{exp}} &\leq 3.0 \times 10^{-26} e \text{ cm}
 \end{aligned}$$

- Pattern of mixing and mixing induced CP violation determined by GIM: **Tiny effects in the up quark sector**
 - $\Delta C = 2$ is very small
 - Mixing with third generation is small: charm physics basically “two family”
 - \rightarrow CP violation in charm is small in the SM
- **Fully consistent with particle physics observations**
- ... but inconsistent with matter-antimatter asymmetry

??? Many Open Questions ???

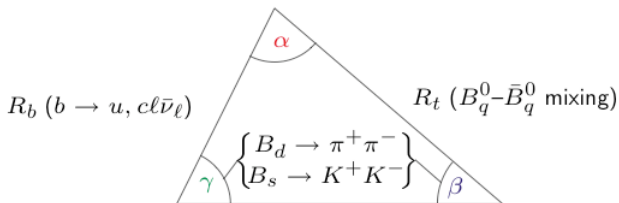
- **Our Understanding of Flavour is unsatisfactory:**
 - 22 (out of 27) free Parameters of the SM originate from the Yukawa Sector (including Lepton Mixing)
 - Why is the CKM Matrix hierarchical?
 - Why is CKM so different from the PMNS?
 - Why are the quark masses (except the top mass) so small compared with the electroweak VEV?
 - Why do we have three families?
- Why is CP Violation in Flavour-diagonal Processes not observed? (e.g. z.B. electric dipolmoments of electron and neutron)
- Where is the CP violation needed to explain the matter-antimatter asymmetry of the Universe?

Rare Decays: Small CKM Elements

- Focus on Non-leptonic Decays
- Small CKM Elements: **Non-Charmed Decays**

Roadmap of Bottom Decays (R. Fleischer)

$B \rightarrow \pi\pi$ (isospin), $B \rightarrow \rho\pi$, $B \rightarrow \rho\rho$



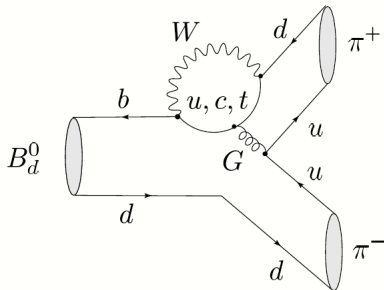
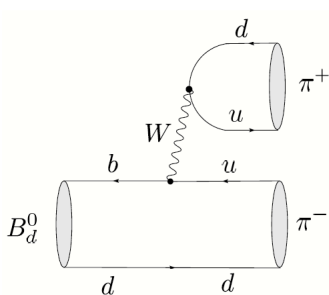
$B \rightarrow \pi K$ (penguins)

$B_d \rightarrow \psi K_S$ ($B_s \rightarrow \psi\phi : \phi_s \approx 0$)

$\left. \begin{array}{l} B_u^\pm \rightarrow K^\pm D \\ B_d \rightarrow K^{*0} D \\ B_c^\pm \rightarrow D_s^\pm D \end{array} \right\} \text{only trees}$

$B_d \rightarrow \phi K_S$ (pure penguin)

$\left. \begin{array}{l} B_d \rightarrow D^{(*)\pm} \pi^\mp : \gamma + 2\beta \\ B_s \rightarrow D_s^\pm K^\mp : \gamma + \phi_s \end{array} \right\} \text{only trees}$

$B \rightarrow \pi\pi$ 

- Interplay between “Tree” and “Penguin”
- Tree $\sim V_{ub}^* V_{ud} \sim \lambda^3$
- Penguin $\sim V_{tb}^* V_{td} \sim \lambda^3$
- Expect sizable direct CP violation

Isospin Relations in $B \rightarrow \pi\pi$

- Isospin relation: $\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}$
- In combination with

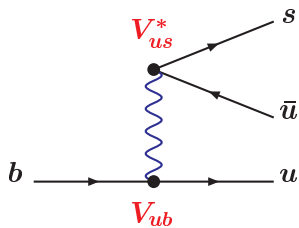
$$\frac{1}{\sqrt{2}}A^{+-} = Te^{i\gamma} + Pe^{-i\beta} \quad T : \text{tree amplitude}$$

$$A^{00} = Ce^{i\gamma} - Pe^{-i\beta} \quad P : \text{penguin amplitude}$$

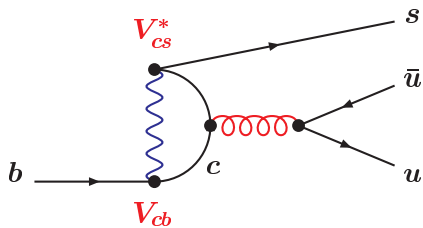
$$A^{+0} = (C + T)e^{i\gamma} \quad C : \text{Color suppressed amplitude}$$

gives a good handle on α (Gronau, London, Wyler)

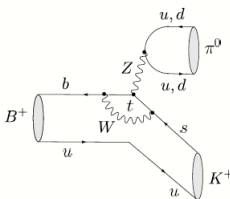
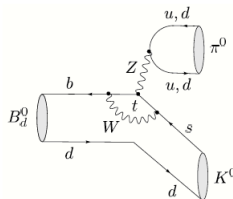
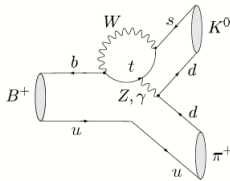
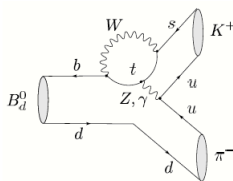
- P is (expected to be) sizable
- Separate measurement of $B_d \rightarrow \pi^0\pi^0$ and $\bar{B}_d \rightarrow \pi^0\pi^0$

$B \rightarrow \pi K$ 

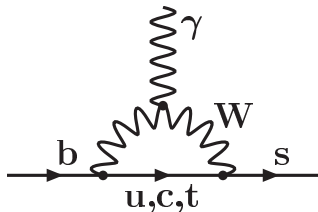
• ...



● Electroweak penguins can be significant!



Rare Decays: FCNC's



- Example: $b \rightarrow s\gamma$

$$\mathcal{A}(b \rightarrow s\gamma) = V_{ub} V_{us}^* f(m_u) + V_{cb} V_{cs}^* f(m_c) + V_{tb} V_{ts}^* f(m_t)$$

- In case of degenerate masses up-type masses:

$$\mathcal{A}(b \rightarrow s\gamma) = f(m) [V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^*] = 0$$

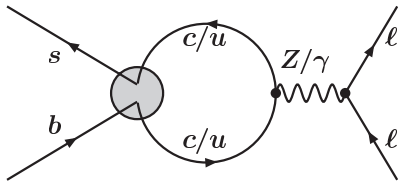
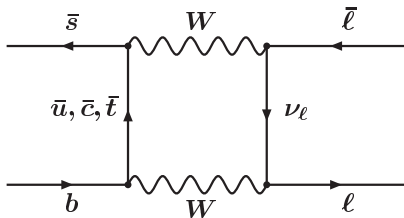
- Loop induced Processes: **Sensitivity to “new physics”**
- Exclusive as well as inclusive decays of the form
 $b \rightarrow s\gamma$ and $b \rightarrow sll$
- $b \rightarrow sll$ contains a lot of information through various observables
 - Lepton invariant mass
 - Lepton energy spectra
 - Forward Backward Asymmetries
 - ...

$B \rightarrow X_s \gamma$

- Complete NNLO Calculation has been performed
- Theory Predictions: (from Flächer @ EPS07)
 - Misiak et al. [hep-ph/0609232](https://arxiv.org/abs/hep-ph/0609232)
 - ❖ $\text{BF}(B \rightarrow X_s \gamma) = (3.15 \pm 0.23) \cdot 10^{-4}$ for $E_\gamma > 1.6 \text{ GeV}$
 - ❖ dedicated error analysis resulting in 7% error
 - Becher et al. [hep-ph/0610067](https://arxiv.org/abs/hep-ph/0610067)
 - ❖ $\text{BF}(B \rightarrow X_s \gamma) = (2.98 \pm 0.26) \cdot 10^{-4}$ for $E_\gamma > 1.6 \text{ GeV}$
 - ❖ larger perturbative uncertainty resulting in 9% error
 - Andersen et al. [hep-ph/0609250](https://arxiv.org/abs/hep-ph/0609250)
 - ❖ $\text{BF}(B \rightarrow X_s \gamma) = (3.47 \pm 0.48) \cdot 10^{-4}$ for $E_\gamma > 1.6 \text{ GeV}$
 - ❖ 11% uncertainty from variation of renormalisation scale
- HFAG Average: $(3.55 \pm 0.26) \times 10^{-4}$

$$B \rightarrow X_s \ell^+ \ell^-$$

- In addition to the diagrams for $B \rightarrow X_s \gamma$



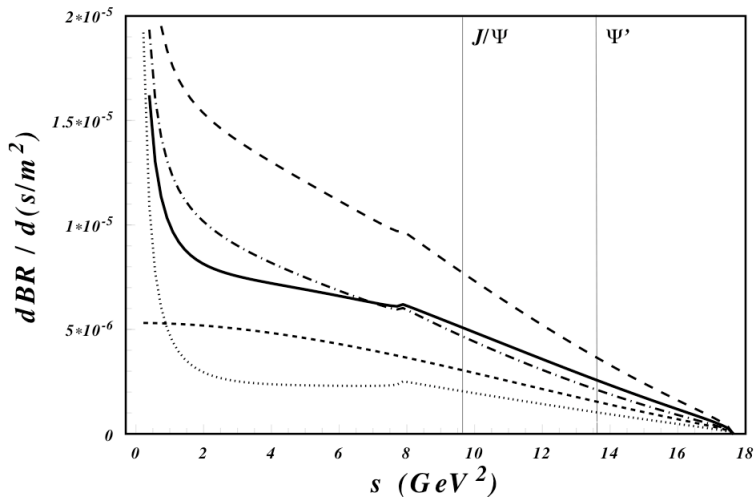
Observables in $B \rightarrow X_s \ell^+ \ell^-$

- Doubly differential rate: $d^2\Gamma / (dM_{\ell\ell}^2 d(\cos\theta))$
- Total rate R (or rate within cuts)
- Forward backward Asymmetry A_{FB}

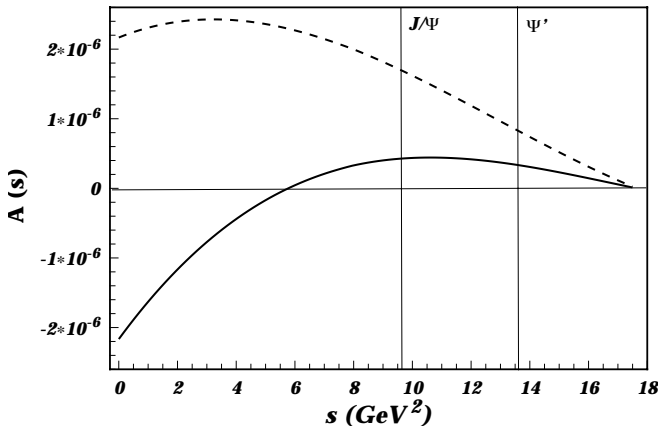
$$A_{FB}(M_{\ell\ell}^2) = \left[\frac{\int_{-1}^0 d(\cos\theta) - \int_0^{-1} d(\cos\theta)}{\int_{-1}^0 d(\cos\theta) + \int_0^{-1} d(\cos\theta)} \right] \frac{d^2\Gamma}{dM_{\ell\ell}^2 d(\cos\theta)}$$

- Observables depend on the Wilson Coefficients
→ Try to extract $C_1 \dots C_{10}$

The lepton invariant mass spectrum of $B \rightarrow X_{sll}$



The forward-backward of $B \rightarrow X_s ll$



Extraction of Wilson Coefficients (Example)

