Introduction to Operator Product Expansion

(Effective Hamiltonians, Wilson coefficients and all that . . .)

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Outline

1. $b \to c d \bar{u}$ decays
   - Born level
   - Quantum-loop corrections
   - Effective Operators
   - Wilson Coefficients
2. $b \to s(d)\, q\bar{q}$ decays
   - Current-current operators
   - Strong penguin operators
   - Electroweak corrections
3. B-$\bar{B}$ Mixing
4. Summary: Effective Theory for b-quark decays
5. Hadronic matrix elements for B-meson decays
   - $B \to D\pi$
   - $B \to \pi\pi$ and $B \to \pi K$

. . . Some introductory remarks . . .

- Physical processes involve
  different typical energy/length scales
- Separate short-distance and long-distance effects:

New physics	:	$\delta x \sim 1/\Lambda_{\rm NP}$
Electroweak interactions	:	$\delta x \sim 1/M_W$
Short-distance QCD(QED) corrections	:	$\delta x \sim 1/M_W \to 1/m_b$
Hadronic effects	:	$\delta x < 1/m_b$

$\longrightarrow$ Sequence of Effective Theories.

$\longrightarrow$ Perturbative and non-perturbative calculations.

$\longrightarrow$ Definition / measurement of hadronic input parameters.

Central Notions to be explained

Disclaimer:

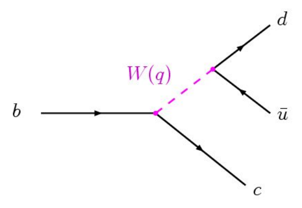
The dynamics of strong interactions in B-decays is very complex and has many faces. I will not be able to cover everything, but I hope that some theoretical and phenomenological concepts become clearer . . .

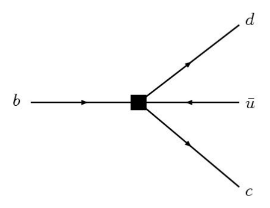
- Factorization
  1. separation of scales in perturbation theory (OPE, ET)
  2. simplification of exclusive hadronic matrix elements (QCDF)
- Operators in the weak effective Hamiltonian
  (current-current, strong penguins, electroweak penguins)
- Matching and Running of Wilson coefficients

Example: $b \to c d \bar{u}$ decays

$b \longrightarrow c d \bar{u}$ decay at Born level

Full theory (SM) $\longrightarrow$ **Fermi model**





$$\left(\frac{g}{2\sqrt{2}}\right)^2 J_\alpha^{(b\to c)} \frac{-g^{\alpha\beta}+\frac{q^\alpha q^\beta}{M_W^2}}{q^2-M_W^2} \bar{J}_\beta^{(d\to u)} \quad \xrightarrow{|q|\ll M_W} \quad \frac{G_F}{\sqrt{2}} J_\alpha^{(b\to c)} g^{\alpha\beta} \bar{J}_\beta^{(d\to u)}$$

- Energy/Momentum transfer limited by mass of decaying b-quark.
- b-quark mass much smaller than W-boson mass.

$$\boxed{|q| \leq m_b \ll m_W}$$

Effective Theory:

- Analogously to muon decay, transition described in terms of current-current interaction, with left-handed charged currents

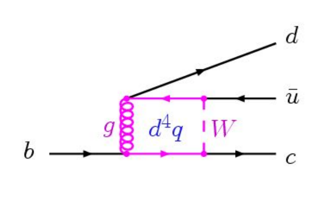
$$J_\alpha^{(b\to c)} = V_{cb}\, [\bar c\, \gamma_\alpha (1-\gamma_5)\, b] \;, \qquad \bar J_\beta^{(d\to u)} = V_{ud}^*\, [\bar d\, \gamma_\beta (1-\gamma_5)\, u]$$

- Effective operators only contain light fields
  ("light" quarks, electron, neutrinos, gluons, photons). $\surd$

- Effect of large scale M_W in effective Fermi coupling constant:

$$\boxed{\frac{g^2}{8M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}} \simeq 1.16639 \cdot 10^{-5}\, \mathrm{GeV}^{-2}}$$

Quantum-loop corrections to $b \to cd\bar{u}$ decay



- Momentum q of the W-boson is an internal loop parameter that is integrated over and can take values between $-\infty$ and $+\infty$.

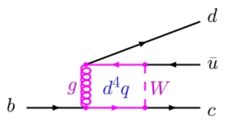
$\Rightarrow$ We cannot simply expand in $|q|/M_W$!

$\Rightarrow$ Need a method to separate the cases $|q| \geq M_W$ and $|q| \ll M_W$.

$\longrightarrow$ OPE / Factorization

IR and UV regions in the Effective Theory

full theory $=$ IR region $(M_W \to \infty)$ $+$ UV region $(m_{b,c} \to 0)$



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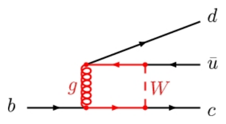

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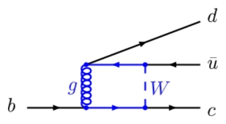

$$I(\alpha_s; \frac{m_b}{M_w}, \frac{m_c}{m_b})/G_F \simeq I_{IR}(\alpha_s; \frac{m_b}{\mu}, \frac{m_c}{m_b}) + I_{UV}(\alpha_s; \frac{\mu}{m_W})$$

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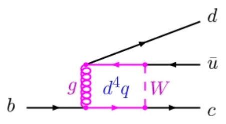

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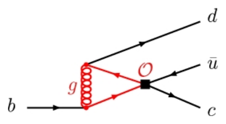

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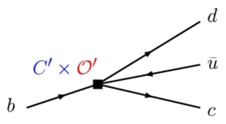
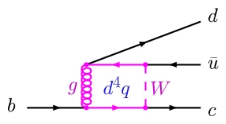
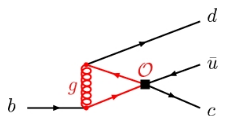
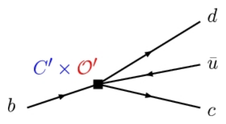


$$I(\alpha_s;\, \tfrac{m_b}{M_w},\, \tfrac{m_c}{m_b})/G_F \simeq \langle \mathcal{O} \rangle^{\rm loop}(\alpha_s;\, \tfrac{m_b}{\mu},\, \tfrac{m_c}{m_b}) + C'(\alpha_s;\, \tfrac{\mu}{m_W}) \times \langle \mathcal{O}' \rangle^{\rm tree}$$

IR and UV regions in the Effective Theory

full theory	=	IR region $(M_W \to \infty)$	+	UV region $(m_{b,c} \to 0)$
	$\simeq$		+	
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IR region ($\updownarrow$): 1-loop matrix element of operator $\mathcal{O}$ in Eff. Th.

- independent of M_W
- UV divergent $\to \mu$

UV region ($\updownarrow$): 1-loop coefficient for new operator $\mathcal{O}'$ in ET

- independent of $m_{b,c}$
- IR divergent $\to \mu$

For the curious: Operator Product Expansion

- Study time-ordered product of two general operators:

$$\int d^4p\, e^{ipx}\, T\left(\phi(x)\phi(0)\right)$$

- For small values of x, the product can be expanded into a set of local composite (renormalized) operators $\mathcal{O}_i(0)$ and c-numbered Wilson coefficients $c_i(x)$:

$$T\left(\phi(x)\phi(0)\right) \stackrel{x\to 0}{=} \sum_i c_i(x)\, \mathcal{O}_i(0)$$

- Construction is independent of additional field operators appearing in Green functions / scattering amplitudes
  (i.e. universal for all processes involving the T-product under consideration)
- In momentum space, this corresponds to the limit of large momenta/energies/masses . . .

Effective Operators for $b \to c\bar{u}$

- short-distance QCD corrections preserve chirality;
- quark-gluon vertices induce second colour structure.

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\, V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu)\, \mathcal{O}_i + \text{h.c.} \qquad (b \to c d\bar{u})$$

- Current-current operators: $(b \to c d\bar{u},$ analogously for $b \to q q' \bar{q}''$ decays)

$$\begin{aligned} \mathcal{O}_1 &= (\bar{d}^i_L \gamma_\alpha u^j_L)(\bar{c}^j_L \gamma^\alpha b^i_L) \\ \mathcal{O}_2 &= (\bar{d}^i_L \gamma_\alpha u^i_L)(\bar{c}^j_L \gamma^\alpha b^j_L) \end{aligned}$$

- The so-called Wilson coefficients $C_i(\mu)$ contain all information about short-distance physics above the scale μ (SM)

Wilson Coefficients in Perturbation Theory

- 1-loop result:

$$C_i(\mu) = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} + \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{\mu^2}{M_W^2} + \frac{11}{6} \right) \left\{ \begin{array}{c} 3 \\ -1 \end{array} \right\} + \mathcal{O}(\alpha_s^2)$$

- Wilson coefficients depend on the renormalization scale μ

"Matching"

For $\mu \sim M_W$ the logarithmic term is small, and $C_i(M_W)$ can be calculated in fixed-order perturbation theory, since $\frac{\alpha_s(M_W)}{\pi} \ll 1$.

Here M_W is called the matching scale.

Anomalous Dimensions

- In order to compare with experiment / hadronic models, the matrix elements of ET operators are needed at low-energy scale $\mu \sim m_b$
  - Only the combination
    $$\sum_i C_i(\mu)\, \langle \mathcal{O}_i \rangle(\mu)$$
    is μ-independent (in perturbation theory).

  $\Rightarrow$ Need Wilson coefficients at low scale !

- Scale dependence can be calculated in perturbation theory:
  - Loop diagrams in ET are UV divergent $\Rightarrow$ anomalous dimensions (matrix):

$$\frac{\partial}{\partial \ln \mu} C_i(\mu) \equiv \gamma_{ji}(\mu)\, C_j(\mu) = \left(\frac{\alpha_s(\mu)}{4\pi}\, \gamma_{ji}^{(1)} + \ldots \right) C_j(\mu)$$

  - $\gamma = \gamma(\alpha_s)$ has a perturbative expansion.

RG Evolution ("running")

In our case:

$$\gamma^{(1)} = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix} \qquad \begin{cases} \text{Eigenvectors: } C_\pm = \frac{1}{\sqrt{2}}(C_2 \pm C_1) \\ \text{Eigenvalues: } \gamma_\pm^{(1)} = +4, \ -8 \end{cases}$$

- Formal solution of differential equation: (separation of variables)

$$\ln \frac{C_\pm(\mu)}{C_\pm(M)} = \int_{\ln M}^{\ln \mu} d\ln\mu' \, \gamma_\pm(\mu') = \int_{\alpha_s(M)}^{\alpha_s(\mu)} \frac{d\alpha_s}{2\beta(\alpha_s)} \gamma_\pm(\alpha_s)$$

- Perturbative expansion of anomalous dimension and β-function:

$$\gamma = \frac{\alpha_s}{4\pi} \gamma^{(1)} + \ldots , \qquad 2\beta \equiv \frac{d\alpha_s}{d\ln\mu} = -\frac{2\beta_0}{4\pi} \alpha_s^2 + \ldots$$

$$C_\pm(\mu) \simeq C_\pm(M_W) \cdot \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)} \right)^{-\gamma_\pm^{(1)}/2\beta_0} \qquad \text{(LeadingLogApprox)}$$

Numerical values for $C_{1,2}$ in the SM [Buchalla/Buras/Lautenbacher 96]

operator:	$\mathcal{O}_1 = (\bar{d}^i_L \gamma_\mu u^i_L)(\bar{c}^j_L \gamma^\mu b^j_L)$	$\mathcal{O}_2 = (\bar{d}^i_L \gamma_\mu T u^i_L)(\bar{c}^j_L \gamma^\mu b^j_L)$
$C_i(m_b)$:	-0.514 (LL) -0.303 (NLL)	1.026 (LL) 1.008 (NLL)

(modulo parametric uncertainties from $M_W, m_b, \alpha_s(M_Z)$ and QED corr.)

(potential) New Physics modifications:

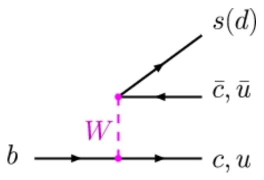
- new left-handed interactions (incl. new phases)

$$C_{1,2}(M_W) \to C_{1,2}(M_W) + \delta_{\rm NP}(M_W, M_{\rm NP})$$

- new chiral structures $\Rightarrow$ extend operator basis (LR,RR currents)

Next Example: $b \to s(d)\, q\bar{q}$ decays

$b \to s(d)\, q\bar{q}$ decays – Current-current operators



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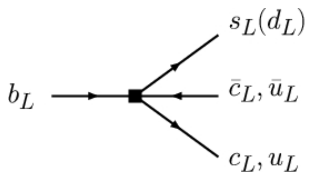


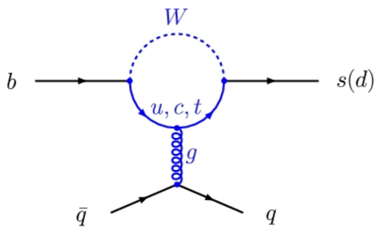
- Now, there are two possible flavour structures:

$$V_{ub} V_{us(d)}^* \, (\bar{u}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu u_L) \equiv \lambda_u \, \mathcal{O}_2^{(u)} \, ,$$
$$V_{cb} V_{cs(d)}^* \, (\bar{c}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu c_L) \equiv \lambda_c \, \mathcal{O}_2^{(c)} \, ,$$

- Again, α_s corrections induce independent colour structures $\mathcal{O}_1^{(u,c)}$.

$b \to s(d)\, q\bar{q}$ decays – strong penguin operators

- New feature:
  Penguin diagrams induce additional operator structures



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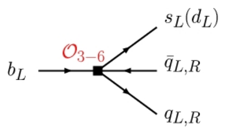

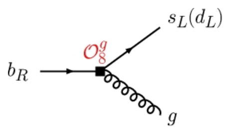

- Strong penguin operators: $\mathcal{O}_{3-6}$
- Chromomagnetic operator: $\mathcal{O}_8^g$
- Wilson coefficients numerically suppressed by α_s / loop factor.

Eff. Hamiltonian for $b \to s(d) q\bar{q}$ decays (QCD only)

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu) \left(\lambda_u \, \mathcal{O}_i^{(u)} + \lambda_c \, \mathcal{O}_i^{(c)} \right) - \frac{G_F}{\sqrt{2}} \lambda_t \sum_{i=3}^{6} C_i(\mu) \, \mathcal{O}_i - \frac{G_F}{\sqrt{2}} \lambda_t \, C_8^g(\mu) \, \mathcal{O}_8^g$$

$$\mathcal{O}_3 = (\bar{s}_L^i \gamma_\mu b_L^i) \sum_{q \neq t} (\bar{q}_L^j \gamma^\mu q_L^j) \,, \qquad \mathcal{O}_4 = (\bar{s}_L^i \gamma_\mu b_L^j) \sum_{q \neq t} (\bar{q}_L^j \gamma^\mu q_L^i) \,,$$

$$\mathcal{O}_5 = (\bar{s}_L^i \gamma_\mu b_L^i) \sum_{q \neq t} (\bar{q}_R^j \gamma^\mu q_R^j) \,, \qquad \mathcal{O}_6 = (\bar{s}_L^i \gamma_\mu b_L^j) \sum_{q \neq t} (\bar{q}_R^j \gamma^\mu q_R^i) \,,$$

$$\mathcal{O}_8^g = \frac{g_s}{8\pi^2} \, m_b \, (\bar{s}_L \, \sigma^{\mu\nu} \, T^A \, b_R) \, G_{\mu\nu}^A \,.$$

- virtual u- and c-contributions (with $m_u = m_c \to 0$) : use $\lambda_u + \lambda_c = -\lambda_t$
- gluon couples to left- and right-handed currents.
- chromomagnetic operator requires one chirality flip ! (m_s is set to zero)

Matching and running for strong penguin operators

- Matching coefficients depend on top mass, $x_t = m_t^2/M_W^2$

$$
\begin{aligned}
C_3(M_W) \simeq C_5(M_W) &= -\frac{\alpha_s}{24\pi}\,\tilde{E}_0(x_t) + \ldots \\
C_4(M_W) \simeq C_6(M_W) &= +\frac{\alpha_s}{8\pi}\,\tilde{E}_0(x_t) + \ldots
\end{aligned}
$$

- Beware! – Different conventions/schemes (BBL,CMM,BBNS)
- Mixing of $\mathcal{O}_{1-6}$ under RG evolution described by 6×6 matrix:

$$
\gamma = \begin{pmatrix}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & \bullet & \bullet & \bullet & \bullet
\end{pmatrix}
$$

Chromomagnetic operator

- Matching:

$$C_8^g(M_W) = -\frac{1}{2}\,E_0'(x_t) + \ldots$$

- Usually, one considers scheme-independent linear combination:

$$C_8^{g,\,\mathrm{eff}} = C_8^g(M_W) + \sum_{i=1}^{6} z_i\, C_i(M_W)$$

(z_i : scheme-dependent coefficients)

- RG mixing between C_{1-6} and $C_8^{g,\,\mathrm{eff}}$

Electroweak Corrections

- Penguin and box diagrams with additional γ/Z exchange:
  - $\longrightarrow$ Electroweak Penguin Operators $\mathcal{O}_{7-10}$

$$\mathcal{O}_7 = \frac{2}{3}\,(\bar{s}_L^i \gamma_\mu b_L^i) \sum_{q \neq t} e_q\,(\bar{q}_L^j \gamma^\mu q_L^j)\,, \quad \mathcal{O}_8 = \frac{2}{3}\,(\bar{s}_L^i \gamma_\mu b_L^j) \sum_{q \neq t} e_q\,(\bar{q}_L^j \gamma^\mu q_L^i)\,,$$

$$\mathcal{O}_9 = \frac{2}{3}\,(\bar{s}_L^i \gamma_\mu b_L^i) \sum_{q \neq t} e_q\,(\bar{q}_R^j \gamma^\mu q_R^j)\,, \quad \mathcal{O}_{10} = \frac{2}{3}\,(\bar{s}_L^i \gamma_\mu b_L^j) \sum_{q \neq t} e_q\,(\bar{q}_R^j \gamma^\mu q_R^i)\,.$$

  depends on electromagnetic charge of final state quarks !

  - $\longrightarrow$ Electromagnetic operators $\mathcal{O}_7^\gamma$
    main contribution to $b \to s(d)\gamma$ decays.
  - $\longrightarrow$ Semileptonic operators $\mathcal{O}_{9V}$, $\mathcal{O}_{10A}$
    main contribution to $b \to s\ell^+\ell^-$ decays.
  - $\longrightarrow$ electroweak corrections to matching coefficients

- Isospin effects in non-leptonic decays (e.g. $B \to \pi K$)
  and radiative decays ($b \to s(d)\gamma$, $b \to s\ell^+\ell^-$)
  are particularly sensitive to New Physics !

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  - $\rightarrow$ Electromagnetic operators $\mathcal{O}_7^\gamma$

$$\mathcal{O}_7^\gamma = \frac{e}{8\pi^2} \, m_b \, (\bar{s}_L \, \sigma_{\mu\nu} \, b_R) \, F^{\mu\nu}$$

    main contribution to $b \to s(d)\gamma$ decays.
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$$\begin{aligned} \mathcal{O}_{9V} &= (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell), \\ \mathcal{O}_{10A} &= (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell) \end{aligned}$$

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B-$\bar{B}$ Mixing

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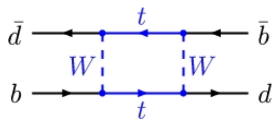
- $\Delta B = 2$ operators require two $W^{\pm}$ exchanges $\rightarrow$ box diagrams:



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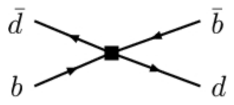


$$H_{\text{eff}}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 \, (V_{tb}^* V_{td})^2 \; C(x_t, \mu) \, (\bar{b}_L \, \gamma^\mu \, d_L)(\bar{b}_L \, \gamma_\mu \, d_L) + \text{h.c.}$$

- only one colour structure due to symmetry of operator
- analogously for B_s-$\bar{B}_s$ mixing

Summary: Effective Theory for b-quark decays

"Full theory" $\leftrightarrow$ all modes propagate
Parameters: $M_{W,Z}, M_H, m_t, m_q, g, g', \alpha_s \ldots$

$\uparrow \mu > M_W$

$C_i(M_W) = C_i\big|_{\rm tree} + \delta_i^{(1)} \frac{\alpha_s(M_W)}{4\pi} + \ldots$

matching: $\mu \sim M_W$

"Eff. theory" $\leftrightarrow$ low-energy modes propagate.
High-energy modes are "integrated out".
Parameters: $m_b, m_c, G_F, \alpha_s, C_i(\mu) \ldots$

$\downarrow \mu < M_W$

$\frac{\partial}{\partial \ln \mu} C_i(\mu) = \gamma_{ji}(\mu)\, C_j(\mu)$

anomalous dimensions

Expectation values of operators $\langle O_i \rangle$ at $\mu = m_b$.
All dependence on M_W absorbed into $C_i(m_b)$

resummation of logs

From $b \to c d \bar{u}$ to $\bar{B}^0 \to D^+ \pi^-$

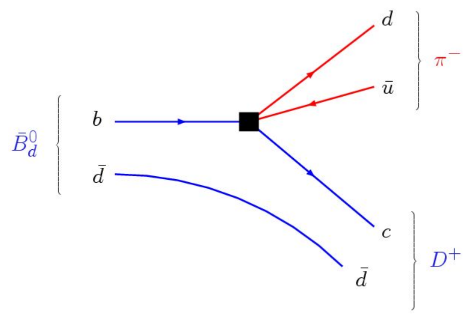
- In experiment, we cannot see the quark transition directly.
- Rather, we observe exclusive hadronic transitions, described by hadronic matrix elements, like e.g.

$$\langle D^+ \pi^- | \mathcal{H}_{\rm eff}^{b \to c d \bar{u}} | \bar{B}_d^0 \rangle = V_{cb} V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu)\, r_i(\mu)$$

$$r_i(\mu) = \langle D^+ \pi^- | \mathcal{O}_i | \bar{B}_d^0 \rangle \Big|_\mu$$

- The hadronic matrix elements r_i contain QCD (and also QED) dynamics below the scale $\mu \sim m_b$.

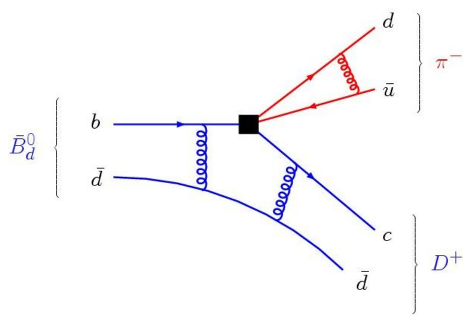
"Naive" Factorization of hadronic matrix elements



$$r_i \quad = \quad \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}$$

- Quantum fluctuations above $\mu \sim m_b$ already in Wilson coefficients

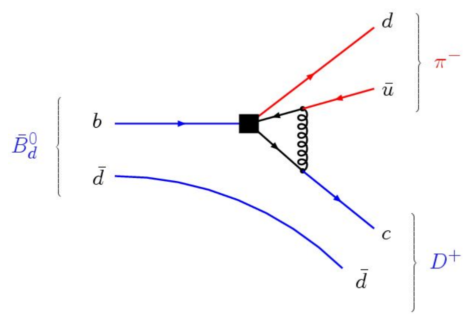
"Naive" Factorization of hadronic matrix elements



$$r_i \quad = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}_{\text{form factor}} \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}_{\text{decay constant}}$$

- Part of (low-energy) gluon effects encoded in simple/universal had. quantities

"Naive" Factorization of hadronic matrix elements



$$r_i(\mu) \;=\; \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}^0_d \rangle}_{\text{form factor}} \; \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}_{\text{decay constant}} + \text{corrections}(\mu)$$

- Gluon cross-talk between π^- and $B \to D$ $\Rightarrow$ QCD corrections

QCD factorization [Beneke/Buchalla/Neubert/Sachrajda 2000]

- light quarks in π^- have large energy (in B rest frame)
- gluons from the $B \to D$ transition see "small colour-dipole"

$\Rightarrow$ corrections to naive factorization dominated by gluon exchange at short distances $\delta x \sim 1/m_b$

New feature: Light-cone distribution amplitudes $\phi_\pi(u)$

- Short-distance corrections to naive factorization given as convolution

$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)} \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi} t_{ij}(u,\mu) + \ldots \right) f_\pi \, \phi_\pi(u,\mu)$$

- $\phi_\pi(u)$: distribution of momentum fraction u of a quark in the pion.
- $t_{ij}(u,\mu)$: perturbative coefficient function (depends on u)

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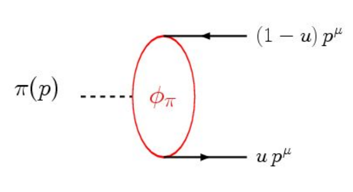
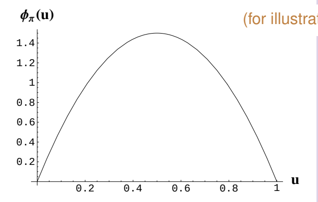
Light-cone distribution amplitude for the pion





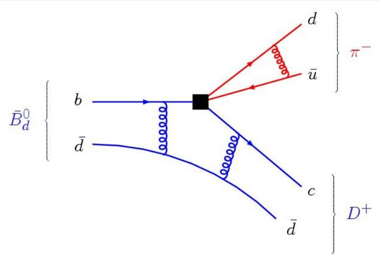
(for illustration only)

- Exclusive analogue of parton distribution function:
  - PDF: probability density (all Fock states)
  - LCDA: probability amplitude (one Fock state, e.g. $q\bar{q}$)
- Phenomenologically relevant $\langle u^{-1}\rangle_\pi \simeq 3.3 \pm 0.3$

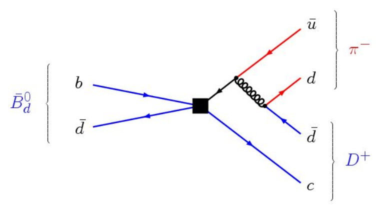
[from sum rules, lattice, exp.]

Complication: Annihilation in $\bar{B}_d \to D^+\pi^-$

Second topology for hadronic matrix element possible:



"Tree" (class-I)

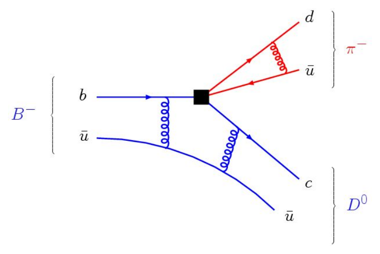


"Annihilation" (class-III)

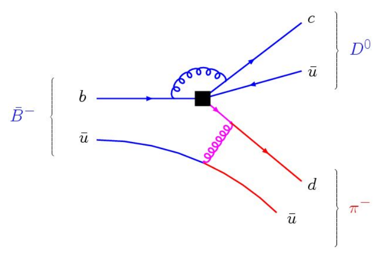
- annihilation is power-suppressed by Λ/m_b
- difficult to estimate (final-state interactions?)

Still more complicated: $B^- \to D^0 \pi^-$

Second topology with spectator quark going into light meson:



"Tree" (class-I)

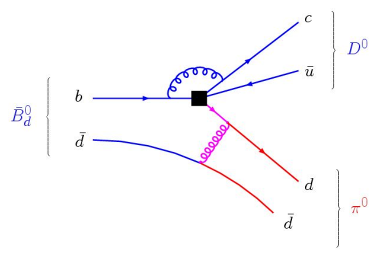


"Tree" (class-II)

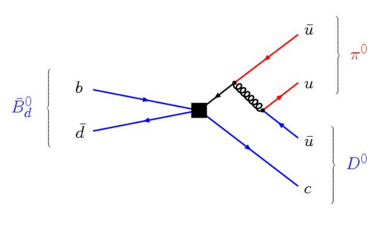
- class-II amplitude does not factorize into simpler objects
  (colour-transparency argument does not apply)
- again, it is power-suppressed compared to class-I topology

Non-factorizable: $\bar{B}^0 \to D^0\pi^0$

In this channel, class-I topology is absent:



"Tree" (class-II)



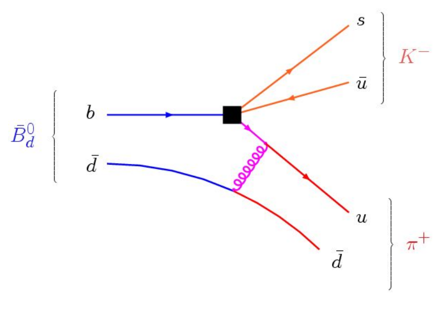
"Annihilation" (class-III)

- The whole decay amplitude is power-suppressed!
- Naive factorization is not even an approximation!

$B \to \pi\pi$ and $B \to \pi K$

$B \to \pi\pi$ and $B \to \pi K$

Naive factorization:



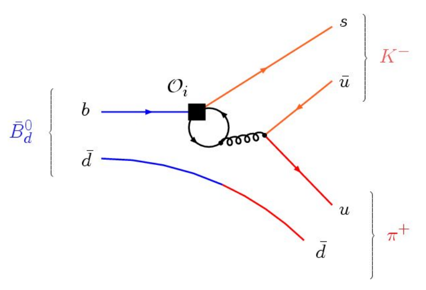
- Both final-state mesons are light and energetic.
- Colour-transparency argument applies for class-I and class-II topologies.
- $B \to \pi(K)$ form factors fairly well known (QCD sum rules)

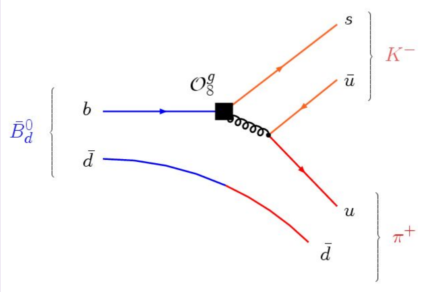
QCDF for $B \to \pi\pi$ and $B \to \pi K$ decays (BBNS 1999)

Factorization formula has to be extended:

- Vertex corrections are treated as in $B \to D\pi$
  - Include penguin (and electroweak) operators from H_{eff}.
  - Take into account new (long-distance) penguin diagrams! ($\to$ Fig.)
- Additional perturbative interactions involving spectator in B-meson ($\to$ Fig.)
  - Sensitive to the distribution of the spectator momentum ω
    $\longrightarrow$ light-cone distribution amplitude $\phi_B(\omega)$

Additional diagrams for hard corrections in QCDF (example)





$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u, \mu)$

$$r_i(\mu)\Big|_{\rm hard} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_K\, \phi_K(u,\mu)$$

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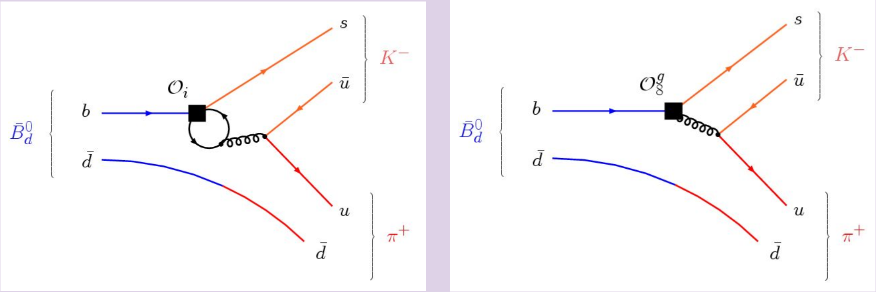

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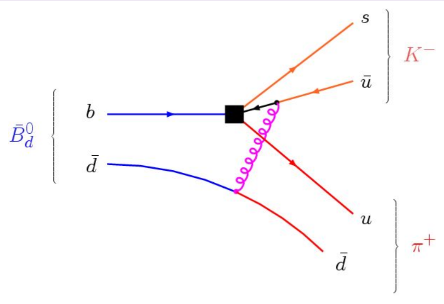
Spectator corrections with hard-collinear gluons in QCDF

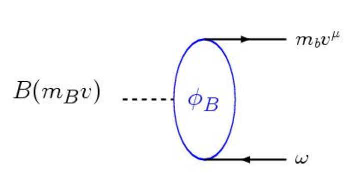


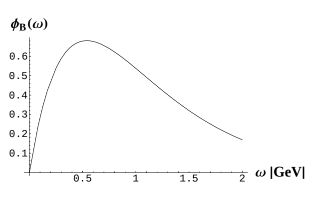
$\longrightarrow$ additive correction to naive factorization

$$\Delta r_i(\mu)\Big|_{\text{spect.}} = \int du\, dv\, d\omega \left(\frac{\alpha_s}{4\pi} h_i(u,v,\omega,\mu) + \ldots\right) \times f_K\, \phi_K(u,\mu)\, f_\pi\, \phi_\pi(v,\mu)\, f_B\, \phi_B(\omega,\mu)$$

Distribution amplitudes for all three mesons involved!

New ingredient: LCDA for the B-meson





- Phenomenologically relevant: $\langle \omega^{-1} \rangle_B \simeq (1.9 \pm 0.2)\ \mathrm{GeV}^{-1}$

(at $\mu = \sqrt{m_b \Lambda} \simeq 1.5$ GeV)

(from QCD sum rules [Braun/Ivanov/Korchemsky])

(from HQET parameters [Lee/Neubert])

- Large logarithms $\ln m_b$ can be resummed using SCET $\surd$

Complications for QCDF in $B \to \pi\pi, \pi K$ etc.

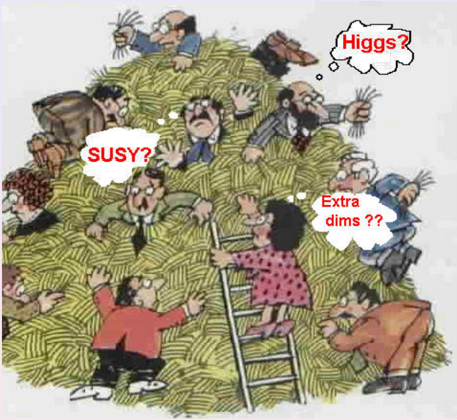
- Annihilation topologies are numerically important. BBNS use conservative model estimates.
- Some power-corrections are numerically enhanced by "chiral factor"

$$\frac{\mu_\pi}{f_\pi} = \frac{m_\pi^2}{2 f_\pi \, m_q}$$

- Many decay topologies interfere with each other.
- Many hadronic parameters to vary.

$\to$ Hadronic uncertainties sometimes quite large.

Summary



[from C. Berger's homepage]

" When looking for new physics, . . .
. . . do not forget about the complexity of the old physics ! "