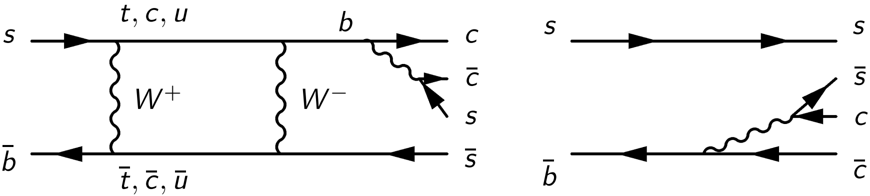
Short recap: $B_s \to J/\Psi\,(\mu\mu)\,\Phi\,(KK)$



How can we extract $\Phi_S\,(=2\beta_s) = \Phi_{SM} + \Phi_{NP} \approx \Phi_{NP}$ and $\Delta\Gamma = \Gamma_H - \Gamma_L$? $\to$

Maximum-Likelihood-Fit

What is a Likelihood?

Data $\{x_1, x_2, \ldots, x_N\}$
x_i drawn from **P**robability **D**ensity **F**unction $P(x_i; a)$ with Parameter a
Likelihood = Probability for data set $\{x_1, x_2, \ldots, x_N\}$, product of $P(x_i; a)$

$$\begin{aligned} \mathscr{L}(x_1, \ldots, x_N; a) &= P(x_1; a), \cdot P(x_2; a) \cdot \ldots \cdot P(x_N; a) \\ &= \prod_{i=1}^{N} P(x_i; a) \end{aligned}$$

Principle of Maximum Likelihood: Find a for which $\mathscr{L}$ is maximal

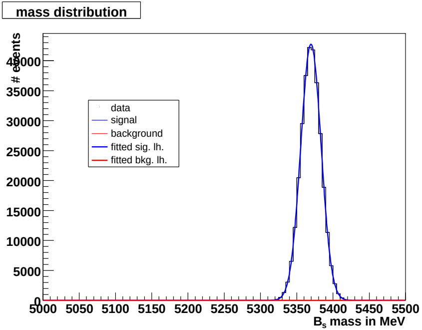
$$\begin{aligned} \frac{d}{da}\mathscr{L}(x_1, \ldots, x_N; a) &= 0 \\ \frac{d}{da}\ln(\mathscr{L}) &= \sum_{i=1}^{N} \frac{d}{da} \ln P(x_i; a) = 0 \quad \text{(used in practice)} \end{aligned}$$

Not always possible analytically → use numerical maximisation

An Example: Gaussian Mass Distribution

Parameter $a = M$, Data $\{m_1, \ldots, m_N\}$

P.D.F.: $P_i = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(m_i - M)^2}{2\sigma^2}}$



$$\begin{aligned}
\frac{d}{dM} \ln(\mathscr{L}) &= \sum_{i=1}^{N} \frac{d}{dM} \ln P(m_i; M) \\
&= \sum_{i=1}^{N} \frac{d}{dM} \left(\ln \frac{1}{\sigma\sqrt{2\pi}} - \frac{(m_i - M)^2}{2\sigma^2} \right) \\
&= \sum_{i=1}^{N} \frac{m_i - M}{\sigma^2} = 0 \\
\Rightarrow \mathbf{M} &= \frac{\mathbf{1}}{\mathbf{N}} \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{m_i}
\end{aligned}$$

What about errors?

Solve $\ln \mathscr{L}(a) = \ln \mathscr{L}_{\max} - \frac{1}{2}$ for a!

Example $P_i = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(m_i - M)^2}{2\sigma^2}}$:

$$\begin{aligned}
\ln \mathscr{L}(M + \Delta M) &= \ln \mathscr{L}_{\max}(M) - \frac{1}{2} \\
\sum_{i=1}^{N} -\frac{(m_i - \Delta_M - M)^2}{2\sigma^2} &= \sum_{i=1}^{N} -\frac{(m_i - M)^2}{2\sigma^2} - \frac{1}{2} \\
N\frac{\Delta_M^2}{2\sigma^2} + 2N\frac{\Delta_M}{2\sigma^2} M - 2\Delta_M \sum_{i=1}^{N} m_i &= \frac{1}{2} \qquad \Big| \sum_i m_i = N m_{B_s} \\
\Delta_M^2 &= \frac{\sigma^2}{N} \\
\mathbf{\Delta_M} &= \pm \frac{\sigma}{\sqrt{\mathbf{N}}}
\end{aligned}$$

This also works for multiple parameters $\rightarrow$ Confidence regions

The Pros and Cons of Likelihoods

LH-Estimator (value of a for which $\mathscr{L}$ maximal)

- is optimal (minimal Variance) for large N
- can be time-consuming to compute
- can be biased for small N!
- need to know p.d.f. shape beforehand
- χ^2-method can be derived from ML if p.d.f.s are gaussian

Booktip: Statistics - A Guide to the Use of Statistical Methods in the Physical Sciences, R. Barlow

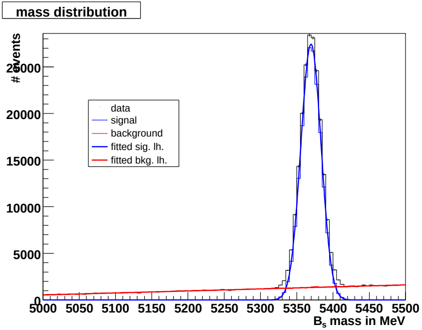
The Reconstructed B_s-Mass

Used to separate signal/background events

For Signal: Use Gaussian peak (always normalize correctly!)

$$P_{\mathrm{sig}}^{\mathrm{M}} = \frac{1}{\sqrt{2\pi}\sigma_m} e^{-\frac{(m_i - M)^2}{2\sigma_m^2}}$$

For Background: Use Linear or Exponential distribution



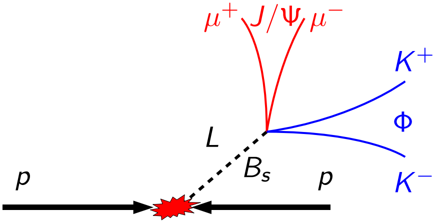
$$\mathscr{L} = \prod_i^{\text{all events}} P_i(m_i)$$

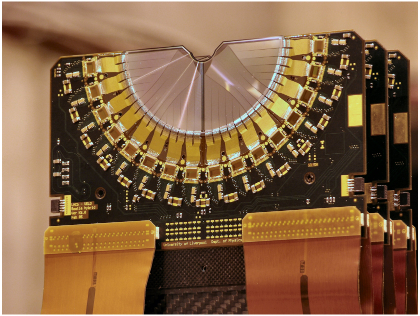
$$P_i = f_{\mathrm{sig}} P_{\mathrm{sig}}^{M}(m_i) + (1 - f_{\mathrm{sig}}) P_{\mathrm{bkg}}^{M}(m_i)$$

Depending on mass m_i, event is Signal/Background-like

The Proper Time 1: What is the Proper Time?

Defined as $t = \frac{L}{p} m_{B_s}$, $\sigma_t = \frac{\sigma_L}{p} m_{B_s}$





Good secondary Vertex resolution needed!

$\rightarrow$ VE$_{\text{rtex}}$LO$_{\text{cator}}$: $\sigma_{ct} = 0.03\text{ps} \approx 150\mu m$

Momentum resolution negligible

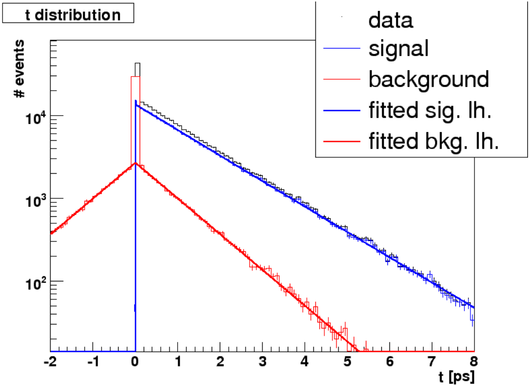
The Proper Time 2: Model

For Signal: B_s-decays

$$P_i \quad \propto \quad e^{-\Gamma_L t_i} \text{ or } e^{-\Gamma_H t_i}$$

For Background: Mainly prompt J/Ψ (use sideband)

$$P_i \quad \propto \quad \delta(t_i)$$



Not realistic

Finite decay length resolution

$\Rightarrow$ Finite proper time resolution!

The Proper Time 3: Convolution

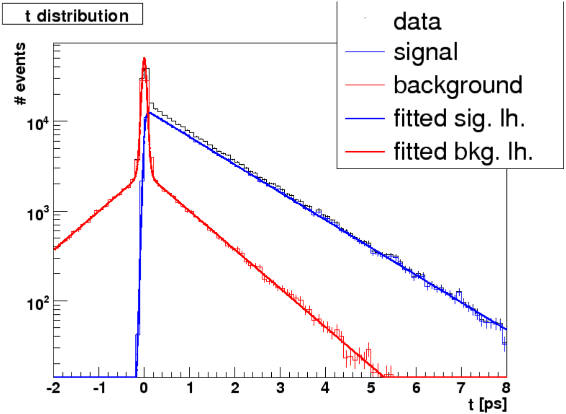
$\Rightarrow$ Need to convolute proper time probabilities with gaussian resolution

$$P_i^{real}(t) = P_i^{ideal}(t) \otimes \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{(t-t')^2}{2\sigma_t^2}}$$

$$e^{-\Gamma t}$$

$$\Downarrow$$

$$\frac{1}{2} e^{-\Gamma t} e^{\frac{1}{2}\sigma_t^2\Gamma^2} erfc\left(\frac{\sigma_t\Gamma}{\sqrt{2}} - \frac{t}{\sigma_t\sqrt{2}}\right)$$



The Transversity Angles 1

Final state is an admixture of CP-eigenstates

$B_s(J=0) \rightarrow J/\Psi(J=1)\Phi(J=1) \rightarrow$ final state L=0,1,2

$CP(J/\Psi\Phi) = CP(J/\Psi)CP(\Phi)(-1)^L = (-1)^L$

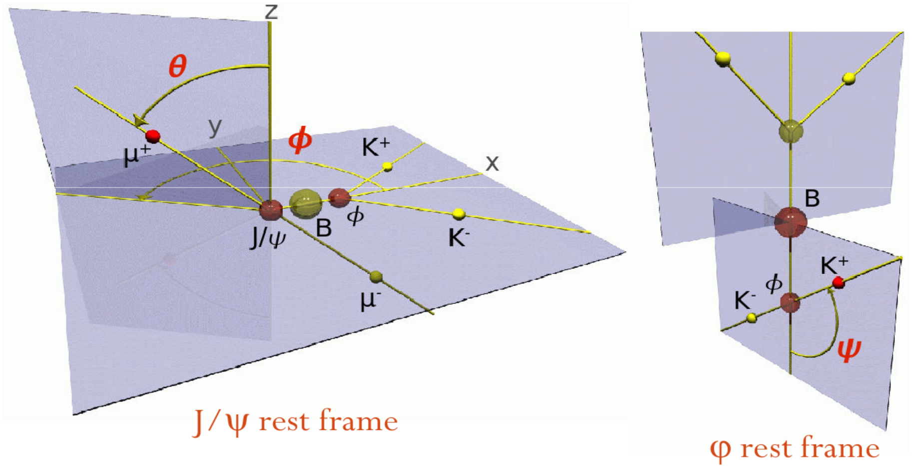
L	CP	Decay Amplitude
0	even	$A_{\parallel}$
1	odd	$A_{\perp}$
2	even	A_0

Angular analysis for separation of final state

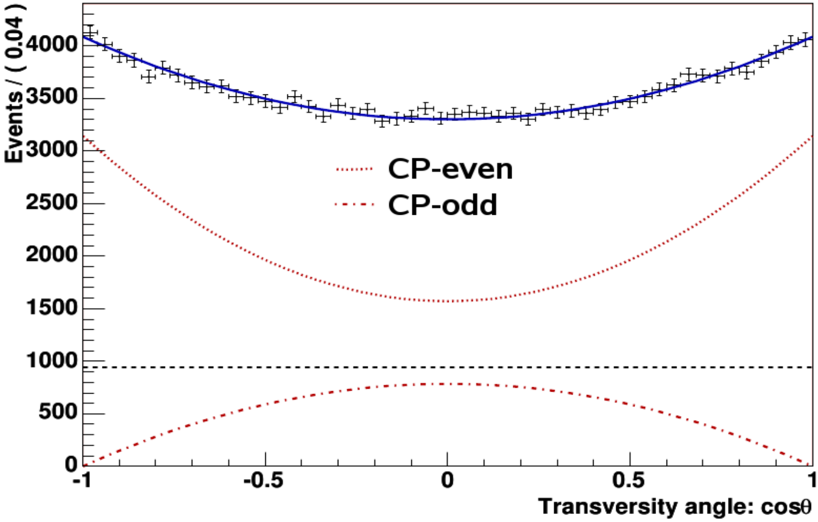
Need 3 angles to describe the final state

$\rightarrow$ **Transversity Base** $\vec{\rho} = (\Theta, \Psi, \Phi)$

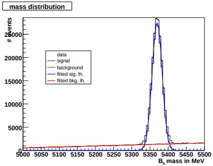
The Transversity Angles 2



The Transversity Angles 3: Separation



Now Put Everything Together



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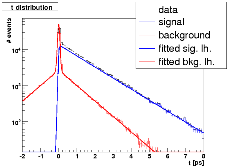
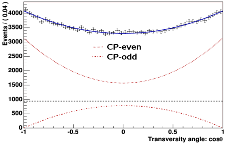


$\otimes$



Measured Quantities per event: $m_i, t_i, \vec{\rho}_i = (\Theta_i, \Psi_i, \Phi_i)$

Physical Parameters: $\Phi_s, \Gamma, \Delta\Gamma, |A_0|, |A_{||}|, |A_\perp|, \delta_1, \delta_2, \Delta m$

$$\mathscr{L}_{\text{tot}} = \prod_i^{\text{all events}} P_i$$

$$P_i = f_{\text{sig}} P_{\text{sig}} + (1 - f_{\text{sig}}) P_{\text{bkg}}$$

$$P_{\text{sig}} = P_{\text{sig}}^{t/\rho}(t_i, \vec{\rho}_i) \times P_{\text{sig}}^{\text{M}}(m_i)$$

$$P_{\text{bkg}} = P_{\text{bkg}}^{t/\rho}(t_i, \vec{\rho}_i) \times P_{\text{bkg}}^{\text{M}}(m_i)$$

Unfortunately, $P_{\text{sig}}^{t/\rho}$ cannot be separated in proper time/angles!

The Time/Angular Probability: Angular Dependance

$$P_{\text{sig}}^{t/\rho} = \overbrace{\frac{1+\xi D}{1+|\xi|}}^{\text{Prob. } B_s \text{ id.}} \overbrace{\sum_{k=1}^{6} A_{(k)}(t_i) f_{(k)}(\vec{\rho}_i)}^{|\langle J/\Psi\Phi|B_s\rangle|^2} + \overbrace{\frac{1-\xi D}{1+|\xi|}}^{\text{Prob. } \bar{B}_s \text{ id.}} \overbrace{\sum_{k=1}^{6} \bar{A}_{(k)}(t_i) f_{(k)}(\vec{\rho}_i)}^{|\langle J/\Psi\Phi|\bar{B}_s\rangle|^2}$$

$f_1 \ldots f_6$ are only dependent on the measured angles

k	$A_{(k)}(t)$	$\bar{A}_{(k)}(t)$	$f_{(k)}(\vec{\rho})$
1	$\|A_0(t)\|^2$	$\|\bar{A}_0(t)\|^2$	$\frac{9}{32\pi} 2\cos^2\psi\,(1-\sin^2\Theta\cos^2\phi)$
2	$\|A_\|(t)\|^2$	$\|\bar{A}_\|(t)\|^2$	$\frac{9}{32\pi}\sin^2\psi\,(1-\sin^2\Theta\sin^2\phi)$
3	$\|A_\perp(t)\|^2$	$\|\bar{A}_\perp(t)\|^2$	$\frac{9}{32\pi}\sin^2\psi\sin^2\Theta$
4	$Im(A_\|^*(t)A_\perp(t))$	$Im(\bar{A}_\|^*(t)\bar{A}_\perp(t))$	$-\frac{9}{32\pi}\sin^2\psi\sin 2\Theta\sin\phi$
5	$Re(A_0^*(t)A_\|(t))$	$Re(\bar{A}_0^*(t)\bar{A}_\|(t))$	$\frac{9}{32\pi\sqrt{2}}\sin 2\psi\sin^2\Theta\sin 2\phi$
6	$Im(A_0^*(t)A_\perp(t))$	$Im(\bar{A}_0^*(t)\bar{A}_\perp(t))$	$\frac{9}{32\pi\sqrt{2}}\sin 2\psi\sin 2\Theta\cos\phi$

Amplitudes on next slide

The Time/Angular Probability: Tagged

$$P_{\text{sig}}^{t/\rho} = \overbrace{\frac{1+\xi D}{1+|\xi|}}^{\text{Tagging}} \sum_{k=1}^{6} A_{(k)}(t_i) f_{(k)}(\vec{\rho}_i) + \overbrace{\frac{1-\xi D}{1+|\xi|}}^{\text{Tagging}} \sum_{k=1}^{6} \bar{A}_{(k)}(t_i) f_{(k)}(\vec{\rho}_i)$$

$$|A_0(t)|^2 = \frac{|A_0(0)|^2}{2}\left[(1+\cos\Phi_s)e^{-\Gamma_L t} + (1-\cos\Phi_s)e^{-\Gamma_H t} + 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$|A_{\|}(t)|^2 = \frac{|A_{\|}(0)|^2}{2}\left[(1+\cos\Phi_s)e^{-\Gamma_L t} + (1-\cos\Phi_s)e^{-\Gamma_H t} + 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$|A_{\perp}(t)|^2 = \frac{|A_{\perp}(0)|^2}{2}\left[(1-\cos\Phi_s)e^{-\Gamma_L t} + (1+\cos\Phi_s)e^{-\Gamma_H t} - 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$Im(A_{\|}^*(t)A_{\perp}(t)) = +|A_{\|}(0)||A_{\perp}(0)|[e^{-\Gamma t}(\sin\delta_1\cos(\Delta m t) - \cos\delta_1\sin(\Delta m t)\cos\Phi_s) - \frac{1}{2}(e^{-\Gamma_H t} - e^{-\Gamma_L t})\cos\delta_1\sin\Phi_s]$$

$$Re(A_0^*(t)A_{\|}(t)) = \frac{1}{2}|A_0(0)||A_{\|}(0)|\cos(\delta_2-\delta_1)\left[(1+\cos\Phi_s)e^{-\Gamma_L t} + (1-\cos\Phi_s)e^{-\Gamma_H t} + 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$Im(A_0^*(t)A_{\perp}(t)) = +|A_0(0)||A_{\perp}(0)|[e^{-\Gamma t}(\sin\delta_2\cos(\Delta m t) - \cos\delta_2\sin(\Delta m t)\cos\Phi_s) - \frac{1}{2}(e^{-\Gamma_H t} - e^{-\Gamma_L t})\cos\delta_2\sin\Phi_s]$$

Terms depending on Φ_s | Δm only with Tagging | 2-fold ambiguity in $\Delta\Gamma$

The Time/Angular Probability: UnTagged

$$P_{\text{sig}}^{t/\rho} = \overbrace{\frac{1}{2}}^{\text{No Tagging}} \sum_{k=1}^{6} A_{(k)}(t_i) f_{(k)}(\vec{\rho}_i) + \overbrace{\frac{1}{2}}^{\text{No Tagging}} \sum_{k=1}^{6} \bar{A}_{(k)}(t_i) f_{(k)}(\vec{\rho}_i)$$

$$|A_0(t)|^2 = \frac{|A_0(0)|^2}{2}\left[(1+\cos\Phi_s)e^{-\Gamma_L t} + (1-\cos\Phi_s)e^{-\Gamma_H t} + 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$|A_{\parallel}(t)|^2 = \frac{|A_{\parallel}(0)|^2}{2}\left[(1+\cos\Phi_s)e^{-\Gamma_L t} + (1-\cos\Phi_s)e^{-\Gamma_H t} + 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$|A_{\perp}(t)|^2 = \frac{|A_{\perp}(0)|^2}{2}\left[(1-\cos\Phi_s)e^{-\Gamma_L t} + (1+\cos\Phi_s)e^{-\Gamma_H t} - 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$Im(A_{\parallel}^*(t)A_{\perp}(t)) = +|A_{\parallel}(0)||A_{\perp}(0)|[e^{-\Gamma t}(\sin\delta_1\cos(\Delta m t) - \cos\delta_1\sin(\Delta m t)\cos\Phi_s) - \frac{1}{2}(e^{-\Gamma_H t} - e^{-\Gamma_L t})\cos\delta_1 \sin\Phi_s]$$

$$Re(A_0^*(t)A_{\parallel}(t)) = \frac{1}{2}|A_0(0)||A_{\parallel}(0)|\cos(\delta_2-\delta_1)\left[(1+\cos\Phi_s)e^{-\Gamma_L t} + (1-\cos\Phi_s)e^{-\Gamma_H t} + 2e^{-\Gamma t}\sin(\Delta m t)\sin\Phi_s\right]$$

$$Im(A_0^*(t)A_{\perp}(t)) = +|A_0(0)||A_{\perp}(0)|[e^{-\Gamma t}(\sin\delta_2\cos(\Delta m t) - \cos\delta_2\sin(\Delta m t)\cos\Phi_s) - \frac{1}{2}(e^{-\Gamma_H t} - e^{-\Gamma_L t})\cos\delta_2 \sin\Phi_s]$$

Less sensitivity on Φ_s

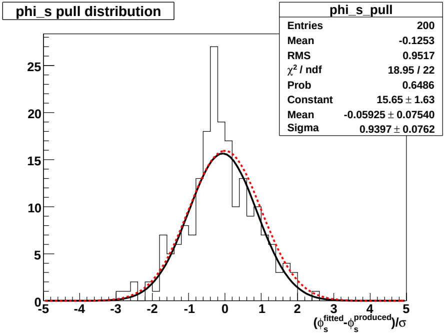
4-fold ambiguity in $\Phi_s, \Delta\Gamma$

Toy Studies: Fit Validation

- Is the Fit biased?
- Does it under/overestimate the errors?

$\Rightarrow$ Do Toy Study

1. Create large number of simulated data samples
2. Run Fit on those data samples
3. For Parameter a Pull $= \frac{a_{\text{fitted}} - a_{\text{generated}}}{\Delta a_{\text{fitted}}}$



$\Rightarrow$ **Expectation: Gaussian distribution with mean 0 and width 1**

(Expected) Performance Comparison

Quantity	CDF	D0	LHCb
N_{sig} σ_t ϵD^2	2.02k 0.1ps 3.6% /1.4%	1.97k 0.1ps 4.7%	131k (2fb^{-1}) 0.03ps 8%
$\Delta\Gamma$ Φ_s	[-0.4,0.4]ps^{-1} [0.32,2.82] / [0.40,1.20]	(0.19 ± 0.07)ps^{-1} -0.57 ± 0.30	(0.10 ± 0.008)ps^{-1} -0.04 ± 0.02

(Very) Preliminary Results

$\Delta\Phi_s$ Tagged

N_sig	$\Phi_s = -0.04$	$\Phi_s = -0.60$
6.4k		0.14
19.2k		0.08
180.0k	0.02	

$\Delta\Phi_s$ Untagged (δ_1, δ_2 fixed!)

N_sig	$\Phi_s = -0.04$	$\Phi_s = -0.60$
6.4k		
19.2k		
180.0k	0.08	