Physics beyond the Standard Model

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Scope

The lecture gives an introduction to physics beyond the Standard Model associated with new particles in the 100 GeV–10 TeV range.

The concept is bottom-up, starting from the Standard Model and its problems, with emphasis on the *standard way beyond the Standard Model*: Grand unification (and supersymmetry).

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The concept is bottom-up, starting from the Standard Model and its problems, with emphasis on the *standard way beyond the Standard Model*: Grand unification (and supersymmetry).

Not covered:
Search strategies for new particles at colliders, theories of gravitation, large extra dimensions, strongly interacting Higgs sectors, . . .

Contents

The theorist's toolbox

A theory's particles and their interactions are encoded in the Lagrangian $\mathcal{L}$. To construct a Lorentz-invariant theory specify:

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- the internal symmetries of $\mathcal{L}(\phi_j, \psi_k, A_l^\mu)$: Transforming

$$\phi_j \to U_{jj'}\phi_{j'}, \quad \psi_k \to U'_{kk'}\psi_{k'}, \quad A_l^\mu \to U''_{ll'}A_{l'}^\mu$$

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  The set G of matrices U form a *group*, meaning that $\mathbb{1} \in G$, with $U^{(1)}, U^{(2)} \in G$ also $U^{(1)}U^{(2)} \in G$, and for each $U \in G$ there is an inverse $U^{-1} \in G$. The corresponding matrices U', U'' fulfill the same multiplication law, e.g. $U^{(1)}U^{(2)} = U^{(3)} \Rightarrow U^{(1)\prime}U^{(2)\prime} = U^{(3)\prime}$. That is, the sets $\{U\}$, $\{U'\}$, and $\{U''\}$ are all *representations* of the symmetry group G.

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  leaves $\mathcal{L}$ invariant, $\mathcal{L} \to \mathcal{L}$. The set G of matrices U form a *group*.
- the representations of G according to which the fields $\phi_j(x), \psi_k(x), A_l^\mu(x)$ transform.
- whether $\mathcal{L}$ shall be *renormalisable by power-counting* or not.

More on symmetries

Example:

$$\text{SU(2)}=\{U \in \mathbb{C}^{2\times 2} : U^\dagger U = \mathbb{1} \text{ and } \det U = 1\}$$

$U(\vec{\phi}) = \exp[i\phi_j\sigma^j/2]$ is an SU(2) rotation with angle $\phi \equiv |\vec{\phi}|$ around the axis $\vec{\phi}/\phi$. Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

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Application: Weak isospin: The left-handed (left-chiral) fermion fields of the Standard Model,

$$L^1=\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \; L^2=\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \; L^3=\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \quad Q^1=\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; Q^2=\begin{pmatrix} c_L \\ s_L \end{pmatrix}, \; Q^3=\begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

transform under the weak SU(2) group according to the defining representation, e.g. $L^j \to \exp[i\phi_k\sigma^k/2]L^j$, they are doublets.

The right-handed fermion fields $e_R, \mu_R, \tau_R, u_R, d_R, s_R, c_R, b_R, t_R$ are singlets of SU(2), they live in the trivial representation: e.g. $e_R \to e_R$.

$SU(2)=\{\exp[i\phi_j\sigma^j/2] : \phi_{1,2,3}\in\mathbb{R}\}$

The Pauli matrices are the generators of SU(2), they satisfy the commutation relations

$$\left[\frac{\sigma^k}{2},\frac{\sigma^l}{2}\right]=i\epsilon_{klm}\frac{\sigma^m}{2}$$

with the Levi-Civita tensor defined by
$\epsilon_{123}=\epsilon_{231}=\epsilon_{312}=1,\ \ \epsilon_{lkm}=-\epsilon_{klm}$.

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$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1, \;\; \epsilon_{lkm} = -\epsilon_{klm}$.
The SU(2) matrices $U(\vec{\phi})$ are continuous functions of the parameters ϕ_j, making SU(2) a *Lie group* of matrices.
The generators of a Lie group span a vector space, the *Lie algebra*. The Lie algebra su(2) of SU(2) is spanned by
$\frac{\sigma^1}{2}, \frac{\sigma^2}{2}, \frac{\sigma^3}{2}$ and therefore consists of all hermitian 2×2 matrices with trace zero. (su(2) is a *real* Lie algebra, meaning that only linear combinations with real coefficents are allowed.)

Any set of matrices T^1, T^2, T^3 satisfying the commutation relations

$$\left[T^k, T^l\right] = i\epsilon_{klm} T^m$$

form a *representation* of su(2). The $3^3 = 27$ numbers ϵ_{klm} are the *structure constants* of su(2).

Important: adjoint representation

$$\left[T^k_{\mathrm{su}(2)}\right]_{lm} := -i\epsilon_{klm}$$

$$T^1_{\mathrm{su}(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^2_{\mathrm{su}(2)} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad T^3_{\mathrm{su}(2)} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The generators are related to quantum numbers (charges) of the fields:
Doublets: Take L^1 as an example:

$$\frac{(\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2}{4} L^1 = \frac{1}{2}\left(\frac{1}{2} + 1\right) L^1, \quad \frac{\sigma^3}{2} L^1 = \frac{1}{2}\begin{pmatrix} \nu_{eL} \\ -e_L \end{pmatrix}.$$

That is, the weak isospin quantum number I_W of the doublets is found as $I_W = 1/2$, the third component of the weak isospin is $I_W^3 = \pm 1/2$ for neutrino and electron, respectively.

Here I_W and I_W^3 are defined in analogy to the spin quantum numbers s and s_3 in quantum mechanics.

The adjoint representation has $I_W = 1$:

$$\left(T^1_{\mathrm{su}(2)}\right)^2 + \left(T^2_{\mathrm{su}(2)}\right)^2 + \left(T^3_{\mathrm{su}(2)}\right)^2 = 1\,(1+1)\,\mathbb{1}$$

Eigenvectors of $T^3_{\mathrm{su}(2)}$:

$$T^3_{\mathrm{su}(2)} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad T^3_{\mathrm{su}(2)} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0, \quad T^3_{\mathrm{su}(2)} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} = -\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}.$$

We find eigenvalues $I^3_W = 1, 0, -1$, i.e. the adjoint representation is a triplet.

Gauge principle

If the group parameters (ϕ_j in the case of SU(2)) depend on the space-time coordinate $x = (t, \vec{x})$, one calls the symmetry *local* or *gauged*.

Problem with derivatives, from extra term:

$\partial_\mu \exp\left[-i\phi(x)\right] = -i(\partial_\mu \phi(x)) \exp\left[-i\phi(x)\right]$

The group of phase transformation $\psi(x) \to e^{-i\phi}\psi$ is called U(1), for “unitary 1×1 matrices”. The structure constants of U(1) vanish, thus different U(1) transformations always commute and U(1) is called *Abelian*.

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The gauge group of Quantum Electrodynamics is U(1).

Ensure gauge invariance with a *gauge field A_μ* and *covariant derivative*

$$D_\mu = \partial_\mu + i g_e A_\mu$$

with $A_\mu \to A_\mu + \partial_\mu \phi(x)/g_e$. Need also kinetic term (i.e. bilinear term) for A^μ, conveniently expressed in terms of the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. With the electron field $\psi = e$ find *quantum electrodynamics*:

$$\mathcal{L}_{\text{QED}} = \overline{e}\left[i \not{D} - m\right] e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

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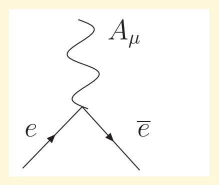
Interpretation: A_μ is the *photon field*. It mediates the electromagnetic force between electrons. $g_e \approx 0.30$ accompanying A_μ is the electromagnetic coupling constant.



Next SU(2): Consider lepton doublet $L^1 = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$: The covariant derivative involves three gauge fields $W^1_\mu, W^2_\mu, W^3_\mu$:

$$D_\mu = \partial_\mu - igW^a_\mu \frac{\sigma^a}{2}$$

and

$$\mathcal{L} = \bar{L} i \not{D} L - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$

Novel feature of *non-Abelian* gauge theory:

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{abc} W^b_\mu W^c_\nu$$

Thanks to the third term in $F^a_{\mu\nu}$ the lagrangian contains *self-interactions* of the gauge bosons.

Gauge bosons live in the adjoint representation $\left[T^k_{\mathrm{su}(2)}\right]_{lm} := -i\epsilon_{klm}$:

Three parameters needed to describe an SU(2) rotation multiply three Pauli matrices.

$\Rightarrow$ Adjoint representation consists of 3×3 matrices.

$\Rightarrow$ three W-bosons!

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One parameter needed to describe a U(1) phase rotation $\exp\left[-i\phi(x)\right]$. The structure constant is zero, because U(1) is Abelian.

$\Rightarrow$ Adjoint representation consists of the 1×1 matrix 0.

$\Rightarrow$ one gauge boson A_μ!

Recall the eigenvectors of of $T^3_{\mathrm{su}(2)}$:

$$\begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} = \frac{W^+}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} + \frac{W^-}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} + W^3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

with $W^\pm = \dfrac{W^1 \mp iW^2}{\sqrt{2}}$.

Hence

$$T^3_{\mathrm{su}(2)} \begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} = \frac{W^+}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} - \frac{W^-}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

and we realise that W^+, W^-, W^3 have the I^3_W quantum numbers $1, -1, 0,$ respectively.

Everything learned from U(1) and SU(2) generalises to other Lie groups as well:

We'll encounter SU(N) gauge theories with $N = 2, 3, 4, 5$:
The Lie algebra su(N) is spanned by $N^2 - 1$ traceless hermitian $N \times N$ matrices, therefore there are $N^2 - 1$ gauge bosons and the adjoint representation consists of $(N^2 - 1) \times (N^2 - 1)$ matrices.

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Other popular models of new physics involve SO(N), the group of real orthogonal $N \times N$ matrices with determinant equal to 1.
SO(N) has $N(N - 1)/2$ generators, which are traceless imaginary antisymmetric matrices.
SO(N) matrices describe rotations in $\mathbb{R}^n$.

Simple groups:

subgroup U of group G: $g_1, g_2 \in U \;\Rightarrow\; g_1 g_2 \in U$,
also: $g_2^{-1} g_1 g_2 \in U$

invariant subgroup U of G: $g_1 \in U, g_2 \in G \;\Rightarrow\; g_2^{-1} g_1 g_2 \in U$

A group G is called simple, if it has no invariant Lie subgroups (other than G and 1). The SU(N) and SO(N) groups (except for SO(4)) are simple.
E.g. SO(2) is a subgroup of SO(3), but it is not invariant.

With $g_1, g_1' \in G_1$ and $g_2, g_2' \in G_2$ the pairwise multiplication $(g_1, g_2) \cdot (g_1', g_2') = (g_1 g_1', g_2 g_2')$ defines another group, the direct product $G_1 \times G_2$.
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Physical significance: A direct product of gauge groups describes independent interactions. E.g. the gauge fields of $SU(2)$ (i.e. the W-bosons) carry no color or hypercharge.

Compare this with the electric charge:

The W-bosons carry electric charges!

$U(1)_{\mathrm{em}}$ is a non-invariant subgroup of $SU(2) \times U(1)_Y$.

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- completely fixes the form of the self-interaction of the gauge bosons (in terms of the structure constants), i.e. the three-boson and four-boson couplings,
- fixes the interaction of gauge bosons, once the group representations of the fermion fields are specified,
- involves only a single coupling constant g for the boson-fermion and all boson-boson couplings.

Electroweak interaction

Gauge group: $SU(2) \times U(1)_Y$

doublets: $Q_L^j = \begin{pmatrix} u_L^j \\ d_L^j \end{pmatrix}$ und $L^j = \begin{pmatrix} \nu_L^j \\ \ell_L^j \end{pmatrix}$

$j = 1, 2, 3$ labels the generation.

Examples: $Q_L^3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $L^1 = \begin{pmatrix} \nu^{eL} \\ e_L \end{pmatrix}$

singlets: u_R^j, d_R^j and e_R^j.

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Important: Only left-handed fields couple to the W boson.

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- three gauge interactions
- Yukawa interaction of Higgs with quarks and leptons
- Higgs self-interaction

Spontaneous symmetry breaking

Higgs doublet field $H = \begin{pmatrix} H_1^+ \\ H_2^0 \end{pmatrix}$ with hypercharge quantum number $y = 1/2$.
(Beware of different normalisations of y in the literature!)

The classical Higgs potential is chosen such that it develops minima with $H \neq 0$. To quantise the theory around this minimum identify the value v at which $|H|^2$ is minimal with the vacuum expectation value of the quantised Higgs field.

These minima are related by an $SU(2) \times U(1)_Y$ gauge transformation. Use this to choose $\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$ with

$v = 174\,\text{GeV} = \dfrac{246\,\text{GeV}}{\sqrt{2}}$.

Spontaneous symmetry breaking $SU(2) \times U(1)_Y \to U(1)_{\text{em}}$:

$$T^3 \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{\sigma^3}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0, \qquad Y \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$$

but

$$\left[T^3 + Y\mathbb{1}\right] \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{\sigma^3 + \mathbb{1}}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

and we recognise the electric-charge operator

$Q = T^3 + Y$ Gell-Mann–Nishijima relation

Note: The mass parameter μ^2 of the Higgs potential is the only dimensionful parameter of the SM Lagrangian. Its value is chosen to give the correct $v = \sqrt{\sqrt{2}/(4G_F)}$, with the Fermi constant G_F determined from muon decay. All masses of elementary particles are proportional to v.

Yukawa interaction

Higgs doublet $H = \begin{pmatrix} G^+ \\ v + \frac{h^0 + iG^0}{\sqrt{2}} \end{pmatrix}$ with $v = 174\,\mathrm{GeV}$.

Charge-conjugate doublet: $\widetilde{H} = \begin{pmatrix} v + \frac{h^0 - iG^0}{\sqrt{2}} \\ -G^- \end{pmatrix}$

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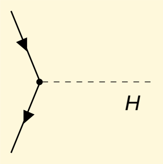

Yukawa lagrangian:

$$-L_Y = Y^d_{jk}\, \overline{Q}^j_L\, H\, d^k_R \; + \; Y^u_{jk}\, \overline{Q}^j_L\, \widetilde{H}\, u^k_R \; + \; Y^l_{jk}\, \overline{L}^j_L\, H\, e^k_R \; + \; \text{h.c.}$$

Here neutrinos are (still) massless.
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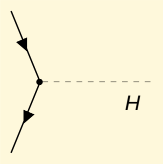

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Here neutrinos are (still) massless.
The Yukawa matrices Y^f are arbitrary complex 3×3 matrices.
The mass matrices $M^f = Y^f v$ are not diagonal!

$\Rightarrow$ $u^j_{L,R}, d^j_{L,R}$ do not describe physical quarks!
We must find a basis in which Y^f is diagonal!

Any matrix can be diagonalised by a bi-unitary transformation. Start with

$$\widehat{Y}^u = S_Q^\dagger Y^u S_u \quad \text{with } \widehat{Y}^u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad \text{and } y_{u,c,t} \geq 0$$

This can be achieved via

$$Q_L^j = S_{jk}^Q Q_L^{k\prime}, \qquad u_R^j = S_{jk}^u u_R^{k\prime}$$

with unitary 3×3 matrices S^Q, S^u.
This transformation leaves L_{gauge} invariant ("flavour-blindness of the gauge interactions")!

Next diagonalise Y^d:

$$\widehat{Y}^d = V^\dagger S_Q^\dagger Y^d S_d \quad \text{with } \widehat{Y}^d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad \text{and } y_{d,s,b} \geq 0$$

with unitary 3×3 matrices V, S^d.
Via $d_R^j = S_{jk}^d d_R^{k\prime}$ we leave L_{gauge} unchanged, while

$$-L_Y^{\text{quark}} = \overline{Q}_L V \widehat{Y}^d H \, d_R \; + \; \overline{Q}_L \widehat{Y}^u \widetilde{H} \, u_R \; + \; \text{h.c.}$$

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with unitary 3×3 matrices V, S^d.
Via $d_R^j = S_{jk}^d d_R^{k\prime}$ we leave L_{gauge} unchanged, while

$$-L_Y^{\text{quark}} = \overline{Q}_L V \widehat{Y}^d H\, d_R + \overline{Q}_L \widehat{Y}^u \widetilde{H}\, u_R + \text{h.c.}$$

To diagonalise $M^d = V \widehat{Y}^d v$ transform

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This breaks up the $SU(2)$ doublet Q_L.

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The transformation $d_L^j = V_{jk} d_L^{k\prime}$ changes the W-boson couplings in $\mathcal{L}_{\text{gauge}}$:

$$\mathcal{L}_W = \frac{g_2}{\sqrt{2}} \left[\overline{u}_L V \gamma^\mu \, d_L \, W_\mu^+ \; + \; \overline{d}_L V^\dagger \gamma^\mu \, u_L \, W_\mu^- \right]$$

The Z-boson couplings stay flavour-diagonal because of $V^\dagger V = 1$.

V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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No lepton-flavour violation!

$\Rightarrow$ Add a ν_R to the SM to mimick the quark sector or add a Majorana mass term $Y^M \frac{\overline{L}HH^T L^c}{M}$.

The lepton mixing matrix is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

Methodology of new physics searches

Renormalisability, power counting and decoupling:

The Standard-Model Lagrangian is renormalisable by power counting, meaning that it has no interaction terms beyond mass dimension 4.

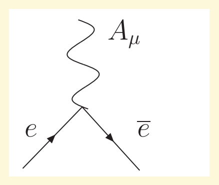
Counting rule:

Lagrangian:	$[\mathcal{L}] = 4$
fermion field:	$[\psi] = 3/2$
boson filed:	$[A_\mu] = [\phi] = 1$

Example:

Photon-electron coupling $\overline{e}\gamma^\mu e A_\mu g_e$:

$$2[e] + [A_\mu] = 2\frac{3}{2} + 1 = 4 \quad \Rightarrow \quad [g_e] = 0$$



In a renormalisable theory one can systematically calculate quantum corrections (loop corrections); the results are applicable to arbitarily high energies.

If a theory has couplings with negative dimension, one can still calculate quantum corrections, but the results are only meaningful for energies well below the mass scale inferred from these couplings.

If the Standard Model is an effective theory superseded by a more complete theory at higher energies, higher-dimension terms are perfectly allowed. Such terms arise automatically, if very heavy, yet undiscovered particles are present: At low energies the interaction mediated by such a particle appears point-like, e.g.

$$\frac{g^{\mu\nu}}{p^2 - M^2} \to -\frac{g^{\mu\nu}}{M^2} \qquad \text{for} \quad p^2 \ll M^2$$

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Famous example: Fermi theory valid for $p^2 \ll M_W^2$ and $G_F \propto 1/M_W^2$ has dimension -2.

Decoupling theorem of (Appelquist and Carazzone), applied to the Standard Model: If the Standard-Model Lagrangian is complemented by couplings to a new field with mass $M \gg v$, then all physical effects of this new field measurable at energies below M are encoded in higher-dimensional (i.e. dimension-5 and dimension-6) operators added to to the Standard-Model Lagrangian.

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The decoupling theorem does not hold, if masses are increased by increasing a coupling constant! A sequential fourth fermion generation involves masses proportional to Yukawa couplings, $m_{f'} = y_{f'} v$, and in a loop process involving $y_{f'}$ the increase of $y_{f'}$ can compensate for the $1/m_{f'}$ suppression of the loop integral!

Where to look for new physics?

Accidental features:

Properties of the Standard Model, which are not the consequences of the gauge symmetry $SU(3) \times SU(2) \times U(1)_Y$, are vulnerable to effects of new physics:

i) Effects of the chosen particle content of the Standard Model, in particular accidental symmetries,

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i) Effects of the chosen particle content of the Standard Model, in particular accidental symmetries,

ii) Transition amplitudes which are suppressed, because certain Standard-Model parameters are small. The smallness of parameters is often linked to approximate global symmetries.

Examples for i):

The Standard Model possesses only one Higgs doublet.

- Adding more Higgs doublets leads to flavour-changing neutral Higgs couplings like $\bar{s}dH^{0'}$ with dramatic consequences: Flavour-changing neutral current (FCNC) processes like $\mathrm{K}-\overline{\mathrm{K}}$ mixing, which only occur at loop level in the Standard Model, can now appear at tree-level: Enhancement factor of $16\pi^2 M_W^2/M_{\mathrm{new}}^2$.

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- Adding Higgs fields in other SU(2) representations (e.g. adding a Higgs triplet) will spoil the tree-level relation
$$1 = \frac{M_W^2}{M_Z^2 \cos\theta_W} = \frac{M_W^2}{M_Z^2\sqrt{1 - g_Y^2/g_2^2}}$$
by new tree-level contributions. (Loop corrections occur already in the Standard Model.)

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  The Standard Model has failed this test, because neutrinos have been found to oscillate!

Examples for ii):

- A CP-violating parameter $\theta_{\rm QCD}$ is experimentally found to be smaller than 10^{-10}, from searches for electric dipole moments (EDMs). Consequently EDMs probe models of new physics with new sources of flavour-diagonal CP-violation.

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- A CP-violating parameter $\theta_{\rm QCD}$ is experimentally found to be smaller than 10^{-10}, from searches for electric dipole moments (EDMs). Consequently EDMs probe models of new physics with new sources of flavour-diagonal CP-violation.
- Only one Yukawa coupling is large, $y_t \approx 1$. especially flavour-changing elements of the Yukawa matrices are small, exhibiting approximate flavour symmetries.

  $\Rightarrow$ FCNC processes are excellent for new physics searches.

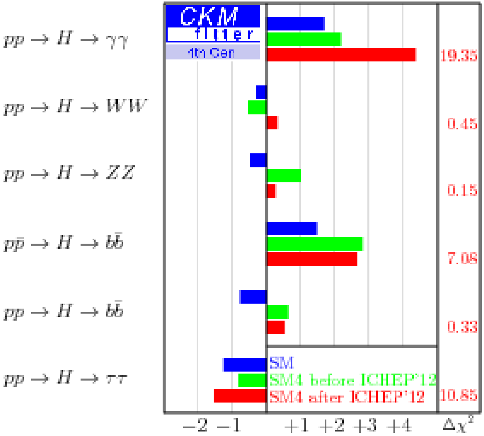
Model-independent approach

Compare the warehouse (nature) with the inventory list (Standard Model):

- How many Higgs doublets?
- How many fermion generations?

There are no theory reasons for having three generations, not even anthropic ones.

Fourth generation:



Deviations of Higgs signal strength from best-fit value:
blue: 3 generations,
red: 4 generations

A sequential fourth generation is ruled out at 5.3σ.

see: 1209.1101

Agnostic approach:

Add all possible dimension-5 and dimension-6 terms to $\mathcal{L}_{\mathrm{SM}}$:

Only one dimension-5 term:

$$L_M = \frac{1}{2}\left[Y^M M^{-1}\right]_{jk} \overline{L}_j \widetilde{H} H^\dagger L_k^c$$

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With $L_k^c = \begin{pmatrix} e_{L,k}^c \\ -\nu_{L,k}^c \end{pmatrix}$ and the Higgs vev find the neutrino Majorana mass term:

$$L_{\rm Maj} = -\frac{v^2}{2}\left[\left[Y^M M^{-1}\right]_{jk} \overline{\nu}_{L,j}\nu_{L,k}^c + \left[Y^M M^{-1}\right]_{kj}^\dagger \overline{\nu}_{L,k}^c \nu_{L,j}\right]$$

Transforming $\nu_{L,j} = U_{jl}\nu_{L,l}$ allows us to diagonalise $\left[Y^M M^{-1}\right]$ and puts the PMNS matrix U into the couplings of the W-boson.
$L_{\rm Maj}$ breaks lepton number L by two units and permits neutrinoless 2β decays.

Imposing baryon number conservation there are 59 dimension-6 terms, 15 without fermion fields, 19 with two fermion fields, and 25 with four fermion fields, not counting the flavour structure.
One finds 5 operators violating baryon number, i.e. transforming quarks into leptons ($\longrightarrow$ proton decay).

Iskrzynski et al., JHEP 1010 (2010) 085.

Physics beyond the Standard Model

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Do we need any physics?
Do we need new physics at the TeV scale?

Physics beyond the Standard Model: phenomena

- Gravity. It is associated with the Planck scale $M_P = G_N^{-1/2} \approx 10^{19}\,\mathrm{GeV}$.

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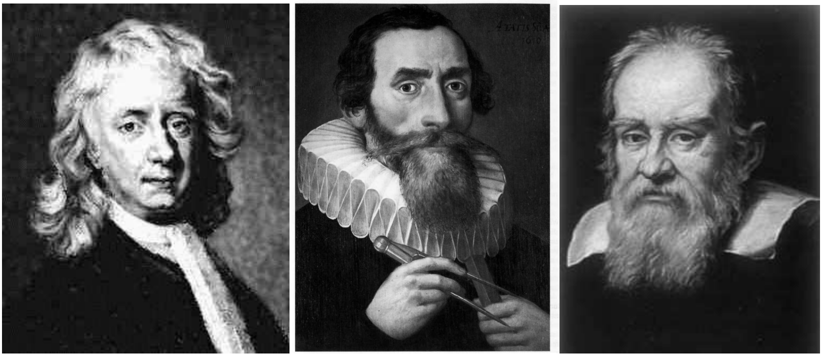
Pioneers of physics beyond the Standard Model:



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  We don't understand 95% of the universe's energy budget.

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  $\Rightarrow$ We don't understand the remaining 5% either!

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- Charge quantisation: $Q(\nu) = 0$ and $Q(e) = 3Q(d)$ to all digits behind the decimal point.

Charge quantisation

$$Q \quad = \quad T_3 \; + \; Y$$

↗ ↑ ↖

electric charge — three-component of the weak isospin — hypercharge

right-handed fermions: $T_3 = 0$
left-handed up-type fermions: $T_3 = 1/2$
left-handed down-type fermions: $T_3 = -1/2$

$U(1)_Y$ coupling: $\propto g_Y Y$.
The normalisations of the coupling g_Y and the charges are arbitrary.

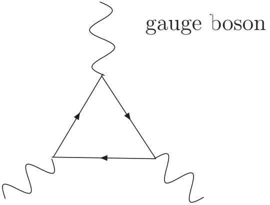
Y and therefore Q of any fermion could be any real number: π, $\sqrt{2}$, 1.602, $\ldots$

But: E.g. $Q(\nu) = 0$ and $Q(e) = 3Q(d)$ to all digits behind the decimal point, because neutrinos and atoms are electrically neutral.

Why is Y quantised?

Anomaly cancellation

Anomaly: breaking of a symmetry through quantum effects



The gauge symmetry $SU(3) \times SU(2)_L \times U(1)_Y$ must be anomaly-free...

...and it is thanks to

$$\sum_f Y(f) = 3[6Y(q_L) + 6Y(q_R) + 2Y(\ell_L) + Y(\ell_R)] = 0.$$

In the Standard Model (without $\nu_{R,k}$ fields) the requirement of anomaly cancellation fixes the hypercharges, and charges are correctly quantised.

The Standard Model (without neutrino masses) has four "accidental" global $U(1)$ symmetries:

baryon number B, lepton numbers L_e, L_μ, L_τ

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Once we add $\nu_{R,k}$ fields, we encounter a fine-tuning of Y:

We could add any multiple of $B - L$ to Y!

But out of these ∞ many choices all but one leads to charged atoms and neutrinos!

Why neutrino masses imply BSM physics:

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If we stay with the fields of the Standard Model, we must accomodate the neutrino masses through

$$L_{\mathrm{Maj}} = -\frac{v^2}{2}\left[\left[Y^M M^{-1}\right]_{jk} \overline{\nu}_{L,j}\nu^c_{L,k} + \left[Y^M M^{-1}\right]^\dagger_{kj} \overline{\nu}^c_{L,k}\nu_{L,j}\right].$$

But this non-renormalisable term implies a breakdown of the theory at and above the mass scale implied by M^{-1}, which is $10^{10\pm 6}$ GeV.

M^{-1} could stem from a Majorana mass term of very heavy singlet $\nu_{R,k}$ fields (when combined with Dirac mass terms this gives the famous see-saw mechanism).

But M^{-1} could also stem from heavy SU(2) Higgs triplet fields or from SU(2)-triplet fermion fields.

Towards unification of forces

In the Standard Model the hypercharge Y is tuned from the experimentally observed electric charges.

fermions:	$\begin{pmatrix} u_L, u_L, u_L \\ d_L, d_L, d_L \end{pmatrix}$	u_R, u_R, u_R	d_R, d_R, d_R	$\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}$	e_R
hypercharge Y:	$1/6$	$2/3$	$-1/3$	$-1/2$	-1

Is there a symmetry argument for Y?

$B-L$: baryon number minus lepton number

fermion:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	$\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}$	e_R	$\nu_{e,R}$
$Y-(B-L)/2$:	0	$1/2$	$-1/2$	0	$-1/2$	$1/2$

$\Rightarrow$ Is $Y-(B-L)/2$ the z-component of a right-handed isospin?

The magic relation $Y = T_3^R + (B-L)/2$ with a right-handed weak isospin T_3^R allows us to embed

$$U(1)_Y \quad \subset \quad SU(2)_R \times U(1)_{B-L}$$

Nice: The spontaneous symmetry breaking $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SU(3) \times SU(2)_L \times U(1)_Y$ also breaks U_{B-L} and induces Majorana masses for neutrinos. Covariant derivative for U_{B-L} with $B-L$ gauge field:

$$D_\mu = \partial_\mu - i g_{B-L} B'_\mu \frac{B-L}{2}$$

and $g_{B-L} = g_Y$

The saga continues:
Consider an $SU(3) \times U(1)_{B-L}$ transformation with $U_3 \in SU(3)$ and $\exp[i\phi(B-L)] \in U(1)_{B-L}$:

$$\begin{pmatrix} Q_L^r \\ Q_L^g \\ Q_L^b \\ L \end{pmatrix} \to \begin{pmatrix} & U_3 e^{i\phi/3} & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix} \begin{pmatrix} Q_L^r \\ Q_L^g \\ Q_L^b \\ L \end{pmatrix} =: U_4 \begin{pmatrix} Q_L^r \\ Q_L^g \\ Q_L^b \\ L \end{pmatrix}$$

Note that $\det U = 1$! Since $U_4 \in SU(4)$, we can embed $SU(3) \times U(1)_{B-L} \subset SU(4)$. This works, because $\mathrm{tr}\,(B-L) = 0$ here, i.e. the $B-L$ quantum numbers of the left-handed quarks and leptons doublets sum to zero. (Same holds for the right-handed doublets.)

Therefore the Left-right model with gauge group $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ can be further unified to the Pati-Salam model with gauge group

$$SU(4) \times SU(2)_L \times SU(2)_R .$$

Title of the 1974 Pati-Salam paper:

Lepton number as the fourth color

There are $4^2 - 1$ generators $T^a = \lambda^a/2$, i.e. 15 gauge fields A^a_μ. By convention, A^{15}_μ is the $B - L$ gauge field. The normalisation of the coupling g_{B-L} is fixed now by the value of g_3, but needs to be rescaled compared to the conventional definition:

$$T^{15} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} = \sqrt{\frac{3}{8}}(B - L)$$

Since the $SU(4)$ and $SU(3)$ couplings are the same, we find $g_{B-L} = \sqrt{\frac{3}{2}} g_3$.
No U(1) factor in the gauge group: The charge quantisation problem is solved!

SU(4) gauge bosons: A_μ^a with $a = 1, \dots 8$ are the gluons. A_μ^{15} is the $B - L$ gauge boson, the remaining 6 gauge bosons are the "“new gluons” linking the quarks to the leptons. They have electric charge $2/3$ and couple e.g. to electron and down-quark; another coupling is to neutrino and up-quark. But: No proton decay!

Typical experimental signature: decays violating both lepton and quark flavour, e.g. $K^0 \to \mu^+ e^-$.

SU(5)

The fermions also magically fit into $SU(5)$ multiplets:

$$\underline{5}^* \equiv \begin{pmatrix} d^c \\ d^c \\ d^c \\ e_L \\ -\nu_{e,L} \end{pmatrix} \qquad \underline{10} \equiv \begin{pmatrix} 0 & u^c & -u^c & u_L & d_L \\ -u^c & 0 & u^c & u_L & d_L \\ u^c & -u^c & 0 & u_L & d_L \\ -u_L & -u_L & -u_L & 0 & e^c \\ -d_L & -d_L & -d_L & -e^c & 0 \end{pmatrix}$$

Here the superscript c denotes antiparticle fields of right–handed fermions.

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- two of the four $SU(3)$ triplets are $SU(2)$ singlets and the other two combine to $SU(2)$ doublets,
- the remaining three colourless fields form a singlet and a doublet with respect to $SU(2)$.

Gauge bosons

There are $5^2 - 1 = 24$ gauge bosons A^a_μ, with 8 gluons, the 3 W-bosons, and the hypercharge boson $A^{24}_\mu = B_\mu$.

The coupling is rescaled as $g_Y =: -\sqrt{\frac{3}{5}} g_1 = -\sqrt{\frac{3}{5}} g$ in terms of the SU(5) coupling g.

Note that the three couplings are equal,

$$g_1 = g_2 = g_3 = g,$$

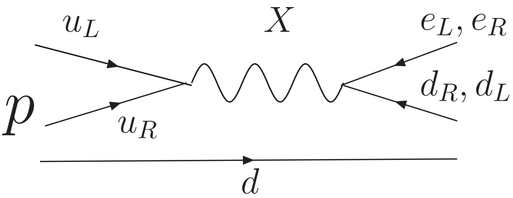
at and above the GUT scale M_{GUT} at which the SU(5) is an unbroken symmetry. The couplings run with energy (renormalisation group evolution) and are very different at the low energies probed by experiment.

The couplings g_1, g_2, g_3 indeed converge and intersect around $M_{\text{GUT}} \approx 10^{15}$ GeV, but the unification is imperfect.

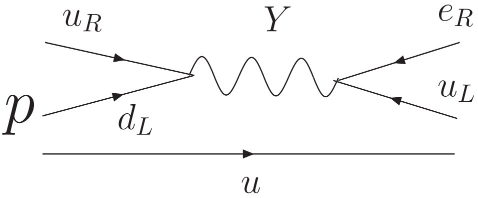
Proton decay

The remaining 12 real gauge bosons form a weak doublet $\begin{pmatrix} X_\mu^a \\ Y_\mu^a \end{pmatrix}$, $a = 1, 2, 3$ of complex color triplets (just like left-handed quark doublets). The electric charges are $4/3$ for X_μ^a and $1/3$ for Y_μ^a.

An important feature of SU(5) is the possibility of proton decay mediated by the X and Y bosons.



$p \to e^+\pi^0$



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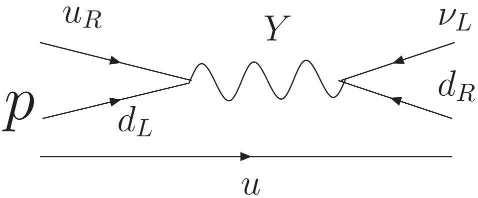

$p \to \overline{\nu}\pi^+$

SO(10)

Even better: The 15 fermion fields of each Standard Model generation and an extra right-handed neutrino field fit into a $\underline{16}$ of

$$SO(10) \supset SU(5)$$

In an $SO(10)$ GUT $U(1)_{B-L}$ is gauged and broken at the $SO(10)$-breaking scale M_{10}.

With appropriate Higgs fields the right-handed neutrino field ν_R gets a Majorana mass of the order of M_{10}. The light neutrino masses come out with (almost) the right size through the see–saw formula:

$$\mathcal{L}_{\rm mass} \supset -(\overline{\nu}_L, \overline{\nu_R^c}) \begin{pmatrix} 0 & Y_D v \\ Y_D^T v & Y_M M_{10} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Three eigenvalues are $\mathcal{O}(M_{10})$, the other three are $\mathcal{O}(v^2/M_{10})$ and the neutrinos are Majorana fermions.

$SO(10)$

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- $U(1)_{B-L}$ is gauged and broken at the $SO(10)$ breaking scale.
  - $\Rightarrow$ attractive mechanism for leptogenesis and baryogenesis.

Ende GUT - alles GUT?

$$SU(3) \times SU(2)_L \times U(1)_Y$$

$$\swarrow \qquad \qquad \searrow$$

$$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \qquad \qquad SU(5)$$

$$\searrow \qquad \qquad \swarrow$$

$$SO(10)$$

$v = 174\,\text{GeV}$ and M_{GUT} are separated by a factor $M_{\text{GUT}}/v \approx 10^{13}$. Quantum corrections of particles with mass M_{GUT} destabilise the electroweak scale, adding a term of order $M_{\text{GUT}}^2/(16\pi^2)$ to v^2 and the Higgs mass M_h^2. Technically, this is no problem, since we can cancel this contribution by a finite counterterm δM_h^2, but this involves fine-tuning of 24 digits. This is the gauge hierarchy problem.

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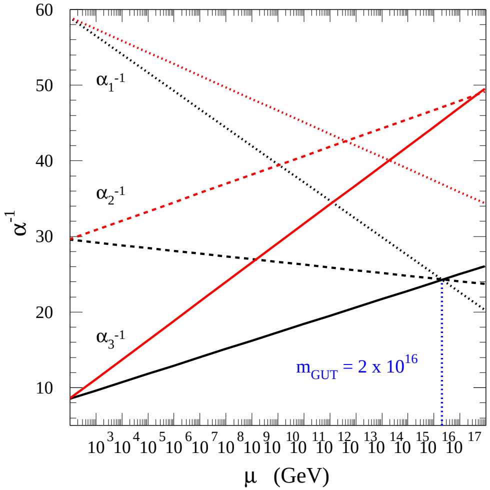
The only known way to solve the fine-tuning problem in a way resulting in a theory valid up to the GUT scale involves supersymmetry.
In supersymmetric theories all Standard-Model fermions have scalar partners, the squarks and sleptons. The superpartners of the bosons are spin-1/2 particles, the gauginos and higgsinos.

Supersymmetry

- tames the quantum corrections to the Higgs mass,
- provides a dark-matter candidate, the lightest supersymmetric particle (LSP),
- improves the unification of gauge couplings required by GUTs,
- can link gravity to the other gauge interactions.

Inverse gauge couplings with and without supersymmetry:



The GUT scale determined from the couplings agrees sufficiently well with the right-handed neutrino mass.

Probing new physics with flavour

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Spectacular: In FCNC transitions of charged leptons the GIM suppression factor is even m_ν^2/M_W^2!

$\Rightarrow$ The SM predictions for charged-lepton FCNCs are essentially zero!

The suppression of FCNC processes in the Standard Model is not a consequence of the $SU(3) \times SU(2)_L \times U(1)_Y$ symmetry. It results from the particle content of the Standard Model and the accidental smallness of most Yukawa couplings. It is absent in generic extensions of the Standard Model.

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Examples:

extra Higgses	$\Rightarrow$	Higgs-mediated FCNC's at tree-level , helicity suppression possibly absent,
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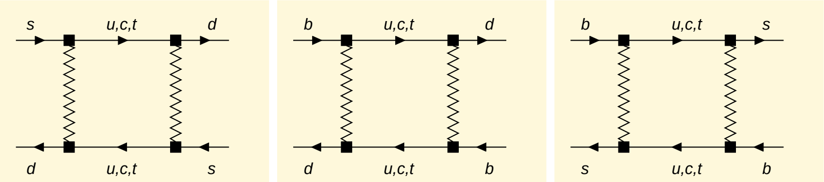
$B_d - \overline{B}_d$ mixing and $B_s - \overline{B}_s$ mixing are sensitive to scales up to $\Lambda \sim 100$ TeV.

Meson-antimeson mixing

Important new-physics analysers are the meson-antimeson mixing amplitudes:



$K - \overline{K}$ mixing $B_d - \overline{B}_d$ mixing $B_s - \overline{B}_s$ mixing

Global analysis of $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing with A. Lenz and the CKMfitter Group (J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker, S. Monteil, V. Niess) arXiv:1008.1593, 1203.0238

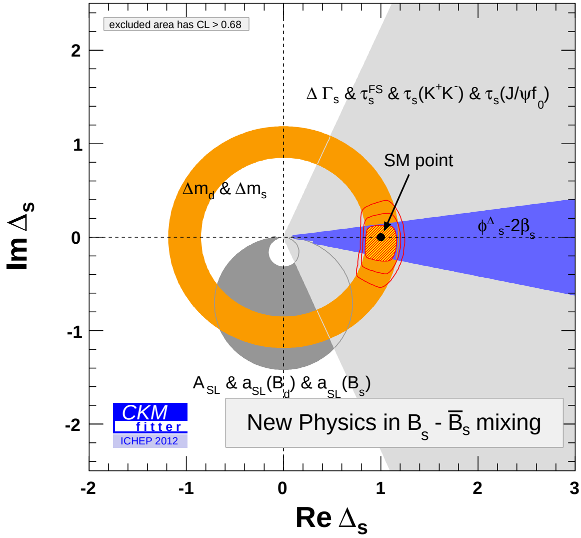
Rfit method: No statistical meaning is assigned to systematic errors and theoretical uncertainties.

We have performed a simultaneous fit to the Wolfenstein parameters and to the new physics parameters Δ_s and Δ_d:

$$\Delta_q \equiv \frac{M_{12}^q}{M_{12}^{q,\mathrm{SM}}}, \qquad \Delta_q \equiv |\Delta_q| e^{i\phi_q^\Delta}.$$

Trouble maker: A_{SL} measurement by DØ.
Small tension form world average of $B^+ \to \tau^+ \nu_\tau$.

CKMfitter September 2012 update of 1203.0238:



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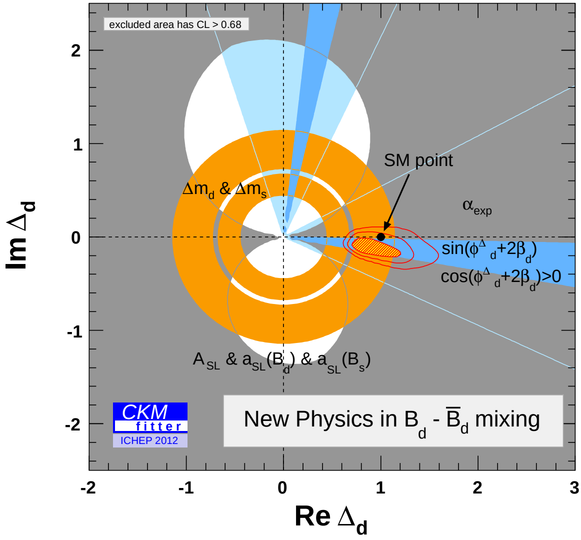

$A_{\rm SL}$ and WA for $B(B \to \tau\nu)$ prefer small $\phi_d^\Delta < 0$.

Pull value for $A_{\rm SL}$: 3.3σ

$\Rightarrow$ Scenario with NP in M_{12}^q only cannot accomodate the DØ measurement of $A_{\rm SL}$.

The Standard Model point $\Delta_s = \Delta_d = 1$ is disfavoured by 1σ, down from the 2010 value of 3.6σ.

Supersymmetry and flavour

The Minimal Supersymmetric Standard Model (MSSM) has many new sources of flavour violation, all in the supersymmetry-breaking sector.

No problem to get big effects in a certain FCNC amplitudes, but rather to suppress the big effects elsewhere.

Squark mass matrix

Diagonalise the Yukawa matrices Y^u_{jk} and Y^d_{jk}

$\Rightarrow$ quark mass matrices are diagonal, super-CKM basis

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E.g. Down-squark mass matrix:

$$M^2_{\tilde{d}} = \begin{pmatrix} \left(M^{\tilde{d}}_{1L}\right)^2 & \Delta^{\tilde{d}\,LL}_{12} & \Delta^{\tilde{d}\,LL}_{13} & \Delta^{\tilde{d}\,LR}_{11} & \Delta^{\tilde{d}\,LR}_{12} & \Delta^{\tilde{d}\,LR}_{13} \\ \Delta^{\tilde{d}\,LL}_{12}{}^{*} & \left(M^{\tilde{d}}_{2L}\right)^2 & \Delta^{\tilde{d}\,LL}_{23} & \Delta^{\tilde{d}\,RL}_{12}{}^{*} & \Delta^{\tilde{d}\,LR}_{22} & \Delta^{\tilde{d}\,LR}_{23} \\ \Delta^{\tilde{d}\,LL}_{13}{}^{*} & \Delta^{\tilde{d}\,LL}_{23}{}^{*} & \left(M^{\tilde{d}}_{3L}\right)^2 & \Delta^{\tilde{d}RL}_{13}{}^{*} & \Delta^{\tilde{d}\,RL*}_{23} & \Delta^{\tilde{d}\,LR}_{33} \\ \Delta^{\tilde{d}\,LR}_{11}{}^{*} & \Delta^{\tilde{d}RL}_{12} & \Delta^{\tilde{d}RL}_{13} & \left(M^{\tilde{d}}_{1R}\right)^2 & \Delta^{\tilde{d}\,RR}_{12} & \Delta^{\tilde{d}\,RR}_{13} \\ \Delta^{\tilde{d}\,LR}_{12}{}^{*} & \Delta^{\tilde{d}\,LR*}_{22} & \Delta^{\tilde{d}RL}_{23} & \Delta^{\tilde{d}\,RR}_{12}{}^{*} & \left(M^{\tilde{d}}_{2R}\right)^2 & \Delta^{\tilde{d}\,RR}_{23} \\ \Delta^{\tilde{d}\,LR}_{13}{}^{*} & \Delta^{\tilde{d}\,LR}_{23}{}^{*} & \Delta^{\tilde{d}\,LR}_{33}{}^{*} & \Delta^{\tilde{d}\,RR}_{13}{}^{*} & \Delta^{\tilde{d}\,RR}_{23}{}^{*} & \left(M^{\tilde{d}}_{3R}\right)^2 \end{pmatrix}$$

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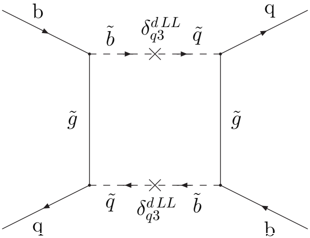
Not diagonal! $\Rightarrow$ new FCNC transitions.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q
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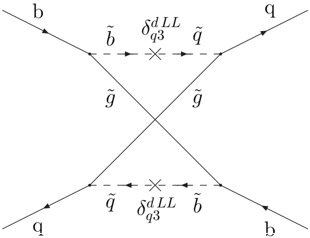
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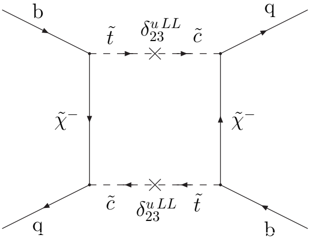
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$\tilde{\chi}^-$
$\tilde{c}$
$\delta^{u\,LL}_{23}$
$\tilde{t}$
q
b

The stringent lower bounds on the squark masses set by ATLAS and CMS make supersymmetry an imperfect solution to the fine-tuning problem involving M_{GUT} and the electroweak scale v.

At the same time the supersymmetric flavour problem is alleviated.

$\Rightarrow$ Minimal Flavour Violation less compelling.

Model-independent analyses constrain

$$\delta_{ij}^{q\,XY} = \frac{\Delta_{ij}^{\tilde{q}\,XY}}{\frac{1}{6}\sum_s \left[M_{\tilde{q}}^2\right]_{ss}} \qquad \text{with } XY = LL, LR, RR \text{ and } q = u, d$$

using data on FCNC (and also charged-current) processes.

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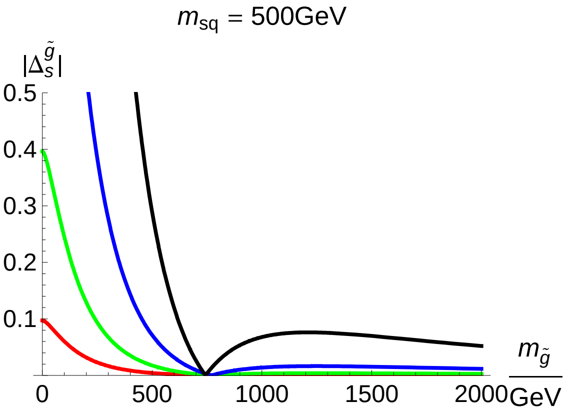
- To derive meaningful bounds on $\delta_{ij}^{q\,LR}$ chirally enhanced higher-order contributions must be taken into account.

A. Crivellin, UN, 2009

Ratio of gluino and Standard-Model contribution to $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing:



The gluino contribution vanishes for $M_{\tilde{g}} \approx 1.5 M_{\tilde{q}}$, independently of the size of $\Delta_{23}^{d\,LL}$ (curves correspond to 4 different values).

Conclusions

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- There is physics beyond the Standard Model: lepton-flavour violation seen in neutrino oscillations, dark matter, the surplus of matter over antimatter in the universe, and gravity.
- The quantum numbers of the Standard-Model fermions point towards grand unification, with preferred gauge group SO(10).
- GUTs explain small but non-zero neutrino masses in a natural way.

Conclusions

- The convergence of the gauge couplings is largely improved in the MSSM, which alleviates the fine-tuning problem induced by the gauge hierarchy. With the LHC lower bounds on squark masses the answer of the MSSM to the fine-tuning problem is imperfect.

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- FCNC processes are sensitive probes of new physics, especially of the supersymmetry-breaking sector.

A pinch of new physics in $B - \overline{B}$ mixing?