Higgs Theory

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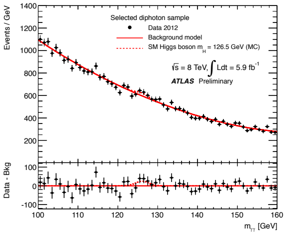
Higgs discovery

Best of ATLAS [and CMS]

– 'silver channel' $H \to \gamma\gamma$

local significance 4.5σ (ATLAS), 4.1σ (CMS)
correct background treatment beneficial



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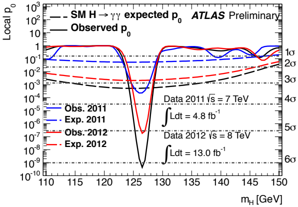

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- 'golden channel' $H \to ZZ \to 4\ell$

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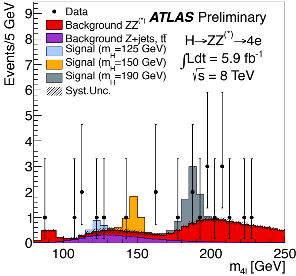
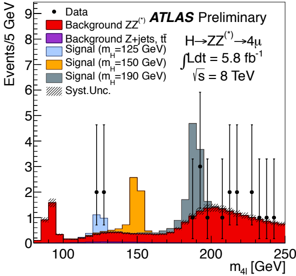
  low event count





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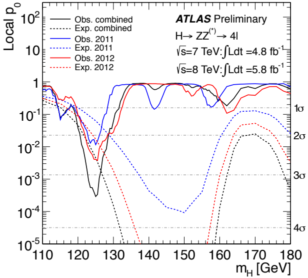

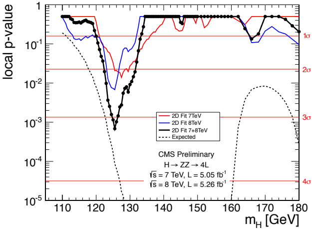

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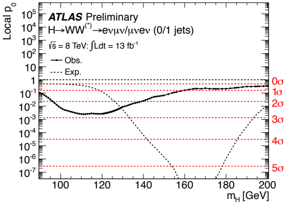
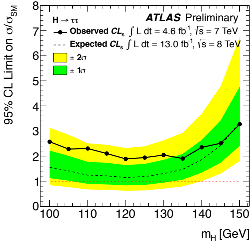
– remaining WW and $\tau\tau$, bb

broad excess, bb not sensitive to SM rates





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Massive Photon and Goldstone theorem

How to make a photon massive (or why $2 \neq 3$)

$$\begin{aligned}\mathscr{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 f^2 A_\mu^2 + \frac{1}{2} (\partial_\mu \phi)^2 - ef A_\mu \partial^\mu \phi \\ &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 f^2 \left(A_\mu - \frac{1}{ef} \partial_\mu \phi \right)^2\end{aligned}$$

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$$\begin{aligned}F_{\mu\nu}\Big|_B &= \partial_\mu B_\nu - \partial_\nu B_\mu = \partial_\mu \left(A_\nu - \frac{1}{ef} \partial_\nu \phi \right) - \partial_\nu \left(A_\mu - \frac{1}{ef} \partial_\mu \phi \right) \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}\Big|_A\end{aligned}$$

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$\Rightarrow$ Goldstone's theorem

If a global symmetry group is spontaneously broken into a group of lower rank, its broken generators correspond to physical Goldstone modes. These scalar fields transform non-linearly under the larger and linearly under the smaller group. This way they are massless and cannot form a potential, because the non-linear transformation only allows derivative terms in the Lagrangian.

One common modification of this situation is an explicit breaking of the smaller symmetry group. In that case the Goldstone modes become pseudo–Goldstones and acquire a mass of the size of this hard-breaking term.

Massive Photon and Goldstone theorem

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$\Rightarrow$ Higgs mechanism

In the special case that the spontaneously broken symmetry is a local gauge symmetry the Goldstone theorem does not apply. Instead of becoming massless scalars the Goldstone modes are then 'eaten' by the additional degrees of freedom of the massive gauge bosons. The gauge boson mass is given by the vacuum expectation value breaking the larger symmetry. A massive additional scalar degree of freedom, the Higgs boson, appears if there are more Goldstone modes than degrees of freedom for the massive gauge bosons.

Sigma model

Fermion masses and $SU(2)_L$ invariance

$$U(x) = \exp\left(i\alpha^a(x)\frac{\tau_a}{2} \right) \equiv e^{i(\alpha\cdot\tau)/2}$$

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$$L_L \stackrel{U}{\to} UL_L \qquad Q_L \stackrel{U}{\to} UQ_L$$
$$L_R \stackrel{U}{\to} L_R \qquad Q_R \stackrel{U}{\to} Q_R$$

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$$\overline{Q}_L\Sigma m_Q Q_R \stackrel{U}{\to} \overline{Q}_L U^{-1}\Sigma^{(U)} m_Q Q_R \stackrel{!}{=} \overline{Q}_L \Sigma m_Q Q_R \qquad \Leftrightarrow \qquad \Sigma \to \Sigma^{(U)} = U\Sigma$$

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Invariant Lagrangian: masses and potential

$$\mathscr{L}_{D3} = -\overline{Q}_L \Sigma m_Q Q_R - \overline{L}_L \Sigma m_L L_R + \text{h.c.} + \dots$$

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$$\mathscr{L}_{D2} = - \frac{v^2}{4} \; \text{Tr}[V_\mu V^\mu] + \Delta\rho \frac{v^2}{8} \; \text{Tr}[T V_\mu] \; \text{Tr}[T V^\mu]$$

$$V_\mu \equiv \Sigma (D_\mu \Sigma)^\dagger \qquad\qquad T \equiv \Sigma \tau_3 \Sigma^\dagger$$

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$$V_\mu \equiv \Sigma (D_\mu \Sigma)^\dagger \qquad T \equiv \Sigma \tau_3 \Sigma^\dagger$$

$$\mathscr{L}_\Sigma = -\frac{\mu^2 v^2}{4} \text{Tr}(\Sigma^\dagger \Sigma) - \frac{\lambda v^4}{16} \left(\text{Tr}(\Sigma^\dagger \Sigma) \right)^2 + \cdots$$

Sigma model

Choice of fields: unitary gauge $\Sigma(x) = 1\!\!1$

$$
\begin{aligned}
V_\mu = \Sigma(D_\mu \Sigma)^\dagger &= 1\!\!1 (D_\mu \Sigma)^\dagger \\
&= -igW_\mu^a \frac{\tau_a}{2} + ig' B_\mu \frac{\tau_3}{2} \\
&= -igW_\mu^+ \frac{\tau_+}{\sqrt{2}} - igW_\mu^- \frac{\tau_-}{\sqrt{2}} - igW_\mu^3 \frac{\tau_3}{2} + ig' B_\mu \frac{\tau_3}{2} \\
&= -i \frac{g}{\sqrt{2}} \left(W_\mu^+ \tau_+ + W_\mu^- \tau_- \right) - ig_Z Z_\mu \frac{\tau_3}{2}
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$$\begin{aligned} \mathrm{Tr}[V_\mu V^\mu] &= -2 \frac{g^2}{2} W_\mu^+ W^{-\,\mu} \, \mathrm{Tr}(\tau_+ \tau_-) - \frac{g_Z^2}{4} Z_\mu Z^\mu \, \mathrm{Tr}(\tau_3^2) \\ &= -g^2 W_\mu^+ W^{-\,\mu} - \frac{g_Z^2}{2} Z_\mu Z^\mu \end{aligned}$$

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T &= \Sigma \tau_3 \Sigma^\dagger = \tau_3 \\
\text{Tr}(T V_\mu) &= \text{Tr} \left(-i g_Z Z_\mu \frac{\tau_3^2}{2} \right) = -i g_Z Z_\mu \\
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$$\begin{aligned}\mathscr{L}_{D2} &= \frac{v^2}{4}\left(-g^2\,W^+_\mu W^{-\mu} - \frac{g_Z^2}{2}\,Z_\mu Z^\mu\right) - \Delta\rho\,\frac{v^2}{8}\left(-g_Z^2 Z_\mu Z^\mu\right)\\ &= -\frac{v^2g^2}{4}\,W^+_\mu W^{-\mu} - \frac{v^2g_Z^2}{8}\,Z_\mu Z^\mu + \Delta\rho\,\frac{v^2g_Z^2}{8}\,Z_\mu Z^\mu\\ &= -\frac{v^2g^2}{4}\,W^+_\mu W^{-\mu} - \frac{v^2g_Z^2}{8}\,(1-\Delta\rho)\,Z_\mu Z^\mu\end{aligned}$$

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&= -\frac{v^2 g^2}{4} W_\mu^+ W^{-\,\mu} - \frac{v^2 g_Z^2}{8}\,(1-\Delta\rho)\,Z_\mu Z^\mu
\end{aligned}$$

$\Rightarrow$ gauge boson masses

$$\boxed{m_W = \frac{gv}{2}} \qquad\qquad \boxed{m_Z = \sqrt{1-\Delta\rho}\;\frac{g_Z v}{2} \overset{\Delta\rho=0}{=} \frac{g_Z v}{2} = \frac{gv}{2c_w}}$$

Sigma model

Other forms of $\Sigma(x)$ including Goldstones $\vec{w}$

minimum requirement

$$\frac{1}{2} \langle \mathrm{Tr}(\Sigma^\dagger(x)\Sigma(x)) \rangle = 1 \qquad \Leftarrow \qquad \Sigma^\dagger(x)\Sigma(x) = \mathbb{1} \qquad \forall x$$

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$$\Sigma(x) = \frac{1}{\sqrt{1+\frac{w_a w_a}{v^2}}}\left(\mathbb{1} - \frac{i}{v}\vec{w}(x)\right) \qquad \text{with} \quad \vec{w}(x) = w_a(x)\tau_a$$

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$$\begin{aligned}
\Sigma(x) &= \exp\left(-\frac{i}{v}\vec{w}(x)\right) \\
&= \mathbb{1} - \frac{i}{v}\vec{w} + \frac{1}{2}\frac{(-1)}{v^2} w_a\tau_a w_b\tau_b + \frac{1}{6}\frac{i}{v^3} w_a\tau_a w_b\tau_b w_c\tau_c + \mathcal{O}(w^4) \\
&= \mathbb{1} - \frac{i}{v}\vec{w} - \frac{1}{2v^2} w_a w_a \mathbb{1} + \frac{i}{6v^3} w_a w_a \vec{w} + \mathcal{O}(w^4) \\
&= \left(1 - \frac{1}{2v^2} w_a w_a + \mathcal{O}(w^4)\right)\mathbb{1} - \frac{i}{v}\left(1 - \frac{1}{6v^2} w_a w_a + \mathcal{O}(w^4)\right)\vec{w}
\end{aligned}$$

needed for $W_L W_L$ scattering

Quantized sigma field

Including all degrees of freedom

$$\Sigma \to \left(1+\frac{H}{v}\right)\Sigma \quad \text{with} \quad \frac{1}{2}\left\langle \text{Tr}(\Sigma^\dagger \Sigma)\right\rangle = \left\langle \left(1+\frac{H}{v}\right)^2 \right\rangle \equiv 1 \quad \Leftrightarrow \quad \langle H \rangle = 0$$

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$$\Sigma = \left(1+\frac{H}{v}\right)\mathbb{1} - \frac{i}{v}\vec{w} = \frac{1}{v}\begin{pmatrix} v+H-iw_3 & -w_2-iw_1 \\ w_2-iw_1 & v+H+iw_3 \end{pmatrix} = \frac{\sqrt{2}}{v}\,(\tilde{\phi}\phi)$$

$$\text{with} \quad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -w_2-iw_1 \\ v+H+iw_3 \end{pmatrix} \qquad \tilde{\phi} = -i\tau_2\,\phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H-iw_3 \\ w_2-iw_1 \end{pmatrix}$$

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$$\Sigma = \left(1 + \frac{H}{v}\right) \mathbb{1} - \frac{i}{v} \vec{w} = \frac{1}{v} \begin{pmatrix} v + H - iw_3 & -w_2 - iw_1 \\ w_2 - iw_1 & v + H + iw_3 \end{pmatrix} = \frac{\sqrt{2}}{v} (\tilde{\phi} \phi)$$

with $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -w_2 - iw_1 \\ v + H + iw_3 \end{pmatrix} \qquad \tilde{\phi} = -i\tau_2 \, \phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H - iw_3 \\ w_2 - iw_1 \end{pmatrix}$

Higgs Lagrangian

$$\mathscr{L}_{D3} \to -y_f \frac{(v + H)}{\sqrt{2}} \, \overline{\psi}_f \psi_f \supset -\frac{y_f}{\sqrt{2}} \, H \overline{\psi}_f \psi_f$$

Quantized sigma field

Including all degrees of freedom

$$\Sigma = \left(1 + \frac{H}{v}\right) 1\!\!1 - \frac{i}{v}\vec{w} = \frac{1}{v}\begin{pmatrix} v + H - iw_3 & -w_2 - iw_1 \\ w_2 - iw_1 & v + H + iw_3 \end{pmatrix} = \frac{\sqrt{2}}{v}\,(\tilde{\phi}\phi)$$

$$\text{with} \quad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -w_2 - iw_1 \\ v + H + iw_3 \end{pmatrix} \qquad \tilde{\phi} = -i\tau_2\,\phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H - iw_3 \\ w_2 - iw_1 \end{pmatrix}$$

Higgs Lagrangian

$$\mathscr{L}_{D3} \to -y_f\frac{(v+H)}{\sqrt{2}}\,\overline{\psi}_f\psi_f \supset -\frac{y_f}{\sqrt{2}}\,H\overline{\psi}_f\psi_f$$

$$\begin{aligned}
\mathscr{L}_{D2} &= -\frac{(v+H)^2 g^2}{4}\,W^+_\mu W^{-\mu} - \frac{(v+H)^2 g_Z^2}{8}\,(1+\Delta\rho)\,Z_\mu Z^\mu \\
&\supset -\frac{2vHg^2}{4}\,W^+_\mu W^{-\mu} - \frac{2vHg_Z^2}{8}\,(1+\Delta\rho)\,Z_\mu Z^\mu \\
&= -gm_W\,HW^+_\mu W^{-\mu} - \frac{g_Z m_Z}{2}\,(1+\Delta\rho)\,HZ_\mu Z^\mu
\end{aligned}$$

Quantized sigma field

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$$\mathscr{L}_\Sigma = -\frac{\mu^2 v^2}{2}\left(1+\frac{H}{v}\right)^2 - \frac{\lambda v^4}{4}\left(1+\frac{H}{v}\right)^4 + \dots$$

Quantized sigma field

For Uli Nierste: two Higgs doublets

dividing $v^2 = v_u^2 + v_d^2$

$$\mathscr{L}_{D2} = -\frac{v_u^2}{2} \text{ Tr}\left[V_\mu^{(u)} V^{(u)\mu} \right] - \frac{v_d^2}{2} \text{ Tr}\left[V_\mu^{(d)} V^{(d)\mu} \right]$$

Quantized sigma field

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fermion masses (type-II 2HDM)

$$\mathscr{L}_{D3} = -\overline{Q}_L m_{Qu} \Sigma_u \frac{\mathbb{1} + \tau_3}{2} Q_R - \overline{Q}_L m_{Qd} \Sigma_d \frac{\mathbb{1} - \tau_3}{2} Q_R + \dots$$

Quantized sigma field

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Higgs fields

$$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} \text{Re} H_u^+ + i\,\text{Im} H_u^+ \\ v_u + \text{Re} H_u^0 + i\,\text{Im} H_u^0 \end{pmatrix} \qquad \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} v_d + \text{Re} H_d^0 + i\,\text{Im} H_d^0 \\ \text{Re} H_d^- + i\,\text{Im} H_d^- \end{pmatrix}$$

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supersymmetric potential

$$\begin{aligned} V = & \frac{|\mu|^2 + m_{H_u}^2}{2}\left(|H_u^+|^2 + |H_u^0|^2\right) + \frac{|\mu|^2 + m_{H_d}^2}{2}\left(|H_d^0|^2 + |H_d^-|^2\right) \\ & + \frac{b}{2}\left(H_u^+ H_d^- - H_u^0 H_d^0 + \text{h.c.}\right) \\ & + \frac{g^2 + g'^2}{16}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^-|^2 - |H_d^0|^2\right)^2 + \frac{g^2}{4}|H_u^+ H_d^0 + H_u^0 H_d^-|^2 \end{aligned}$$

Quantized sigma field

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$$\mathscr{L}_{D3} = -\overline{Q}_L m_{Qu} \Sigma_u \frac{1\!\!1 + \tau_3}{2} Q_R - \overline{Q}_L m_{Qd} \Sigma_d \frac{1\!\!1 - \tau_3}{2} Q_R + \dots$$

Higgs fields

$$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} \text{Re}H_u^+ + i\,\text{Im}H_u^+ \\ v_u + \text{Re}H_u^0 + i\,\text{Im}H_u^0 \end{pmatrix} \qquad \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} v_d + \text{Re}H_d^0 + i\,\text{Im}H_d^0 \\ \text{Re}H_d^- + i\,\text{Im}H_d^- \end{pmatrix}$$

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$$\begin{aligned} V = & \frac{|\mu|^2 + m_{H_u}^2}{2} \left(|H_u^+|^2 + |H_u^0|^2\right) + \frac{|\mu|^2 + m_{H_d}^2}{2} \left(|H_d^0|^2 + |H_d^-|^2\right) \\ & + \frac{b}{2}\left(H_u^+ H_d^- - H_u^0 H_d^0 + \text{h.c.}\right) \\ & + \frac{g^2 + g'^2}{16}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^-|^2 - |H_d^0|^2\right)^2 + \frac{g^2}{4}|H_u^+ H_d^0 + H_u^0 H_d^-|^2 \end{aligned}$$

masses

$$\left(\mathcal{M}^2\right)_{jk} = \frac{\partial^2 V}{\partial H_j^0 \partial H_k^0}\bigg|_{\text{minimum}}$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \; , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

Higgs potential

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first operator, wave function renormalization

$$\begin{aligned} \mathcal{O}_1 &= \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \\ &= \frac{1}{2} \partial_\mu \left(\frac{(\tilde{H} + v)^2}{2} \right) \partial^\mu \left(\frac{(\tilde{H} + v)^2}{2} \right) \\ &= \frac{1}{2} \, (\tilde{H} + v)^2 \, \partial_\mu \tilde{H} \, \partial^\mu \tilde{H} \end{aligned}$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi)\, \partial^\mu (\phi^\dagger \phi) \; , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

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$$\begin{aligned} \mathcal{O}_1 &= \frac{1}{2} \partial_\mu (\phi^\dagger \phi)\, \partial^\mu (\phi^\dagger \phi) \\ &= \frac{1}{2} \partial_\mu \left(\frac{(\tilde{H}+v)^2}{2} \right) \partial^\mu \left(\frac{(\tilde{H}+v)^2}{2} \right) \\ &= \frac{1}{2}\, (\tilde{H}+v)^2\, \partial_\mu \tilde{H}\, \partial^\mu \tilde{H} \end{aligned}$$

$$\mathscr{L}_\text{kin} = \frac{1}{2} \partial_\mu \tilde{H} \partial^\mu \tilde{H} \left(1 + \frac{f_1 v^2}{\Lambda^2} \right) \stackrel{!}{=} \frac{1}{2} \partial_\mu H\, \partial^\mu H \qquad \Leftrightarrow \qquad H = \sqrt{1 + \frac{f_1 v^2}{\Lambda^2}}\, \tilde{H}$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \; , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

second operator, potential

$$V = \mu^2 |\phi|^2 + \lambda |\phi|^4 + \frac{f_2}{3\Lambda^2} |\phi|^6$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \; , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

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$$V = \mu^2 |\phi|^2 + \lambda |\phi|^4 + \frac{f_2}{3\Lambda^2} |\phi|^6$$

$$\frac{\partial V}{\partial \, |\phi|^2} = \mu^2 + 2\lambda |\phi|^2 + \frac{3 f_2}{3\Lambda^2} |\phi|^4 \stackrel{!}{=} 0 \qquad \Leftrightarrow \qquad |\phi|^4 + \frac{2\lambda\Lambda^2}{f_2} |\phi|^2 + \frac{\mu^2 \Lambda^2}{f_2} \stackrel{!}{=} 0$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \, , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

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$$\frac{\partial V}{\partial |\phi|^2} = \mu^2 + 2\lambda |\phi|^2 + \frac{3 f_2}{3\Lambda^2} |\phi|^4 \stackrel{!}{=} 0 \qquad \Leftrightarrow \qquad |\phi|^4 + \frac{2\lambda\Lambda^2}{f_2} |\phi|^2 + \frac{\mu^2 \Lambda^2}{f_2} \stackrel{!}{=} 0$$

$$\begin{aligned}
\frac{v^2}{2} &= -\frac{\lambda\Lambda^2}{f_2} \pm \left[\left(\frac{\lambda\Lambda^2}{f_2} \right)^2 - \frac{\mu^2 \Lambda^2}{f_2} \right]^{\frac{1}{2}} = \frac{\lambda\Lambda^2}{f_2} \left[-1 \pm \sqrt{1 - \frac{\mu^2 f_2}{\Lambda^2 \lambda^2}} \right] \\
&= \frac{\lambda\Lambda^2}{f_2} \left[-1 \pm \left(1 - \frac{f_2 \mu^2}{2\lambda^2\Lambda^2} - \frac{f_2^2 \mu^4}{8\lambda^4\Lambda^4} + \mathcal{O}(\Lambda^{-6}) \right) \right] \\
&= \begin{cases} -\dfrac{\mu^2}{2\lambda} - \dfrac{f_2 \mu^4}{8\lambda^3\Lambda^2} + \mathcal{O}(\Lambda^{-4}) = -\dfrac{\mu^2}{2\lambda} \left(1 + \dfrac{f_2 \mu^2}{4\lambda^2\Lambda^2} \right) \equiv \dfrac{v_0^2}{2} \left(1 + \dfrac{f_2 v_0^2}{4\lambda\Lambda^2} \right) \\ -\dfrac{2\lambda\Lambda^2}{f_2^2} + \mathcal{O}(\Lambda^0) \end{cases}
\end{aligned}$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \, , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

mass and couplings

$$\begin{aligned} \mathcal{O}_2 &= -\frac{1}{3} (\phi^\dagger \phi)^3 = -\frac{1}{3} \frac{(\tilde{H} + v)^6}{8} \\ &= -\frac{1}{24} \left(\tilde{H}^6 + 6\tilde{H}^5 v + 15\tilde{H}^4 v^2 + 20\tilde{H}^3 v^3 + 15\tilde{H}^2 v^4 + 6\tilde{H} v^5 + v^6 \right) \end{aligned}$$

Higgs potential

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$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \, , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

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$$\begin{aligned} \mathscr{L}_{\text{mass}} &= -\frac{\mu^2}{2} \tilde{H}^2 - \frac{3}{2} \lambda v^2 \tilde{H}^2 - \frac{f_2}{\Lambda^2} \frac{15}{24} v^4 \tilde{H}^2 \\ &= -\lambda v^2 \left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{f_2 v^2}{2\Lambda^2 \lambda} + \mathcal{O}(\Lambda^{-4}) \right) H^2 \stackrel{!}{=} -\frac{m_H^2}{2} H^2 \end{aligned}$$

$$\Leftrightarrow \qquad m_H^2 = 2\lambda v^2 \left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{f_2 v^2}{2\Lambda^2 \lambda} \right)$$

Higgs potential

Potential including dimension-6 operators

$$\mathscr{L}_{D6} = \sum_{i=1}^{2} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad \text{with} \quad \mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \, \partial^\mu (\phi^\dagger \phi) \, , \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

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$$\begin{aligned} \mathcal{O}_2 = -\frac{1}{3}(\phi^\dagger \phi)^3 &= -\frac{1}{3} \frac{(\tilde{H}+v)^6}{8} \\ &= -\frac{1}{24} \left(\tilde{H}^6 + 6\tilde{H}^5 v + 15\tilde{H}^4 v^2 + 20\tilde{H}^3 v^3 + 15\tilde{H}^2 v^4 + 6\tilde{H} v^5 + v^6 \right) \end{aligned}$$

$$\begin{aligned} \mathscr{L}_\text{mass} &= -\frac{\mu^2}{2}\tilde{H}^2 - \frac{3}{2}\lambda v^2 \tilde{H}^2 - \frac{f_2}{\Lambda^2}\frac{15}{24} v^4 \tilde{H}^2 \\ &= -\lambda v^2 \left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{f_2 v^2}{2\Lambda^2 \lambda} + \mathcal{O}(\Lambda^{-4}) \right) H^2 \stackrel{!}{=} -\frac{m_H^2}{2} H^2 \\ \Leftrightarrow \qquad m_H^2 &= 2\lambda v^2 \left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{f_2 v^2}{2\Lambda^2 \lambda} \right) \end{aligned}$$

$$\begin{aligned} \mathscr{L}_\text{self} = &- \frac{m_H^2}{2v} \left[\left(1 - \frac{f_1 v^2}{2\Lambda^2} + \frac{2 f_2 v^4}{3\Lambda^2 m_H^2} \right) H^3 - \frac{2 f_1 v^2}{\Lambda^2 m_H^2} H \, \partial_\mu H \, \partial^\mu H \right] \\ &- \frac{m_H^2}{8v^2} \left[\left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{4 f_2 v^4}{\Lambda^2 m_H^2} \right) H^4 - \frac{4 f_1 v^2}{\Lambda^2 m_H^2} H^2 \, \partial_\mu \, H \partial^\mu H \right] \, . \end{aligned}$$

Unitary *WW* scattering

Equivalence theorem and Goldstone scattering

$$\epsilon^\mu_{T,1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon^\mu_L = \frac{1}{m_V} \begin{pmatrix} |\vec{p}| \\ 0 \\ 0 \\ E \end{pmatrix} \stackrel{E \gg m_V}{\longrightarrow} \frac{1}{m_V} \begin{pmatrix} |\vec{p}| \\ 0 \\ 0 \\ |\vec{p}| \end{pmatrix} \equiv \frac{1}{m_V}\, p^\mu$$

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relevant Lagrangian in terms of Goldstones

$$V \supset \frac{m_H^2}{2v^2} w_+ w_- w_+ w_- + \frac{m_H^2}{v} H w_+ w_- + \cdots$$

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$$V \supset \frac{m_H^2}{2v^2} w_+ w_- w_+ w_- + \frac{m_H^2}{v} H w_+ w_- + \cdots$$

scattering amplitude

$$\begin{aligned} A &= \frac{-2im_H^2}{v^2} + \left(\frac{-im_H^2}{v} \right)^2 \frac{i}{s - m_H^2} + \left(\frac{-im_H^2}{v} \right)^2 \frac{i}{t - m_H^2} \\ &= -\frac{im_H^2}{v^2} \left[2 + \frac{m_H^2}{s - m_H^2} + \frac{m_H^2}{t - m_H^2} \right] \end{aligned}$$

Unitary *WW* scattering

Optical theorem and unitarity

optical theorem

$$1\!\!1 \stackrel{!}{=} S^\dagger S = (1\!\!1 - iA^\dagger)(1\!\!1 + iA) = 1\!\!1 + i(A - A^\dagger) + A^\dagger A \qquad \Leftrightarrow \qquad A^\dagger A = -i(A - A^\dagger)$$

$$\Rightarrow \quad -i\langle j|A - A^{*\,T}|j\rangle = 2\,\mathrm{Im}A(\theta = 0) \quad \Rightarrow \quad \boxed{\sigma \equiv \frac{1}{2s}\langle j|A^\dagger A|j\rangle = \frac{1}{s}\,\mathrm{Im}A(\theta = 0)}$$

Unitary *WW* scattering

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partial waves

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(c_\theta)\, a_l \qquad \text{with} \qquad \int_{-1}^{1} dx\, P_l(x) P_{l'}(x) = \frac{2}{2l+1}\delta_{ll'}$$

Unitary *WW* scattering

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$$\sigma = \int d\Omega\, \frac{|A|^2}{64\pi^2 s} = \frac{(16\pi)^2}{64\pi^2 s}\, 2\pi \int_{-1}^{1} dc_\theta \sum_l \sum_{l'} (2l+1)(2l'+1)\; a_l a_{l'}^*\; P_l(c_\theta) P_{l'}(c_\theta)$$

$$= \frac{8\pi}{s} \sum_l 2(2l+1)\; |a_l|^2 = \frac{16\pi}{s} \sum_l (2l+1)\; |a_l|^2\; .$$

Unitary *WW* scattering

Optical theorem and unitarity

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partial waves

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(c_\theta)\, a_l \qquad \text{with} \qquad \int_{-1}^{1} dx\, P_l(x) P_{l'}(x) = \frac{2}{2l+1}\delta_{ll'}$$

$$\sigma = \int d\Omega\, \frac{|A|^2}{64\pi^2 s} = \frac{(16\pi)^2}{64\pi^2 s}\, 2\pi \int_{-1}^{1} dc_\theta \sum_l \sum_{l'} (2l+1)(2l'+1)\; a_l a_{l'}^*\; P_l(c_\theta) P_{l'}(c_\theta)$$

$$= \frac{8\pi}{s} \sum_l 2(2l+1)\;|a_l|^2 = \frac{16\pi}{s} \sum_l (2l+1)\;|a_l|^2\;.$$

combined to $\displaystyle \frac{16\pi}{s}(2l+1)\;|a_l|^2 = \frac{1}{s}\;\mathrm{Im}A(\theta=0)\Big|_l = \frac{1}{s}\;16\pi(2l+1)\;\mathrm{Im}\,a_l$

$$(\mathrm{Re}\,a_l)^2 + \left(\mathrm{Im}\,a_l - \frac{1}{2}\right)^2 = \frac{1}{4} \qquad \Rightarrow \qquad \boxed{|\mathrm{Re}\,a_l| < \frac{1}{2}}\;.$$

Unitary *WW* scattering

Higgs mass limit

$$a_0 = \frac{1}{16\pi s} \int_{-s}^{0} dt \, |A| \; = \; \frac{1}{16\pi s} \int_{-s}^{0} dt \, \frac{m_H^2}{v^2} \left[2 + \frac{m_H^2}{s - m_H^2} + \frac{m_H^2}{t - m_H^2} \right]$$

$$= \frac{m_H^2}{16\pi v^2} \left[2 + \frac{m_H^2}{s - m_H^2} - \frac{m_H^2}{s} \log \left(1 + \frac{s}{m_H^2} \right) \right]$$

$$= \frac{m_H^2}{16\pi v^2} \left[2 + \mathcal{O} \left(\frac{m_H^2}{s} \right) \right] \; .$$

Unitary *WW* scattering

Higgs mass limit

$$\begin{aligned} a_0 = \frac{1}{16\pi s} \int_{-s}^{0} dt \, |A| &= \frac{1}{16\pi s} \int_{-s}^{0} dt \, \frac{m_H^2}{v^2} \left[2 + \frac{m_H^2}{s - m_H^2} + \frac{m_H^2}{t - m_H^2} \right] \\ &= \frac{m_H^2}{16\pi v^2} \left[2 + \frac{m_H^2}{s - m_H^2} - \frac{m_H^2}{s} \log \left(1 + \frac{s}{m_H^2} \right) \right] \\ &= \frac{m_H^2}{16\pi v^2} \left[2 + \mathcal{O} \left(\frac{m_H^2}{s} \right) \right] \; . \end{aligned}$$

$$\frac{m_H^2}{8\pi v^2} < \frac{1}{2} \qquad \Leftrightarrow \qquad \boxed{m_H^2 < 4\pi v^2 = (870 \text{ GeV})^2}$$

assuming all Higgs couplings as predicted!

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

Landau pole at large λ

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{2Q}\,\frac{d\,\lambda}{d\,Q} = \frac{1}{16\pi^2}\,12\lambda^2 + \mathcal{O}(\lambda) = \frac{3}{4\pi^2}\lambda^2 + \mathcal{O}(\lambda)$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

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$$\frac{d\,\lambda}{d\log Q^2} = \frac{d}{d\log Q^2}\,\frac{1}{g} = -\frac{1}{g^2}\,\frac{d\,g}{d\log Q^2} \stackrel{!}{=} \frac{3}{4\pi^2}\,\frac{1}{g^2}$$

$$\frac{d\,g}{d\log Q^2} = -\frac{3}{4\pi^2} \qquad g(Q^2) = -\frac{3}{4\pi^2}\log Q^2 + C$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

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$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{2Q}\frac{d\,\lambda}{d\,Q} = \frac{1}{16\pi^2}\,12\lambda^2 + \mathcal{O}(\lambda) = \frac{3}{4\pi^2}\lambda^2 + \mathcal{O}(\lambda)$$

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$$\frac{d\,g}{d\log Q^2} = -\frac{3}{4\pi^2} \qquad g(Q^2) = -\frac{3}{4\pi^2}\log Q^2 + C$$

$$g_0 = \frac{1}{\lambda_0} = -\frac{3}{4\pi^2}\log v^2 + C \quad\Leftrightarrow\quad C = g_0 + \frac{3}{4\pi^2}\log v^2$$

$$g(Q^2) = -\frac{3}{4\pi^2}\log Q^2 + g_0 + \frac{3}{4\pi^2}\log v^2 = -\frac{3}{4\pi^2}\log\frac{Q^2}{v^2} + g_0$$

$$\Leftrightarrow \quad \lambda(Q^2) = \left[-\frac{3}{4\pi^2}\log\frac{Q^2}{v^2} + \frac{1}{\lambda_0}\right]^{-1} = \lambda_0\left[1 - \frac{3}{4\pi^2}\lambda_0\log\frac{Q^2}{v^2}\right]^{-1}$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

Landau pole at large λ

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{2Q}\,\frac{d\,\lambda}{d\,Q} = \frac{1}{16\pi^2}\,12\lambda^2 + \mathcal{O}(\lambda) = \frac{3}{4\pi^2}\lambda^2 + \mathcal{O}(\lambda)$$

$$g_0 = \frac{1}{\lambda_0} = -\frac{3}{4\pi^2}\log v^2 + C \quad\Leftrightarrow\quad C = g_0 + \frac{3}{4\pi^2}\log v^2$$

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pole condition as upper limit on m_H

$$1 - \frac{3}{4\pi^2}\lambda_0\log\frac{Q^2_\text{pole}}{v^2} \stackrel{!}{=} 0 \qquad\Leftrightarrow\qquad \frac{3}{4\pi^2}\lambda_0\log\frac{Q^2_\text{pole}}{v^2} = 1$$

$$\Leftrightarrow\qquad \log\frac{Q^2_\text{pole}}{v^2} = \frac{4\pi^2}{3\lambda_0}$$

$$\Leftrightarrow\qquad Q_\text{pole} = v\;\exp\frac{2\pi^2}{3\lambda_0} = v\;\exp\frac{4\pi^2 v^2}{3m_H^2}$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

stability bound avoiding $\lambda < 0$

$$\frac{d\,\lambda}{d\log Q^2} = \frac{1}{16\pi^2}\left[-3\frac{4m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2 + g_1^2\right)^2\right) + \mathcal{O}(\lambda)\right]$$

$$\lambda(Q^2) \sim \lambda(v^2) + \frac{1}{16\pi^2}\left[-\frac{12m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2 + g_1^2\right)^2\right)\right]\log\frac{Q^2}{v^2}$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

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$$\lambda(v^2) = \frac{m_H^2}{2v^2} \stackrel{!}{=} -\frac{1}{16\pi^2}\left[-\frac{12m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2 + g_1^2\right)^2\right)\right]\log\frac{Q^2_\text{stable}}{v^2}$$

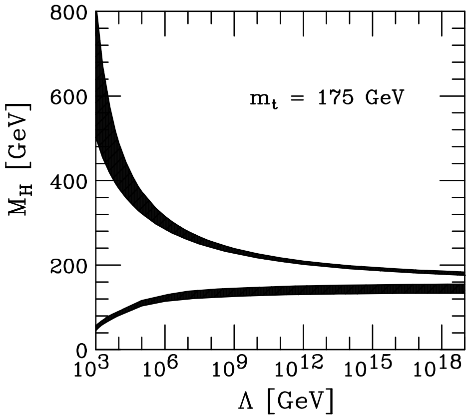
$$\frac{m_H^2}{v^2} = \frac{1}{8\pi^2}\left[\frac{12m_t^4}{v^4} - \frac{3}{16}\left(2g_2^4 + \left(g_2^2 + g_1^2\right)^2\right)\right]\log\frac{Q^2_\text{stable}}{v^2}$$

$$m_H = \begin{cases} 70\text{ GeV} & \text{for} \quad Q_\text{stable} = 10^3\text{ GeV} \\ 130\text{ GeV} & \text{for} \quad Q_\text{stable} = 10^{16}\text{ GeV} \end{cases} .$$

Higgs renormalization group

Constraints from scale dependent potential

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$



Top-Higgs renormalization group

Two-dimensional IR fixed point

renormalization group equation for λ

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4\right)$$

IR fixed point

$$\lim_{\log Q^2 \to -\infty} \lambda(Q^2) = \lambda_* = 0 \qquad \lim_{\log Q^2 \to -\infty} \frac{d\lambda}{d\log Q^2} = \lim_{\log Q^2 \to -\infty} \frac{3\lambda^2}{4\pi^2} = 0$$

Top-Higgs renormalization group

Two-dimensional IR fixed point

renormalization group equation for λ

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4\right)$$

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renormalization group equation for y_t

$$\frac{d\,y_t^2}{d\,\log Q^2} = \frac{9}{32\pi^2}\;y_t^4$$

Top-Higgs renormalization group

Two-dimensional IR fixed point

renormalization group equation for λ

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4\right)$$

renormalization group equation for y_t

$$\frac{d\,y_t^2}{d\,\log Q^2} = \frac{9}{32\pi^2}\;y_t^4$$

renormalization group equation for $R = \lambda/y_t^2$

$$\begin{aligned}
\frac{dR}{d\log Q^2} &= \frac{d\lambda}{d\log Q^2}\;\frac{1}{y_t^2} + \lambda\frac{(-1)}{y_t^4}\;\frac{dy_t^2}{d\log Q^2} \\
&= \frac{1}{16\pi^2 y_t^2}\left(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4\right) - \frac{9\lambda}{32\pi^2} \\
&= \frac{1}{16\pi^2}\;\left(12\lambda R + \frac{3}{2}\lambda - 3y_t^2\right) \\
&= \frac{\lambda}{16\pi^2}\;\left(12R + \frac{3}{2} - 3\frac{1}{R}\right) \\
&= \frac{3\lambda}{32\pi^2 R}\;\left(8R^2 + R - 2\right) \stackrel{!}{=} 0 \qquad \Leftrightarrow \qquad \frac{m_H^2}{2v^2}\;\frac{v^2}{2m_t^2}\Bigg|_\text{IR} = \frac{m_H^2}{4m_t^2}\Bigg|_\text{IR} \simeq 0.44
\end{aligned}$$

Top-Higgs renormalization group

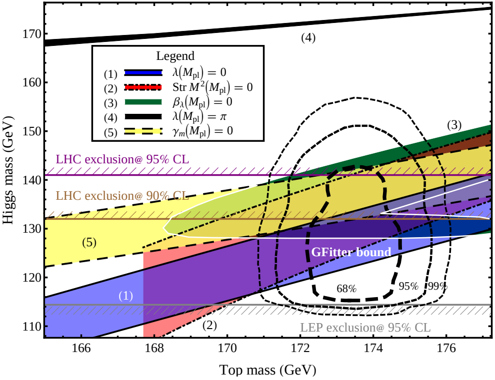
Two-dimensional IR fixed point

renormalization group equation for λ

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4\right)$$

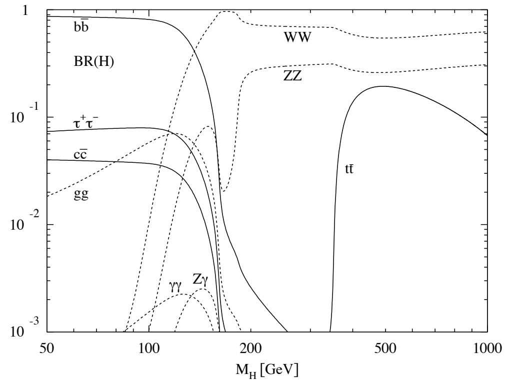
renormalization group equation for y_t

$$\frac{d\,y_t^2}{d\,\log Q^2} = \frac{9}{32\pi^2}\;y_t^4$$



Higgs decays

Branching ratio depending only on masses



– fermionic decays for lighter Higgs
  loop-induced $H \to \gamma\gamma, Z\gamma$ for lighter Higgs
  bosonic decays for heavier Higgs
– perfect spot at $m_H = 126$ GeV
– use HDECAY to compute

Higgs production

Effective *ggH* coupling

tensor structure of the effective coupling

$$\begin{aligned} G^{\mu\nu} G_{\mu\nu} &= -\left(k_{1\,\mu} A_{1\,\nu} - k_{1\,\nu} A_{1\,\mu}\right)\left(k_{2\,\mu} A_{2\,\nu} - k_{2\,\nu} A_{2\,\mu}\right) + \mathcal{O}(A^3) \\ &= -2\left[(k_1 k_2)(A_1 A_2) - (k_1 A_2)(k_2 A_1)\right] + \mathcal{O}(A^3) \\ &= -2(k_1 k_2)\, A_{1\mu} A_{2\nu} \left[g^{\mu\nu} - \frac{k_1^\nu k_2^\mu}{k_1 k_2}\right] + \mathcal{O}(A^3) \\ &= -m_H^2\, A_{1\mu} A_{2\nu} \left[g^{\mu\nu} - \frac{k_1^\nu k_2^\mu}{k_1 k_2}\right] + \mathcal{O}(A^3) \end{aligned}$$

Higgs production

Effective *ggH* coupling

tensor structure of the effective coupling

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projection operator

$$P_T^{\mu\nu} = \frac{1}{\sqrt{2}} \left(g^{\mu\nu} - \frac{k_1^\nu k_2^\mu}{(k_1 k_2)} \right)$$

$$T^{\mu\nu} \sim F\; P_T^{\mu\nu} \qquad \Leftrightarrow \qquad P_{T\,\mu\nu} T^{\mu\nu} \sim P_{T\,\mu\nu} P_T^{\mu\nu}\; F = F\;.$$

Higgs production

Effective ggH coupling

tensor structure of the effective coupling

$$\begin{aligned} G^{\mu\nu} G_{\mu\nu} &= -\left(k_{1\,\mu} A_{1\,\nu} - k_{1\,\nu} A_{1\,\mu}\right) \left(k_{2\,\mu} A_{2\,\nu} - k_{2\,\nu} A_{2\,\mu}\right) + \mathcal{O}(A^3) \\ &= -\,2\left[(k_1 k_2)(A_1 A_2) - (k_1 A_2)(k_2 A_1)\right] + \mathcal{O}(A^3) \\ &= -\,2(k_1 k_2)\, A_{1\mu} A_{2\nu} \left[g^{\mu\nu} - \frac{k_1^\nu k_2^\mu}{k_1 k_2}\right] + \mathcal{O}(A^3) \\ &= -\,m_H^2\, A_{1\mu} A_{2\nu} \left[g^{\mu\nu} - \frac{k_1^\nu k_2^\mu}{k_1 k_2}\right] + \mathcal{O}(A^3) \end{aligned}$$

projection operator

$$P_T^{\mu\nu} = \frac{1}{\sqrt{2}} \left(g^{\mu\nu} - \frac{k_1^\nu k_2^\mu}{(k_1 k_2)}\right)$$

$$T^{\mu\nu} \sim F\, P_T^{\mu\nu} \qquad \Leftrightarrow \qquad P_{T\,\mu\nu} T^{\mu\nu} \sim P_{T\,\mu\nu} P_T^{\mu\nu}\; F = F \; .$$

introduce form factor

$$\begin{aligned} F G^{\mu\nu} G_{\mu\nu} &= -\sqrt{2} m_H^2\, F\, A_{1\mu} A_{2\nu}\; P_T^{\mu\nu} \\ &\propto -\sqrt{2} m_H^2\, A_{1\mu} A_{2\nu} \int \frac{d^4 q}{16\pi^4} \frac{T^{\mu\nu}}{[\dots][\dots][\dots]} \end{aligned}$$

with $$P_{T\,\mu\nu} T^{\mu\nu} = \frac{4 m_t}{\sqrt{2}} \left(-m_H^2 + 3 m_t^2 - \frac{8}{m_H^2} (k_1 q)(k_2 q) - 2(k_1 q) + q^2\right)$$

Higgs production

Effective *ggH* coupling

$$\begin{aligned} F &= - \; i^3 \; (-ig_s)^2 \; \frac{im_t}{v} \; \text{Tr}(T^a T^b) \; \frac{i\pi^2}{16\pi^4} \; \int \frac{d^4 q}{i\pi^2} \, \frac{P_{T\mu\nu} T^{\mu\nu}}{[\dots][\dots][\dots]} \\ &= - \; i^3 \; (-ig_s)^2 \; \frac{im_t}{v} \; \text{Tr}(T^a T^b) \; \frac{i\pi^2}{16\pi^4} \; \frac{8 m_t}{\sqrt{2}} \, (1 + (1-\tau) f(\tau)) \\ &= \frac{g_s^2 m_t}{v} \; \frac{\delta^{ab}}{2} \; \frac{i}{16\pi^2} \; \frac{8 m_t}{\sqrt{2}} \, (1 + (1-\tau) f(\tau)) \\ &= \frac{g_s^2}{v} \; \frac{\delta^{ab}}{2} \; \frac{i}{16\pi^2} \; \frac{8}{\sqrt{2}} \, \frac{m_H^2 \tau}{4} \, (1 + (1-\tau) f(\tau)) \\ &= i g_s^2 \; \delta^{ab} \; \frac{1}{16\sqrt{2}\pi^2} \; \frac{m_H^2}{v} \; \tau \; (1 + (1-\tau) f(\tau)) \\ &= i \alpha_s \; \delta^{ab} \; \frac{1}{4\sqrt{2}\pi} \; \frac{m_H^2}{v} \; \tau \; (1 + (1-\tau) f(\tau)) \end{aligned}$$

with

$$f(\tau) = \begin{cases} \left(\sin^{-1} \sqrt{\dfrac{1}{\tau}} \right)^2 & \tau > 1 \\ -\dfrac{1}{4} \left(\log \dfrac{1 + \sqrt{1-\tau}}{1 - \sqrt{1-\tau}} - i\pi \right)^2 & \tau < 1 \end{cases}$$

Higgs production

Effective ggH coupling

$$\begin{aligned}
F &= - \; i^3 \, (-ig_s)^2 \, \frac{im_t}{v} \; \text{Tr}(T^a T^b) \, \frac{i\pi^2}{16\pi^4} \; \int \frac{d^4q}{i\pi^2} \, \frac{P_{T\mu\nu} T^{\mu\nu}}{[\dots][\dots][\dots]} \\
&= - \; i^3 \, (-ig_s)^2 \, \frac{im_t}{v} \; \text{Tr}(T^a T^b) \, \frac{i\pi^2}{16\pi^4} \, \frac{8m_t}{\sqrt{2}} \, (1 + (1-\tau) f(\tau)) \\
&= \frac{g_s^2 m_t}{v} \, \frac{\delta^{ab}}{2} \, \frac{i}{16\pi^2} \, \frac{8m_t}{\sqrt{2}} \, (1 + (1-\tau) f(\tau)) \\
&= \frac{g_s^2}{v} \, \frac{\delta^{ab}}{2} \, \frac{i}{16\pi^2} \, \frac{8}{\sqrt{2}} \, \frac{m_H^2 \tau}{4} \, (1 + (1-\tau) f(\tau)) \\
&= ig_s^2 \, \delta^{ab} \, \frac{1}{16\sqrt{2}\pi^2} \, \frac{m_H^2}{v} \, \tau \; (1 + (1-\tau) f(\tau)) \\
&= i\alpha_s \, \delta^{ab} \, \frac{1}{4\sqrt{2}\pi} \, \frac{m_H^2}{v} \, \tau \; (1 + (1-\tau) f(\tau))
\end{aligned}$$

giving

$$\mathscr{L}_{ggH} \supset \; \frac{1}{v} \, g_{ggH} \; H \, G^{\mu\nu} G_{\mu\nu} \qquad \text{with} \qquad \frac{1}{v} \, g_{ggH} = -i \, \frac{\alpha_s}{8\pi} \, \frac{1}{v} \, \tau \, [1 + (1-\tau) f(\tau)]$$

Higgs production

Effective coupling at low energies

$$f(\tau) = \left[\sin^{-1}\frac{1}{\sqrt{\tau}}\right]^2 = \left[\frac{1}{\tau^{1/2}} + \frac{1}{6\tau^{3/2}} + \mathcal{O}(\tau^{-5/2})\right]^2 = \frac{1}{\tau} + \frac{1}{3\tau^2} + \mathcal{O}(\tau^{-3}) \stackrel{\tau\to\infty}{\longrightarrow} 0$$

Higgs production

Effective coupling at low energies

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means for the coupling

$$\begin{aligned}
\tau\ [1 + (1-\tau)f(\tau)] &= \tau\ \left[1 + (1-\tau)\left(\frac{1}{\tau} + \frac{1}{3\tau^2} + \mathcal{O}(\tau^{-3})\right)\right] \\
&= \tau\ \left[1 + \frac{1}{\tau} - 1 - \frac{1}{3\tau} + \mathcal{O}(\tau^{-2})\right] \\
&= \tau\ \left[\frac{2}{3\tau} + \mathcal{O}(\tau^{-2})\right] \\
&= \frac{2}{3} + \mathcal{O}(\tau^{-1}) \qquad \text{implying} \qquad g_{ggH} = -i\ \frac{\alpha_s}{12\pi}
\end{aligned}$$

Higgs production

Effective coupling at low energies

$$f(\tau) = \left[\sin^{-1}\frac{1}{\sqrt{\tau}}\right]^2 = \left[\frac{1}{\tau^{1/2}} + \frac{1}{6\tau^{3/2}} + \mathcal{O}(\tau^{-5/2})\right]^2 = \frac{1}{\tau} + \frac{1}{3\tau^2} + \mathcal{O}(\tau^{-3}) \stackrel{\tau\to\infty}{\longrightarrow} 0$$

means for the coupling

$$\begin{aligned}
\tau\,\left[1+(1-\tau)f(\tau)\right] &= \tau\,\left[1+(1-\tau)\left(\frac{1}{\tau}+\frac{1}{3\tau^2}+\mathcal{O}(\tau^{-3})\right)\right] \\
&= \tau\,\left[1+\frac{1}{\tau}-1-\frac{1}{3\tau}+\mathcal{O}(\tau^{-2})\right] \\
&= \tau\,\left[\frac{2}{3\tau}+\mathcal{O}(\tau^{-2})\right] \\
&= \frac{2}{3}+\mathcal{O}(\tau^{-1}) \qquad \text{implying} \qquad g_{ggH} = -i\,\frac{\alpha_s}{12\pi}
\end{aligned}$$

no decoupling of heavy states
only real non-decoupling effect in LHC physics

Operators

Equivalent questions

– what are the Higgs quantum numbers?
– what is the structure of the Higgs Lagrangian?
– can the Higgs give mass to heavy states?

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Heavy flavor inspiration

– for any observed Higgs coupling there exists a renormalizable operator
– except Higgs production in gluon fusion
– except Higgs decay to photons
– except g_{WWH} might mean $HW^{\mu\nu}W_{\mu\nu}$
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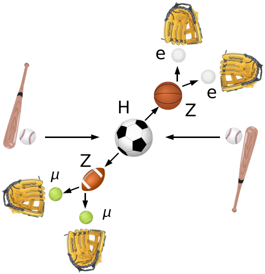
$\Rightarrow$ analyze Higgs kinematics [in as many channels as possible]

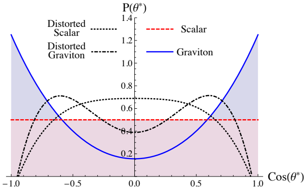


Operators

Model independent angles

– first step: Higgs polar angle for spin-0 vs spin-2 [Alves; Choi etal]

$$\frac{d\Gamma_0}{d\cos\theta^*} \sim P_0(\theta^*) = 1 \qquad\qquad P_2(\theta^*) \sim 1+6\cos^2\theta^* + \cos^4\theta^*$$



Operators

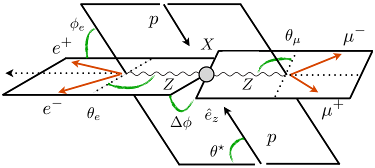
Model independent angles

– $H \to ZZ$ decays [Melnikov etal; Lykken etal; v d Bij etal; Englert, Spannowsky, Takeuchi]
classic Cabibbo–Maksymowicz–Dell'Aquila–Nelson angles

$$\cos\theta_e = \hat{p}_{e^-} \cdot \hat{p}_{Z_\mu}\Big|_{Z_e} \qquad \cos\theta_\mu = \hat{p}_{\mu^-} \cdot \hat{p}_{Z_e}\Big|_{Z_\mu} \qquad \cos\theta^* = \hat{p}_{Z_e} \cdot \hat{p}_{\text{beam}}\Big|_X$$

$$\cos\phi_e = (\hat{p}_{\text{beam}} \times \hat{p}_{Z_\mu}) \cdot (\hat{p}_{Z_\mu} \times \hat{p}_{e^-})\Big|_{Z_e}$$

$$\cos\Delta\phi = (\hat{p}_{e^-} \times \hat{p}_{e^+}) \cdot (\hat{p}_{\mu^-} \times \hat{p}_{\mu^+})\Big|_X$$



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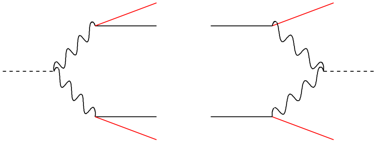
  Breit frame or hadron collider (η, ϕ) [Breit: boost into space-like]



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$$\cos\theta_1 = \hat{p}_{j_1} \cdot \hat{p}_{V_2}\Big|_{V_1\,\text{Breit}} \qquad \cos\theta_2 = \hat{p}_{j_2} \cdot \hat{p}_{V_1}\Big|_{V_2\,\text{Breit}} \qquad \cos\theta^* = \hat{p}_{V_1} \cdot \hat{p}_{d}\Big|_{X}$$

$$\cos\phi_1 = (\hat{p}_{V_2} \times \hat{p}_d) \cdot (\hat{p}_{V_2} \times \hat{p}_{j_1})\Big|_{V_1\,\text{Breit}}$$

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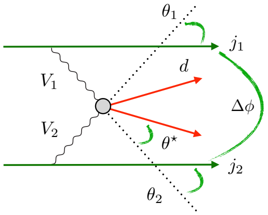

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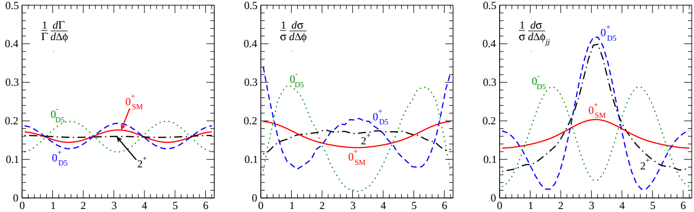


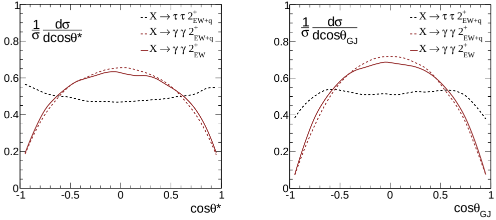
$\Rightarrow$ different approaches with similar physics

Operators

Spin-2 test? [Englert, Mawatari, Netto, TP]

– unitarization affecting all energy variables

– try Gottfried-Jackson angle [$\hat{p}_{X,lab}$ vs $\hat{p}_{d,X}$; Frank, Rauch, Zeppenfeld; Schumi]



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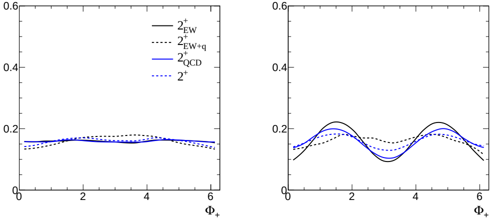
- try Gottfried-Jackson angle [$\hat{p}_{X,lab}$ vs $\hat{p}_{d,X}$; Frank, Rauch, Zeppenfeld; Schumi]
- alternatively $\phi_1 + \phi_2$ [Hagiwara, Li, Mawatari]



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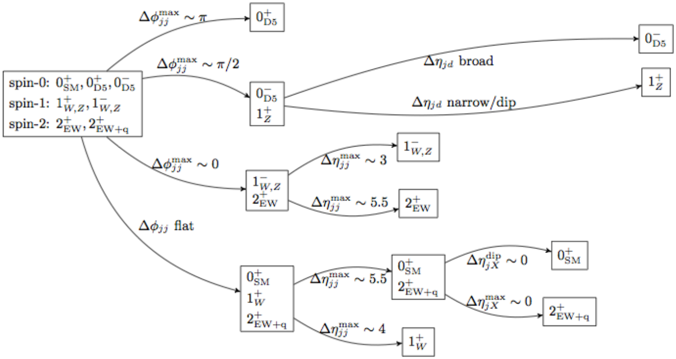
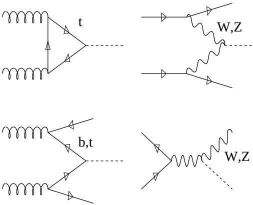
– diagrammatic analysis for WBF [$\Delta\eta_{jj}$ crucial]



$\Rightarrow$ observables in most channels

Where we are going

The model



– assume: we see a scalar [ZZ and WBF correlations]
  it is a narrow resonance
  SM-like D4 structures
  benchmarks useless
– production & decay combinations
– signal strength vs couplings?

$gg \to H$ $qq \to qqH$ $gg \to t\bar{t}H$ $q\bar{q}' \to WH$ plus a little problem	$\leftrightarrow$	$H \to ZZ$ $H \to WW$ $H \to b\bar{b}$ $H \to \tau^+_{\ell h}\tau^-_\ell$ $H \to \gamma\gamma$ $H \to Z\gamma$ ...	$\leftrightarrow$	signal $\times$ trigger backgrounds Gauss/Poisson statistics systematics theory errors

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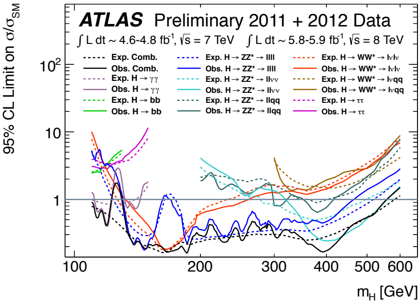

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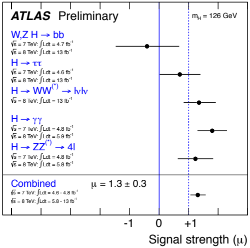

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Why 126 GeV is just perfect [Zeppenfeld et al; Dührssen et al; SFitter 2009/2012]

– parameters: Higgs couplings to $W, Z, t, b, \tau, g, \gamma$ [SM-like D4 operators]

$$\boxed{g_{HXX} = g_{HXX}^{\text{SM}} \; (1 + \Delta_X)} \qquad g_{HWW} > 0$$

– measurements: $GF : H \to ZZ, WW, \gamma\gamma$
  $WBF : H \to ZZ, WW, \gamma\gamma, \tau\tau$
  $VH : H \to b\bar{b}$
  $t\bar{t}H : H \to \gamma\gamma, b\bar{b}$

$\Rightarrow$ perfect application for SFitter

SFitter 1: Markov chains

Probability maps [statistics unexpectedly hard...]

- honest LHC parameters: weak-scale Lagrangean [Higgs, MSSM, dark matter,...]
- likelihood map: data given a model $p(d|m) \sim |\mathcal{M}|^2(m)$
- Bayes' theorem: $p(m|d) = p(d|m)\ p(m)/p(d)$ [$p(d)$ normalization, $p(m)$ prejudice]

Markov chains

- problem in grid: huge phase space, find local best points?
  problem in fit: domain walls, find global best points?
- construct 'representative' poll
- classical: representative set of spin states
  compute average energy on this reduced sample
- BSM or Higgs: map $p(d|m)$ of parameter points
  evaluate whatever you want
- Metropolis-Hastings
  starting probability $p(d|m)$ vs suggested probability $p(d|m')$
  1– accept new point if $p(d|m') > p(d|m)$
  2– or accept with $p(d|m')/p(d|m) < 1$

SFitter 1: Markov chains

Weighted Markov chains [Lafaye, TP, Rauch, Zerwas; Ferrenberg, Swendsen]

- special situation
  measure of ‘representative’: probability itself
- example with 2 bins, probability 9:1
  10 entries needed for good Markov chain
  2 entries needed if weight kept
- binning with weight would double count
  bin with inverse averaging

$$P_{\text{bin}}(p \neq 0) = \frac{\text{bincount}}{\sum_{i=1}^{\text{bincount}} p^{-1}}$$

- good choice for $\mathcal{O}(6)$ dimensions

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Cooling Markov chains [Lafaye, TP, Rauch, Zerwas]

– zoom in on peak structures [inspired by simulated annealing]

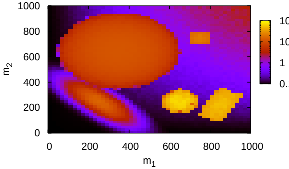
– modified condition
  Markov chain in partitions, numbered by j

$$p(d|m') > p(d|m)\; r^{10/j} \qquad r\epsilon[0,1] \quad \text{random number}$$

– check for parameter coverage with many Markov chains

$\Rightarrow$ exclusive likelihood map first result

SFitter 2: Frequentist vs Bayesian

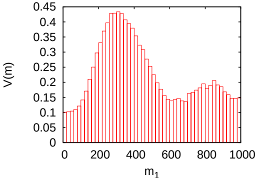


Getting rid of model parameters

– poorly constrained parameters
  uninteresting parameters
  unphysical parameters [JES part of m_t extraction]

– two ways to marginalize likelihood map

1– integrate over probabilities
  normalization etc mathematically correct
  integration measure unclear
  noise accumulation from irrelevant regions
  classical example: convolution of two Gaussians



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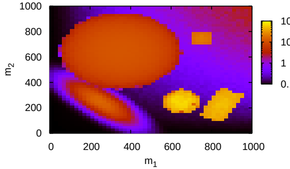

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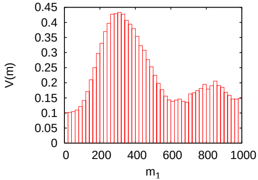
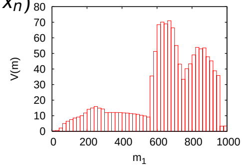


2– profile likelihood $\mathcal{L}(.., x_{j-1}, x_{j+1} ....) \equiv \max_{x_j} \mathcal{L}(x_1, ..., x_n)$
  no integration needed
  no noise accumulation
  not normalized, no comparison of structures
  classical example: best-fit point



– one-dimensional distributions tricky

SFitter 3: Error analysis

Sources of uncertainty

- statistical error: Poisson
  systematic error: Gaussian, if measured
  theory error: not Gaussian
- simple argument
  LHC rate 10% off: no problem
  LHC rate 30% off: no problem
  LHC rate 300% off: Standard Model wrong
- theory likelihood flat centrally and zero far away
- profile likelihood construction: RFit [CKMFitter]

$$-2\log\mathcal{L} = \chi^2 = \vec{\chi}_d^T \, C^{-1} \, \vec{\chi}_d$$

$$\chi_{d,i} = \begin{cases} 0 & |d_i - \bar{d}_i| < \sigma_i^{(\text{theo})} \\ \dfrac{|d_i - \bar{d}_i| - \sigma_i^{(\text{theo})}}{\sigma_i^{(\text{exp})}} & |d_i - \bar{d}_i| > \sigma_i^{(\text{theo})} \end{cases}$$

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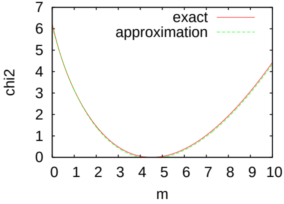
Efficient combination of errors [different from Michael's ATLAS analysis]

- Gaussian $\otimes$ Gaussian: half width added in quadrature
  Gaussian/Poisson $\otimes$ flat: RFit scheme
  Gaussian $\otimes$ Poisson: ??
- approximate formula

$$\frac{1}{\log\mathcal{L}_{\text{comb}}} = \frac{1}{\log\mathcal{L}_{\text{Gauss}}} + \frac{1}{\log\mathcal{L}_{\text{Poisson}}}$$

- modified Minuit gradient fit last step

$\Rightarrow$ error bars from toy measurements



Higgs couplings

Higgs sector at LHC [Zeppenfeld et al; Dührssen et al; SFitter 2009/2012; Contino et al]

– light Higgs around 126 GeV: over 10 channels ($\sigma \times BR$)

– measurements: $GF : H \to ZZ, WW, \gamma\gamma$ [first analyses]
$WBF : H \to ZZ, WW, \gamma\gamma, \tau\tau$ [just starting]
$VH : H \to b\bar{b}$ [BDRS-like analyses only]
$t\bar{t}H : H \to \gamma\gamma, WW, b\bar{b}...$ [useful but later]

– parameters: couplings $W, Z, t, b, \tau, g, \gamma$ [plus eventually Higgs mass]

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Total width

- myths about scaling

$$N = \sigma\, BR \propto \frac{g_p^2}{\sqrt{\Gamma_\text{tot}}}\,\frac{g_d^2}{\sqrt{\Gamma_\text{tot}}} \sim \frac{g^4}{g^2 \dfrac{\sum \Gamma_i(g^2)}{g^2} + \Gamma_\text{unobs}} \stackrel{g^2 \to 0}{\longrightarrow} = 0$$

  gives constraint from $\sum \Gamma_i(g^2) < \Gamma_\text{tot} \to \Gamma_H|_\text{min}$
- $WW \to WW$ unitarity: $g_{WWH} \lesssim g_{WWH}^\text{SM} \to \Gamma_H|_\text{max}$
- SFitter assumption $\Gamma_\text{tot} = \sum_\text{obs} \Gamma_j$ [plus generation universality]

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SFitter ansatz [Dührssen, Klute, Lafaye, TP, Rauch, Zerwas]

– couplings measurement $g_{HXX} = g_{HXX}^{\text{SM}} (1 + \Delta_X)$
D5 couplings $g_{ggH}, g_{\gamma\gamma H}$ free?
electroweak correction currently negligible

– experimental/theory errors on signal and backgrounds
ATLAS and CMS both included

– exclusive likelihood map
each coupling from profile likelihoods
best-fit point with Minuit
complete error analysis

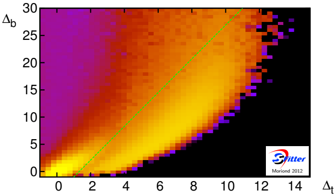
Global/local 7 TeV analysis

Global view on 7 TeV data [Klute, Lafaye, TP, Rauch, Zerwas]

– is there a SM-like solution?
are there alternative solutions?

(1) expected 2011: SM central values, measured error bars

– large-coupling solution separable



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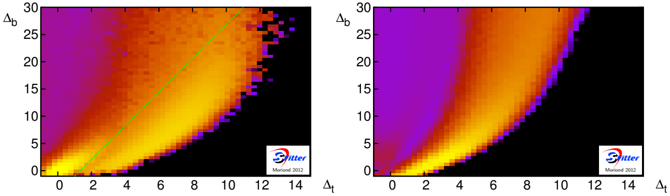

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Local view on 7 TeV data

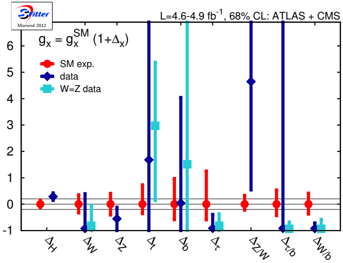
– focus on SM solution where possible

– five couplings from data

  $g_W \sim 0$ while g_Z okay
  g_b and g_t hurt by secondary solution
  g_τ inconclusive in data

– poor man's analysis great: $\Delta_j \equiv \Delta_H$

$\Rightarrow$ pointing towards Standard Model?



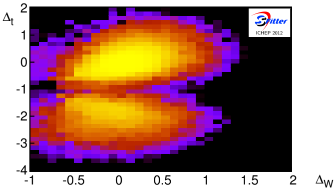
Global/local 8 TeV analysis

Global view on 8 TeV data [Klute, Lafaye, TP, Rauch, Zerwas]

– g_W now improved

(1) expected 2012: SM central values, measured error bars

– two symmetric solutions



Global/local 8 TeV analysis

Global view on 8 TeV data [Klute, Lafaye, TP, Rauch, Zerwas]

– g_W now improved

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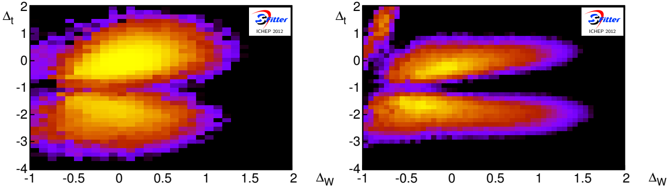
– two symmetric solutions

(2) measured 2012: measured central values and error bars

– alternative solution separated and weakened
improved $\Delta_{W,b,t}$ error bars



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improved $\Delta_{W,b,t}$ error bars

Local view on 8 TeV data [no change from HCP]

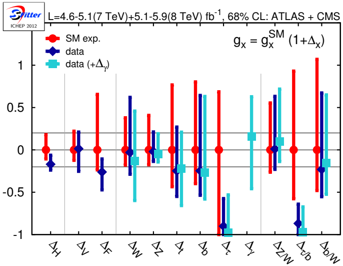
– focus on SM solution

– six couplings from data

$g_{W,Z}$ okay
$g_{t,b}$ indirectly
g_τ poor
g_γ possible

– poor man's analyses great: $\Delta_H, \Delta_V, \Delta_f$

⇒ moving towards Standard Model?



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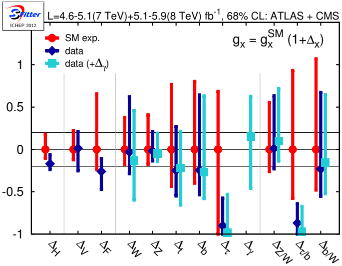
Testing the Higgs

– six couplings determined [g_{ggH} still missing]

– error bars $20 - 50\%$

– central value $\Delta_\gamma = 0.16$

– all good fits



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hypothesis	χ^2_{2012}/dof	solutions
Standard Model	43.3/54	
form factor Δ_H	32.2/53	1
two-parameter $\Delta_{V,f}$	29.0/52	2
independent Δ_x	27.7/49	3
including Δ_γ	27.3/48	2

Global/local 8 TeV analysis

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$\Rightarrow$ what's next?

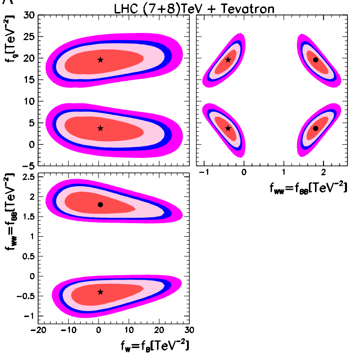
Beyond renormalizable couplings

Anomalous Higgs couplings [Corbett, Eboli, Gonzales-Fraile, Gonzales-Garcia]

– anomalous couplings from D6 operators f_j [index '2' for $W_{\mu\nu}W^{\mu\nu}$]

$$
\begin{aligned}
g_{Hgg} &= -\frac{\alpha_s}{8\pi}\frac{f_g v}{\Lambda^2} & g_{H\gamma\gamma} &= -\frac{gM_W}{\Lambda^2}\frac{s^2(f_{BB}+f_{WW}-f_{BW})}{2}\\
g^{(1)}_{HZ\gamma} &= \frac{gM_W}{\Lambda^2}\frac{s(f_W-f_B)}{2c} & g^{(2)}_{HZ\gamma} &= \frac{gM_W}{\Lambda^2}\frac{s[2s^2 f_{BB}-2c^2 f_{WW}+(c^2-s^2)f_{BW}]}{2c}\\
g^{(1)}_{HZZ} &= \frac{gM_W}{\Lambda^2}\frac{c^2 f_W+s^2 f_B}{2c^2} & g^{(2)}_{HZZ} &= -\frac{gM_W}{\Lambda^2}\frac{s^4 f_{BB}+c^4 f_{WW}+c^2 s^2 f_{BW}}{2c^2}\\
g^{(1)}_{HWW} &= \frac{gM_W}{\Lambda^2}\frac{f_W}{2} & g^{(2)}_{HWW} &= -\frac{gM_W}{\Lambda^2} f_{WW}
\end{aligned}
$$

– asume $f_W = f_B$ [otherwise no convergence]
fit f_{gg}, f_{WW}, f_{BB}
observe usual sign-flip degeneracy
compare to $\Delta\kappa$ and Λ in g_{WWV}



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$$g^{(1)}_{HZZ} = \frac{gM_W}{\Lambda^2}\frac{c^2 f_W+s^2 f_B}{2c^2} \qquad g^{(2)}_{HZZ} = -\frac{gM_W}{\Lambda^2}\frac{s^4 f_{BB}+c^4 f_{WW}+c^2 s^2 f_{BW}}{2c^2}$$

$$g^{(1)}_{HWW} = \frac{gM_W}{\Lambda^2}\frac{f_W}{2} \qquad g^{(2)}_{HWW} = -\frac{gM_W}{\Lambda^2}f_{WW}$$

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compare to $\Delta\kappa$ and Λ in g_{WWV}

A word on benchmarks

– known to 'say more about authors than about physics'

– bottom-up approach crucial

– theory benchmarks really only interesting for authors [I like the Higgs portal]

Top Yukawa

Direct measurement $t\bar{t}H$, $H \to b\bar{b}$ [Atlas-Bonn: Jochen Cammin]

– crucial to understand Higgs sector [details later]

– trigger: $t \to bW^+ \to b\ell^+\nu$
reconstruction and rate: $\bar{t} \to \bar{b}W^- \to \bar{b}jj$

– continuum background $t\bar{t}b\bar{b}$, $t\bar{t}jj$ [weighted by b-tag]

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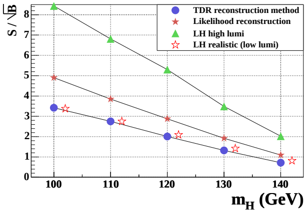
– not a chance:
1– combinatorics: m_H in $pp \to 4b_{tag}\ 2j\ \ell\nu$
2– kinematics: peak-on-peak
3– systematics: $S/B \sim 1/9$



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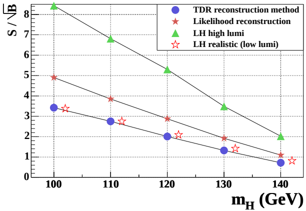

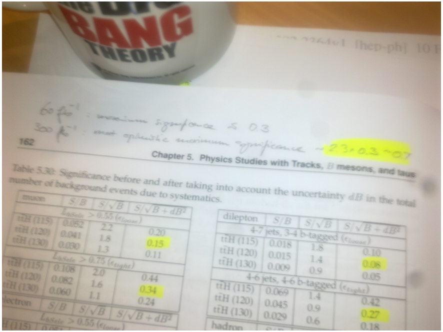

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Chapter 5. Physics Studies with Tracks, B mesons, and taus

Table 5.30: Significance before and after taking into account the uncertainty dB in the total number of background events due to systematics.

muon	S/B	$S/\sqrt{B}$	$S/\sqrt{B+dB^2}$
$L_{bSele} > 0.55$ (ϵ_{loose})			
$t\bar{t}H$ (115)	0.052	2.2	0.20
$t\bar{t}H$ (120)	0.041	1.8	0.15
$t\bar{t}H$ (130)	0.030	1.3	0.11
$L_{bSele} > 0.75$ (ϵ_{tight})			
$t\bar{t}H$ (115)	0.108	2.0	0.44
$t\bar{t}H$ (120)	0.082	1.6	0.34
$t\bar{t}H$ (130)	0.060	1.1	0.24
electron	S/B	$S/\sqrt{B}$	$S/\sqrt{B+dB^2}$
$L_{bSele} > 0.55$ (ϵ_{loose})			

dilepton	S/B	$S/\sqrt{B}$	$S/\sqrt{B+dB^2}$
4-7 jets, 3-4 b-tagged (ϵ_{loose})			
$t\bar{t}H$ (115)	0.018	1.8	0.10
$t\bar{t}H$ (120)	0.015	1.4	0.08
$t\bar{t}H$ (130)	0.009	0.9	0.05
4-6 jets, 4-6 b-tagged (ϵ_{tight})			
$t\bar{t}H$ (115)	0.069	1.4	0.42
$t\bar{t}H$ (120)	0.045	0.9	0.27
$t\bar{t}H$ (130)	0.029	0.6	0.18
hadron			

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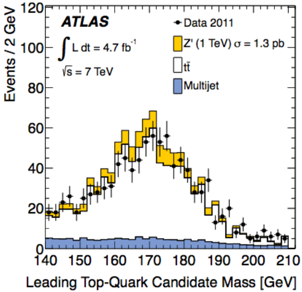
Fat jets analysis [TP, Salam, Spannowsky, Takeuchi]

– require tagged top and Higgs
trigger on lepton
only continuum $t\bar{t}b\bar{b}$ left [with sidebands]

– top tagger working [Atlas-Heidelberg]



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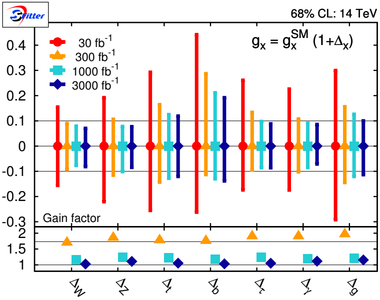
⇒ any good idea welcome



Weak scale models

Higgs portal

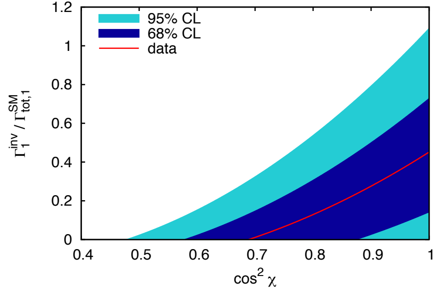
– only few renormalizable links to a new phyics world
  general standard-hidden ansatz $H_1 = \cos\chi\, H_s + \sin\chi\, H_h$

– visible and hidden decays [plus $H_2 \to H_1 H_1$ cascade decays]

$$\Gamma_1^{\text{tot}} = \cos^2\chi\, \Gamma_{\text{tot};1}^{\text{SM}} + \sin^2\chi\, \Gamma_1^{\text{hid}}$$

– constraints on event rate

$$\frac{\sigma[H_1 \to XX^*]}{\sigma[H_1 \to XX^*]^{\text{SM}}} = \frac{\cos^2\chi}{1 + \tan^2\chi\, \dfrac{\Gamma_1^{\text{hid}}}{\Gamma_{\text{tot},1}^{\text{SM}}}}$$



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$\Rightarrow$ invisible Higgs needed for final answer [Eboli & Zeppenfeld]

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$\Rightarrow$ invisible Higgs needed for final answer [Eboli & Zeppenfeld]

Form factor Higgs [Kaplan & Georgi; Contino, Espinosa, Giudice, Grojean, Mühlleitner, Pomarol, Rattazzi]

- simple trick: $\xi \equiv v/f \gtrsim 0.3$ while $m_\rho = g_\rho f \gg f$ [also not calculable]

1– all couplings scaled $g \to g\sqrt{1-\xi}$
- one-parameter fit in SFitter
- from 8 TeV data $\Delta_H = 0 \pm 0.15$

2– gauge couplings $g \to g\sqrt{1-\xi}$
Yukawas $g \to g(1-2\xi)/\sqrt{1-\xi}$
- sign change of Yukawas, $g_{\gamma\gamma H}$ correlated

Weak scale theory

Non-decoupling D6 operators

– SM: chiral fermions $g_{Hgg} \sim \alpha_s/(12\pi v)$

– new particle with charge Q and SU(3) Casimir $C(R)$ [Reece]

$$R_\gamma = \frac{g_{H\gamma\gamma}}{g^{\text{SM}}_{H\gamma\gamma}} = \left[1 + 0.28\xi\left(1 \mp \sqrt{R_g}\right)\right]^2 , \qquad \xi = \frac{3Q^2}{C_2(R)}$$

⇒ end of a fourth chiral generation [Lenz etal]

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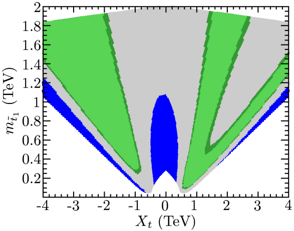
Supersymmetry

– MSSM Higgs mass the best-predicted LHC observable [Hahn etal + Stal]

– production rates mix of form factor and D6 [e.g. Hollik, TP, Rauch, Rzehak]

– stop mass/mixing crucial [m_A = 1 TeV, tan β = 20]



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– SUSY particles in eff couplings [everyone]
stop mixing destructive [Reece]

$$\frac{g_{Hgg}}{g_{Hgg}^{\text{SM}}} = 1 + \frac{1}{4}\left(\frac{m_t^2}{m_{\tilde{t}_1}^2} + \frac{m_t^2}{m_{\tilde{t}_2}^2} - \frac{m_t^2 X_t^2}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}\right)$$

– move towards NMSSM always an option...

⇒ no final verdict on the MSSM (ever?)

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General study [Gupta, Rzehak, Wells]

– modelling Higgs coupling deviations

– deviations allowed by other constraints

	ΔhVV	$\Delta h\bar{t}t$	$\Delta h\bar{b}b$
Mixed-in Singlet	6%	6%	6%
Composite Higgs	8%	tens of %	tens of %
Minimal Supersymmetry	$< 1\%$	3%	$10\%^{(\text{large }\tan\beta)}$, $100\%^{(\text{small }\tan\beta)}$

Weak scale theory

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⇒ end of a fourth chiral generation [Lenz etal]

General study [Gupta, Rzehak, Wells]

– modelling Higgs coupling deviations

– deviations allowed by other constraints

– correlation of Δ_τ and heavy Higgs states

⇒ no final verdict on (too) many models?

High scale theory

What if it is essentially the Standard Model

- many theories decouple in Higgs sector [custodial symmetry, suppressed D6]
- any handle on high-scale physics needed

High scale theory

What if it is essentially the Standard Model

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- any handle on high-scale physics needed

Renormalization group

- Higgs mass related to self coupling: $m_H = v\sqrt{2\lambda}$
  top mass related to Yukawa: $y_t = \sqrt{2} m_t / v$

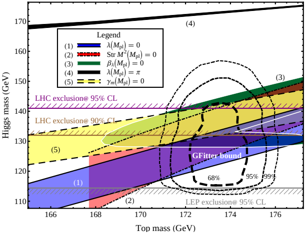
$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2+g_1^2\right) + \frac{3}{16}\left(2g_2^4+(g_2^2+g_1^2)^2\right)\right]$$

- IR fixed point for λ/y_t^2 fixing $m_H^2/m_t^2 = 1/2$ [with gravity: Shaposhnikov, Wetterich]

$$m_H = 126.3 + \frac{m_t - 171.2}{2.1}\times 4.1 - \frac{\alpha_s - 0.1176}{0.002}\times 1.5$$

- Planck-scale conditions [Holthausen, Lim, Lindner]

$\Rightarrow$ Higgs and top crucial in combination



Outlook

LHC Higgs program

– determine coupling structure

– measure pre-factors (i.e. coupling strengths) [ask me in private why by theorists]

– come up with good ideas for top Yukawa

– search for anomalous Higgs decays

– apply to everyone's favorite models [stop calling them benchmarks]