Theory Issues of Precision B Physics in the LHC Era

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- Setting the Stage
- B_s Decay Branching Ratios: $\rightarrow$ Key Application $\boxed{B_s \rightarrow \mu^+\mu^-}$
- Studies of CP Violation: $\boxed{B_s \rightarrow J/\psi\phi,\ B_s \rightarrow J/\psi f_0(980),\ \ldots}$
  - Hadronic Penguin Effects
  - Control Channels
  - Effective B_s Decay Lifetimes

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Setting the Stage

Where Do We Stand?

- Status of Physics @ LHC: $\rightarrow$ discovery of "Higgs-like" particle, but ...
  - No Standard Model (SM) deviations seen at ATLAS and CMS.
  - No solid evidence for New Physics (NP) in the flavour sector at LHCb.

- Implications for the general structure of NP:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm NP}(\varphi_{\rm NP}, g_{\rm NP}, m_{\rm NP}, ...)$$

  - Large characteristic NP scale $\Lambda_{\rm NP}$, i.e. not just $\sim$ TeV, which would be bad news for the direct searches at ATLAS and CMS, or (and?) ...
  - Symmetries prevent large NP effects in FCNCs and the flavour sector; most prominent example: *Minimal Flavour Violation (MFV)*.

- Much more is yet to come: ...

... but prepare to deal with "smallish" NP effects!

Towards New Frontiers in Precision B Physics

- Crucial for resolving smallish NP effects:
  - Have a critical look at theoretical analyses and their approximations:

    $\rightarrow$ key issue: strong interactions: $\rightarrow$ "hadronic" effects

  - Goal: matching between the experimental and theoretical precisions.

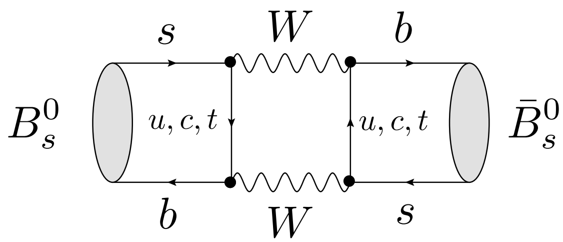
- Key decays for exploring CP violation:

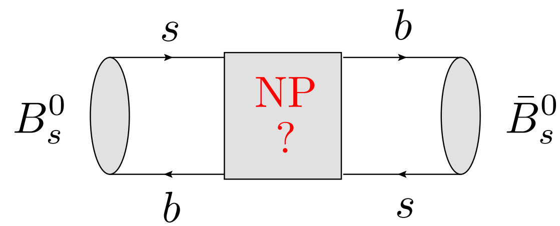
$$B_d \rightarrow J/\psi K_{\rm S},\ B_s \rightarrow J/\psi \phi,\ B_s \rightarrow J/\psi f_0(980)$$

  - Allow measurements of the $B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing phases $\phi_{d,s}$.
  - Uncertainties from doubly Cabibbo-suppressed penguin contributions.
  - These effects are usually neglected; we cannot reliably calculate them...

$\Rightarrow$ How big are they & how can they be controlled?

News on B_s^0–$\bar{B}_s^0$ Mixing





- Quantum mechanics: $\Rightarrow \quad |B_s(t)\rangle = a(t)|B_s^0\rangle + b(t)|\bar{B}_s^0\rangle$
  - Mass eigenstates: $\Delta M_s \equiv M_{\rm H}^{(s)} - M_{\rm L}^{(s)}, \quad \Delta\Gamma_s \equiv \Gamma_{\rm L}^{(s)} - \Gamma_{\rm H}^{(s)}$
  - Time-dependent decay rates: $\Gamma(B_s^0(t) \to f), \quad \Gamma(\bar{B}_s^0(t) \to f)$

- Key feature of the B_s-meson system: $\boxed{\Delta\Gamma_s \neq 0}$
  - Expected theoretically since decades [Review: A. Lenz (2012)].
  - Recently established by LHCb:

$$y_s \equiv \frac{\Delta\Gamma_s}{2\,\Gamma_s} \equiv \frac{\Gamma_{\rm L}^{(s)} - \Gamma_{\rm H}^{(s)}}{2\,\Gamma_s} = 0.088 \pm 0.014 \quad [\to 6\sigma \text{ effect}]$$

$$\tau_{B_s}^{-1} \equiv \Gamma_s \equiv \frac{\Gamma_{\rm L}^{(s)} + \Gamma_{\rm H}^{(s)}}{2} = (0.6580 \pm 0.0085)\,\text{ps}^{-1}$$

B_s Decay Branching Ratios:

$\rightarrow \Bigg\{$ simplest observables, characterizing
the probability of the decay to occur:

- $\Delta\Gamma_s \neq 0 \Rightarrow$ *special care* has to be taken when dealing with the concept of a branching ratio ...

- How to *convert* measured "experimental" B_s branching ratios into "theoretical" B_s branching ratios?

[De Bruyn, R.F., Knegjens, Koppenburg, Merk & Tuning (2012)]

Experiment versus Theory

- Untagged B_s decay rate: $\rightarrow$ *sum of two exponentials:*

$$\langle\Gamma(B_s(t)\to f)\rangle \equiv \Gamma(B^0_s(t)\to f)+\Gamma(\bar{B}^0_s(t)\to f) = R^f_{\rm H}e^{-\Gamma^{(s)}_{\rm H}t}+R^f_{\rm L}e^{-\Gamma^{(s)}_{\rm L}t}$$

$$= \left(R^f_{\rm H}+R^f_{\rm L}\right)e^{-\Gamma_s t}\left[\cosh\left(\frac{y_s\,t}{\tau_{B_s}}\right)+\mathcal{A}^f_{\Delta\Gamma}\sinh\left(\frac{y_s\,t}{\tau_{B_s}}\right)\right]$$

- "Experimental" branching ratio: [I. Dunietz, R.F. & U. Nierste (2001)]

$$\mathrm{BR}\,(B_s\to f)_{\rm exp}\equiv\frac{1}{2}\int_0^\infty\langle\Gamma(B_s(t)\to f)\rangle\,dt$$

$$=\frac{1}{2}\left[\frac{R^f_{\rm H}}{\Gamma^{(s)}_{\rm H}}+\frac{R^f_{\rm L}}{\Gamma^{(s)}_{\rm L}}\right]=\frac{\tau_{B_s}}{2}\left(R^f_{\rm H}+R^f_{\rm L}\right)\left[\frac{1+\mathcal{A}^f_{\Delta\Gamma}\,y_s}{1-y_s^2}\right] \qquad (6)$$

- "Theoretical" branching ratio: [R.F. (1999); S. Faller, R.F. & T. Mannel (2008); ...]

$$\mathrm{BR}\,(B_s\to f)_{\rm theo}\equiv\frac{\tau_{B_s}}{2}\langle\Gamma(B^0_s(t)\to f)\rangle\Big|_{t=0}=\frac{\tau_{B_s}}{2}\left(R^f_{\rm H}+R^f_{\rm L}\right) \qquad (8)$$

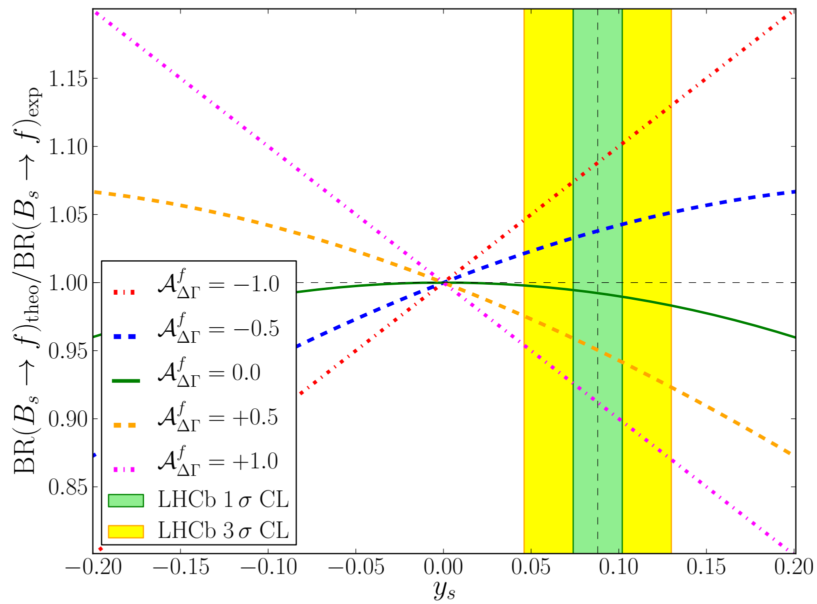
  - By considering $t=0$, the effect of B^0_s–$\bar{B}^0_s$ mixing is "switched off".
  - The advantage of this definition is that it allows a straightforward comparison with the BRs of B^0_d or B^+_u mesons by means of $SU(3)_{\rm F}$.

Conversion of B_s Decay Branching Ratios

- Relation between $\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}}$ and the measured $\mathrm{BR}\,(B_s \to f)_{\mathrm{exp}}$:

$$\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}} = \left[\frac{1-y_s^2}{1+\mathcal{A}^f_{\Delta\Gamma}\,y_s}\right] \mathrm{BR}\,(B_s \to f)_{\mathrm{exp}} \tag{9}$$

- While $y_s = 0.088 \pm 0.014$ has been measured, $\mathcal{A}^f_{\Delta\Gamma}$ depends on the considered decay and generally involves non-perturbative parameters:



$\Rightarrow$ differences can be as large as $\mathcal{O}(10\%)$ for the current value of y_s

[De Bruyn, R.F., Knegjens, Koppenburg, Merk and Tuning (2012)]

- Compilation of theoretical estimates for specific B_s decays:

$B_s \to f$	$\mathrm{BR}(B_s \to f)_{\mathrm{exp}}$	$\mathcal{A}_{\Delta\Gamma}^{f}(\mathrm{SM})$	$\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}}/\mathrm{BR}\,(B_s \to f)_{\mathrm{exp}}$ From Eq. (9)	From Eq. (11)
$J/\psi f_0(980)$	$(1.29^{+0.40}_{-0.28}) \times 10^{-4}$ [18]	0.9984 ± 0.0021 [14]	0.912 ± 0.014	0.890 ± 0.082 [6]
$J/\psi K_{\mathrm{S}}$	$(3.5 \pm 0.8) \times 10^{-5}$ [7]	0.84 ± 0.17 [15]	0.924 ± 0.018	N/A
$D_s^- \pi^+$	$(3.01 \pm 0.34) \times 10^{-3}$ [9]	0 (exact)	0.992 ± 0.003	N/A
$K^+ K^-$	$(3.5 \pm 0.7) \times 10^{-5}$ [18]	-0.972 ± 0.012 [13]	1.085 ± 0.014	1.042 ± 0.033 [19]
$D_s^+ D_s^-$	$(1.04^{+0.29}_{-0.26}) \times 10^{-2}$ [18]	-0.995 ± 0.013 [16]	1.088 ± 0.014	N/A

TABLE I: Factors for converting $\mathrm{BR}\,(B_s \to f)_{\mathrm{exp}}$ (see (6)) into $\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}}$ (see (8)) by means of Eq. (9) with theoretical estimates for $\mathcal{A}_{\Delta\Gamma}^{f}$. Whenever effective lifetime information is available, the corrections are also calculated using Eq. (11).

How can we avoid theoretical input? $\to$

- Effective B_s decay lifetimes:

$$\tau_f \equiv \frac{\int_0^\infty t\,\langle \Gamma(B_s(t) \to f)\rangle\, dt}{\int_0^\infty \langle \Gamma(B_s(t) \to f)\rangle\, dt} = \frac{\tau_{B_s}}{1-y_s^2}\left[\frac{1 + 2\,\mathcal{A}_{\Delta\Gamma}^{f} y_s + y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^{f} y_s}\right]$$

$$\Rightarrow \boxed{\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}} = \left[2 - \left(1 - y_s^2\right)\tau_f/\tau_{B_s}\right]\mathrm{BR}\,(B_s \to f)_{\mathrm{exp}}} \qquad (11)$$

$\to$ advocate the use of this relation for Particle Listings (PDG, HFAG)

$B_s \to VV$ Decays

- Another application is given by B_s decays into two vector mesons:
  - Examples: $B_s \to J/\psi\phi$, $B_s \to K^{*0}\bar{K}^{*0}$, $B_s \to D_s^{*+}D_s^{*-}$, ...
- Angular analysis of the vector-meson decay products has to be performed to disentangle the CP-even $(0, \|)$ and CP-odd $(\perp)$ states (labelled by k):

$$f_{VV,k}^{\rm exp} = \frac{{\rm BR}_{\rm exp}^{VV,k}}{{\rm BR}_{\rm exp}^{VV}}, \quad {\rm BR}_{\rm exp}^{VV} \equiv \sum_k {\rm BR}_{\rm exp}^{VV,k} \;\Rightarrow\; \sum_k f_{VV,k}^{\rm exp} = 1.$$

- Conversion of the "experimental" into the "theoretical" branching ratios:
  - Using *theory info* about $\mathcal{A}_{\Delta\Gamma}^{VV,k} = -\eta_k\sqrt{1 - C_{VV,k}^2}\cos(\phi_s + \Delta\phi_{VV,k})$:

$${\rm BR}_{\rm theo}^{VV} = (1 - y_s^2)\left[\sum_{k=0,\|,\perp} \frac{f_{VV,k}^{\rm exp}}{1 + y_s\mathcal{A}_{\Delta\Gamma}^{VV,k}}\right]{\rm BR}_{\rm exp}^{VV}$$

  - Using *effective lifetime measurements:*

$${\rm BR}_{\rm theo}^{VV} = {\rm BR}_{\rm exp}^{VV} \sum_{k=0,\|,\perp}\left[2 - (1 - y_s^2)\frac{\tau_k^{VV}}{\tau_{B_s}}\right] f_{VV,k}^{\rm exp}$$

[See also LHCb, arXiv:1111.4183; S. Descotes-Genon, J. Matias & J. Virto (2011)]

◇ Key B_s Decay: $B_s^0 \to \mu^+\mu^-$

What is the impact of $\Delta\Gamma_s \neq 0$ on this decay?

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Probing New Physics via the $B_s^0 \to \mu^+\mu^-$ Effective Lifetime

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We have recently seen new upper bounds for $B_s^0 \to \mu^+\mu^-$, a key decay to search for physics beyond the standard model. Furthermore a nonvanishing decay width difference $\Delta\Gamma_s$ of the B_s system has been measured. We show that $\Delta\Gamma_s$ affects the extraction of the $B_s^0 \to \mu^+\mu^-$ branching ratio and the resulting constraints on the new physics parameter space and give formulas for including this effect. Moreover, we point out that $\Delta\Gamma_s$ provides a new observable, the effective $B_s^0 \to \mu^+\mu^-$ lifetime $\tau_{\mu^+\mu^-}$, which offers a theoretically clean probe for new physics searches that is complementary to the branching ratio. Should the $B_s^0 \to \mu^+\mu^-$ branching ratio agree with the standard model, the measurement of $\tau_{\mu^+\mu^-}$, which appears feasible at upgrades of the Large Hadron Collider experiments, may still reveal large new physics effects.

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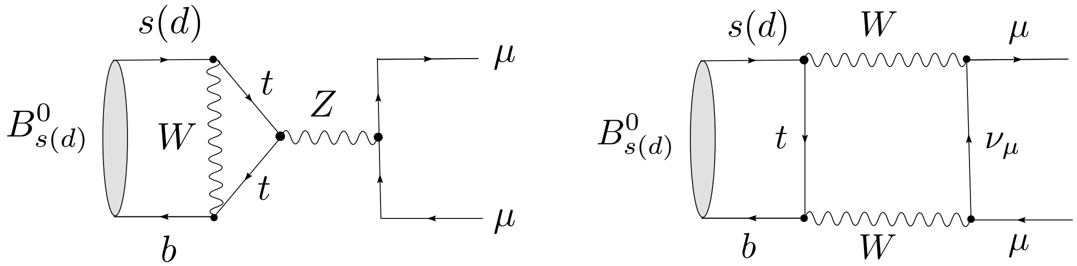
General Features of $B^0_{s(d)} \to \mu^+\mu^-$ Decays

- Only loop contributions in the SM $\oplus$ helicity suppression:



$\Rightarrow$ strongly suppressed & sensitive to New Physics (NP)

- Hadronic sector: only $B_{s(d)}$-decay constant $f_{B_{s(d)}}$ enters:

$\Rightarrow$ $B^0_{s(d)} \to \mu^+\mu^-$ belong to the cleanest rare B decays

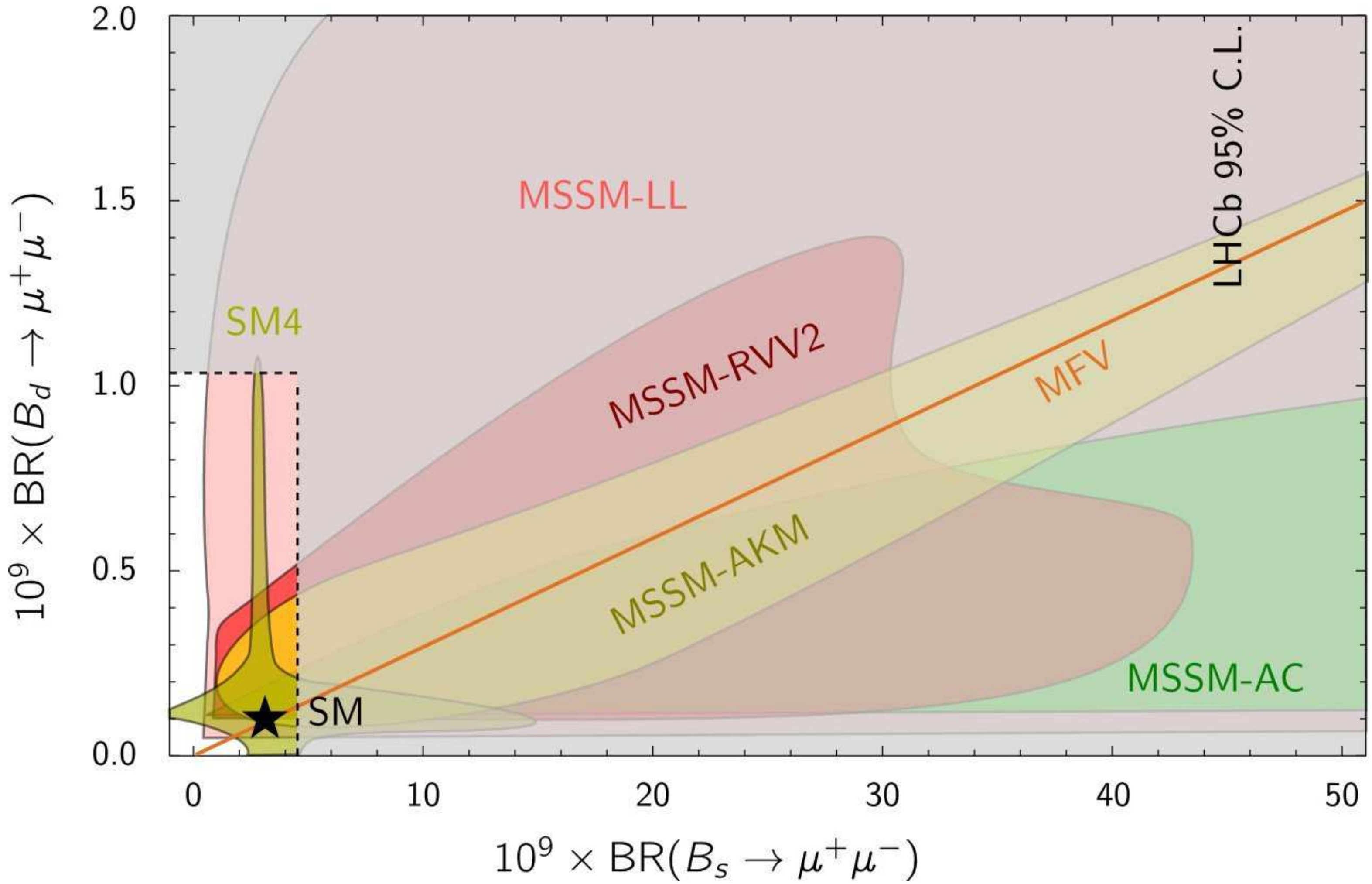
- SM predictions: $\text{BR}(B_s \to \mu^+\mu^-) = (3.23 \pm 0.27) \times 10^{-9}$
  $\text{BR}(B_d \to \mu^+\mu^-) = (1.07 \pm 0.10) \times 10^{-10}$

[Buras, Girrbach, Guadagnoli & Isidori (2012); address also soft photon corrections]

NP may – in principle – enhance BRs significantly...

[Babu & Kolda, Dedes *et al.*, Foster *et al.*, Carena *et al.*, Isidori & Paradisi, ...]

- Situation in different supersymmetric flavour models, showing also the impact of recent LHCb upper bounds on $\mathrm{BR}(B_{s,d} \to \mu^+\mu^-)$:



[D. Straub (2010); A.J. Buras & J. Girrbach (2012)]

The Limiting Factor for the BR($B_s^0 \to \mu^+\mu^-$) Measurement:

- The analysis of $B_s^0 \to \mu^+\mu^-$ relies on normalization channels:

$$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) = \mathrm{BR}(B_q \to X)\frac{\epsilon_X}{\epsilon_{\mu\mu}}\frac{N_{\mu\mu}}{N_X}\frac{f_q}{f_s}$$

  - ϵ factors are total detector efficiencies.
  - N factors denote the observed numbers of events.
  - f_q are *fragmentation functions*, which describe the probability that a b quark will fragment in a B_q meson ($q \in \{u, d, s\}$).

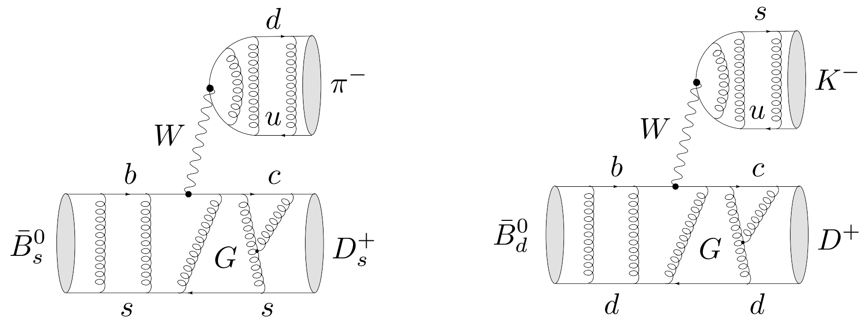
- A closer look shows: f_s/f_d *is the major source of uncertainty:*

$\Rightarrow$ "boring" non-perturbative, hadronic parameter ...

- New method: $\to$ use non-leptonic B decays to *determine* f_s/f_d @ LHCb

$\Rightarrow$ U-spin-related $\bar{B}_s^0 \to D_s^+\pi^-$, $\bar{B}_d^0 \to D^+K^-$ system:

[R.F., Nicola Serra & Niels Tuning (2010)]



- Prime examples for "factorization": [← Bjorken ('89), Dugan & Grinstein ('91); Beneke, Buchalla, Neubert & Sachrajda ('00); Bauer, Pirjol & Steward ('01); ...]
  - Non-fact. $SU(3)$-breaking corrections: tiny (constrainted through data).
  - Factorizable $SU(3)$-breaking corrections:

    → form-factor ratio [QCD sum rule; lattice QCD analyses]:

$$\Rightarrow \textit{ratio of branching ratios can be calculated}$$

$$\Rightarrow \frac{f_s}{f_d} = \underbrace{\frac{N_s}{N_d} \times \frac{\epsilon(\bar{B}^0_d \to D^+K^-)}{\epsilon(\bar{B}^0_s \to D_s^+\pi^-)}}_{\text{experiment}} \times \underbrace{\frac{\mathrm{BR}(\bar{B}^0_d \to D^+K^-)}{\mathrm{BR}(\bar{B}_s \to D_s^+\pi^-)}}_{\text{theory}}$$

- LHCb (using also a variant with $\bar{B}^0_d \to D^+\pi^-$): [PRL (2011)]

$$f_s/f_d = 0.253 \pm 0.017(\text{stat.}) \pm 0.017(\text{syst.}) \pm 0.020(\text{theo.})$$

[excellent agreement with measurements using semileptonic decays]

- Lattice: Fermilab Lattice & MILC [arXiv:1202.6346 [hep-lat]]

- New LHCb analysis of the $B_s^0 \to D_s^- \pi^+$, $B_d^0 \to D^- \pi^+$ strategy:

$\to$ dependence of f_s/f_d on the transverse momentum and pseudo rapidity:

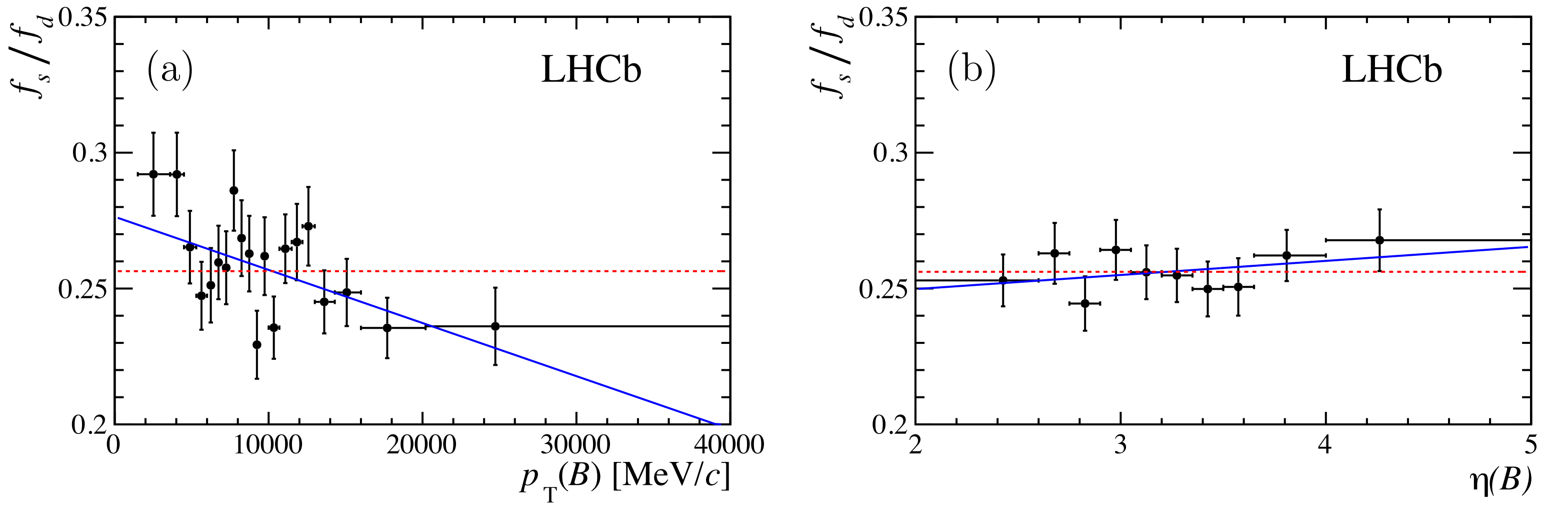


Figure 2: Ratio of fragmentation fractions f_s/f_d as functions of (a) p_{T} and (b) η. The errors on the data points are the statistical and uncorrelated systematic uncertainties added in quadrature. The solid line is the result of a linear fit, and the dashed line corresponds to the fit for the no-dependence hypothesis. The average value of p_{T} or η is determined for each bin and used as the center of the bin. The horizontal error bars indicate the bin size. Note that the scale is zero suppressed.

$$
\begin{aligned}
f_s/f_d\,(p_{\mathrm{T}}) &= (0.256 \pm 0.020) + (-2.0 \pm 0.6) \times 10^{-3}/\,\mathrm{GeV}/c \times (p_{\mathrm{T}} - \langle p_{\mathrm{T}} \rangle) \\
f_s/f_d\,(\eta) &= (0.256 \pm 0.020) + (0.005 \pm 0.006) \times (\eta - \langle \eta \rangle),
\end{aligned}
$$

[LHCb Collaboration, arXiv:1301.5286 [hep-ex]]

Current Experimental Situation of $B_s^0 \to \mu^+\mu^-$:

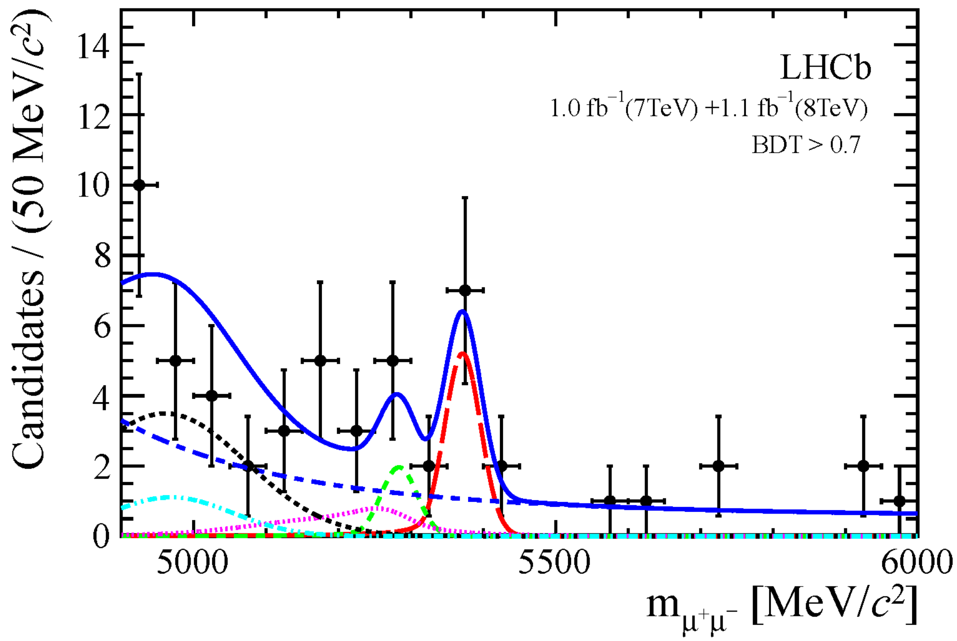
- Tevatron: $\to$ *"legacy"* ...
  - DØ (2013): BR$(B_s^0 \to \mu^+\mu^-) < 15 \times 10^{-9}$ (95% C.L.)
  - CDF (2013): BR$(B_s^0 \to \mu^+\mu^-) < 31 \times 10^{-9}$ (95% C.L.)

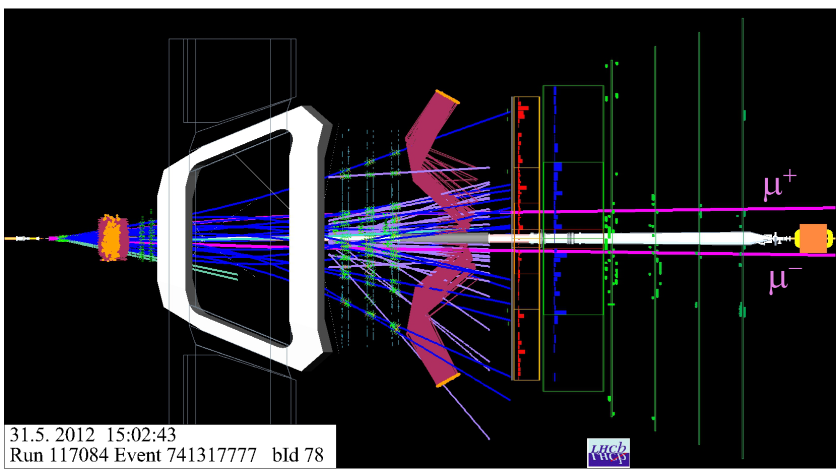
- Large Hardon Collider: $\to$ *future* ...
  - ATLAS (2012): BR$(B_s^0 \to \mu^+\mu^-) < 22 \times 10^{-9}$ (95% C.L.)
  - CMS (2012): BR$(B_s^0 \to \mu^+\mu^-) < 7.7 \times 10^{-9}$ (95% C.L.)
  - Finally *first evidence* for $B_s^0 \to \mu^+\mu^-$ @ LHCb (2012):

$$\text{BR}(B_s^0 \to \mu^+\mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9}$$

$\Rightarrow$ *falls into the SM regime although the error is still very large ...*

- $\Delta\Gamma_s \neq 0$ has been ignored in these considerations:
  - What is the impact for the theoretical interpretation of the data?
  - Can we actually *take advantage* of $\Delta\Gamma_s \neq 0$?


LHCb
1.0 fb^-1(7TeV) +1.1 fb^-1(8TeV)
BDT > 0.7
Candidates / (50 MeV/c^2)
0
2
4
6
8
10
12
14
5000
5500
6000
m_μ+μ- [MeV/c^2]


μ+
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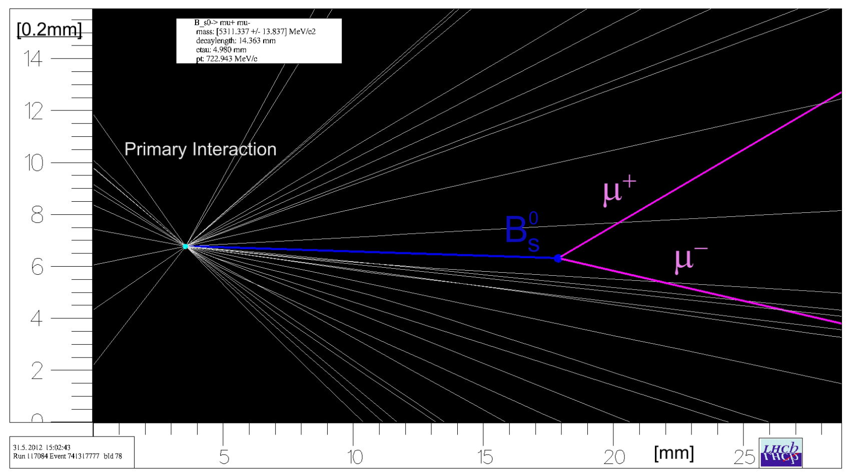
31.5. 2012 15:02:43
Run 117084 Event 741317777 bId 78


[0.2mm]
B_s0-> mu+ mu-
mass: [5311.337 +/- 13.837] MeV/c2
decaylength: 14.363 mm
ctau: 4.980 mm
pt: 722.943 MeV/c
Primary Interaction
B_s^0
μ+
μ−
14
12
10
8
6
4
2
0
5
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15
20
25
[mm]

The General $B_s \to \mu^+\mu^-$ Amplitudes

- Low-energy effective Hamiltonian for $\bar{B}^0_s \to \mu^+\mu^-$: SM $\oplus$ NP

$$\mathcal{H}_{\rm eff} = -\frac{G_{\rm F}}{\sqrt{2}\pi} V_{ts}^* V_{tb} \alpha \left[C_{10} O_{10} + C_S O_S + C_P O_P + C'_{10} O'_{10} + C'_S O'_S + C'_P O'_P \right]$$

[$G_{\rm F}$: Fermi's constant, $V_{qq'}$: CKM matrix elements, α: QED fine structure constant]

- Four-fermion operators, with $P_{L,R} \equiv (1 \mp \gamma_5)/2$ and b-quark mass m_b:

$$\begin{array}{rclrcl} O_{10} & = & (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5 \ell), & O'_{10} & = & (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\gamma_5 \ell) \\ O_S & = & m_b(\bar{s} P_R b)(\bar{\ell}\ell), & O'_S & = & m_b(\bar{s} P_L b)(\bar{\ell}\ell) \\ O_P & = & m_b(\bar{s} P_R b)(\bar{\ell}\gamma_5\ell), & O'_P & = & m_b(\bar{s} P_L b)(\bar{\ell}\gamma_5\ell) \end{array}$$

[Only operators with non-vanishing $\bar{B}^0_s \to \mu^+\mu^-$ matrix elements are included]

- The Wilson coefficients C_i, C'_i encode the short-distance physics:
  - SM case: only $C_{10} \neq 0$, and is given by the *real* coefficient $C_{10}^{\rm SM}$.
  - *Outstanding feature of* $\bar{B}^0_s \to \mu^+\mu^-$: sensitivity to (pseudo-)scalar lepton densities $\to O_{(P)S}$, $O'_{(P)S}$; WCs are still largely unconstrained.

[W. Altmannshofer, P. Paradisi & D. Straub (2011) $\to$ model-independent NP analysis]

$\rightarrow$ *convenient to go to the rest frame of the decaying* $\bar{B}^0_s$ *meson:*

- Distinguish between the $\mu^+_{\rm L}\mu^-_{\rm L}$ and $\mu^+_{\rm R}\mu^-_{\rm R}$ helicity configurations:

$$|(\mu^+_{\rm L}\mu^-_{\rm L})_{\rm CP}\rangle \equiv (\mathcal{CP})|\mu^+_{\rm L}\mu^-_{\rm L}\rangle = e^{i\phi_{\rm CP}(\mu\mu)}|\mu^+_{\rm R}\mu^-_{\rm R}\rangle$$

[$e^{i\phi_{\rm CP}(\mu\mu)}$ is a convention-dependent phase factor $\rightarrow$ cancels in observables]

- General expression for the decay amplitude [$\eta_{\rm L}=+1$, $\eta_{\rm R}=-1$]:

$$A(\bar{B}^0_s \to \mu^+_\lambda\mu^-_\lambda) = \langle \mu^-_\lambda\mu^+_\lambda|\mathcal{H}_{\rm eff}|\bar{B}^0_s\rangle = -\frac{G_{\rm F}}{\sqrt{2}\pi}V^*_{ts}V_{tb}\alpha$$

$$\times f_{B_s}M_{B_s}m_\mu C_{10}^{\rm SM}e^{i\phi_{\rm CP}(\mu\mu)(1-\eta_\lambda)/2}\left[\eta_\lambda P + S\right]$$

- Combination of Wilson coefficient functions [CP-violating phases $\varphi_{P,S}$]:

$$P \equiv |P|e^{i\varphi_P} \equiv \frac{C_{10}-C_{10}'}{C_{10}^{\rm SM}} + \frac{M_{B_s}^2}{2m_\mu}\left(\frac{m_b}{m_b+m_s}\right)\left(\frac{C_P-C_P'}{C_{10}^{\rm SM}}\right) \xrightarrow{\rm SM} 1$$

$$S \equiv |S|e^{i\varphi_S} \equiv \sqrt{1-4\frac{m_\mu^2}{M_{B_s}^2}}\frac{M_{B_s}^2}{2m_\mu}\left(\frac{m_b}{m_b+m_s}\right)\left(\frac{C_S-C_S'}{C_{10}^{\rm SM}}\right) \xrightarrow{\rm SM} 0$$

[f_{B_s}: B_s decay constant, M_{B_s}: B_s mass, m_μ: muon mass, m_s: strange-quark mass]

The $B_s \to \mu^+\mu^-$ Observables

- Key quantity for calculating the CP asymmetries and the untagged rate:

$$\xi_\lambda \equiv -e^{-i\phi_s}\left[e^{i\phi_{\rm CP}(B_s)}\frac{A(\bar{B}^0_s \to \mu^+_\lambda\mu^-_\lambda)}{A(B^0_s \to \mu^+_\lambda\mu^-_\lambda)}\right]$$

$$\Rightarrow A(B^0_s \to \mu^+_\lambda\mu^-_\lambda) = \langle \mu^-_\lambda\mu^+_\lambda|\mathcal{H}^\dagger_{\rm eff}|B^0_s\rangle \text{ is also needed ...}$$

- Using $(\mathcal{CP})^\dagger(\mathcal{CP}) = \hat{1}$ and $(\mathcal{CP})|B^0_s\rangle = e^{i\phi_{\rm CP}(B_s)}|\bar{B}^0_s\rangle$ yields:

$$A(B^0_s \to \mu^+_\lambda\mu^-_\lambda) = -\frac{G_{\rm F}}{\sqrt{2}\pi}V_{ts}V^*_{tb}\alpha f_{B_s}M_{B_s}m_\mu C^{\rm SM}_{10}$$

$$\times\, e^{i[\phi_{\rm CP}(B_s)+\phi_{\rm CP}(\mu\mu)(1-\eta_\lambda)/2]}\left[-\eta_\lambda P^* + S^*\right]$$

- The convention-dependent phases cancel in ξ_λ $[\eta_{\rm L} = +1,\ \eta_{\rm R} = -1]$:

$$\xi_\lambda = -\left[\frac{+\eta_\lambda P + S}{-\eta_\lambda P^* + S^*}\right] \quad \Rightarrow \quad \boxed{\xi_{\rm L}\xi^*_{\rm R} = \xi_{\rm R}\xi^*_{\rm L} = 1}$$

CP Asymmetries:

- Time-dependent rate asymmetry: $\rightarrow$ *requires tagging of B_s^0 and $\bar{B}_s^0$:*

$$\frac{\Gamma(B_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)-\Gamma(\bar{B}_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)}{\Gamma(B_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)+\Gamma(\bar{B}_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)}=\frac{C_\lambda\cos(\Delta M_s t)+S_\lambda\sin(\Delta M_s t)}{\cosh(y_s t/\tau_{B_s})+\mathcal{A}_{\Delta\Gamma}^\lambda\sinh(y_s t/\tau_{B_s})}$$

- Observables (for $\phi_s^{\rm NP}=0$): $\rightarrow$ *theoretically clean (no dependence on f_{B_s}):*

$$C_\lambda\equiv\frac{1-|\xi_\lambda|^2}{1+|\xi_\lambda|^2}=-\eta_\lambda\left[\frac{2|PS|\cos(\varphi_P-\varphi_S)}{|P|^2+|S|^2}\right]\quad\xrightarrow{\rm SM}\quad 0$$

$$S_\lambda\equiv\frac{2\,\mathrm{Im}\,\xi_\lambda}{1+|\xi_\lambda|^2}=\frac{|P|^2\sin 2\varphi_P-|S|^2\sin 2\varphi_S}{|P|^2+|S|^2}\quad\xrightarrow{\rm SM}\quad 0$$

$$\mathcal{A}_{\Delta\Gamma}^\lambda\equiv\frac{2\,\mathrm{Re}\,\xi_\lambda}{1+|\xi_\lambda|^2}=\frac{|P|^2\cos 2\varphi_P-|S|^2\cos 2\varphi_S}{|P|^2+|S|^2}\quad\xrightarrow{\rm SM}\quad 1$$

- Note: $\mathcal{S}_{\rm CP}\equiv S_\lambda$, $\mathcal{A}_{\Delta\Gamma}\equiv\mathcal{A}_{\Delta\Gamma}^\lambda$ are *independent* of the muon helicity λ.

- Difficult to measure the muon helicity: $\Rightarrow$ *consider the following rates:*

$$\Gamma(\overset{(-)}{B}{}^0_s(t) \to \mu^+\mu^-) \equiv \sum_{\lambda=\mathrm{L,R}} \Gamma(\overset{(-)}{B}{}^0_s(t) \to \mu^+_\lambda\mu^-_\lambda)$$

- Corresponding CP-violating rate asymmetry: $\to$ $C_\lambda \propto \eta_\lambda$ *terms cancel:*

$$\frac{\Gamma(B^0_s(t) \to \mu^+\mu^-) - \Gamma(\bar{B}^0_s(t) \to \mu^+\mu^-)}{\Gamma(B^0_s(t) \to \mu^+\mu^-) + \Gamma(\bar{B}^0_s(t) \to \mu^+\mu^-)} = \frac{\mathcal{S}_{\mathrm{CP}} \sin(\Delta M_s t)}{\cosh(y_s t/\tau_{B_s}) + \mathcal{A}_{\Delta\Gamma} \sinh(y_s t/\tau_{B_s})}$$

- Practical comments:

  – It would be most interesting to measure this CP asymmetry since a non-zero value immediately signaled CP-violating NP phases.
  [See, e.g., Buras & Girrbach ('12) for Minimal $U(2)^3$ models [Barbieri *et al.*])]

  – Unfortunately, this is challenging in view of the tiny branching ratio and as B^0_s, $\bar{B}^0_s$ *tagging* and time information are required.

[Previous studies of CP asymmetries of $B^0_{s,d} \to \ell^+\ell^-$ (assuming $\Delta\Gamma_s = 0$):
Huang and Liao (2002); Dedes and Pilaftsis (2002), Chankowski *et al.* (2005)]

Untagged Rate and Branching Ratio:

- The first measurement concerns the "experimental" branching ratio:

$$\mathrm{BR}\left(B_s \to \mu^+\mu^-\right)_{\rm exp} \equiv \frac{1}{2}\int_0^\infty \langle \Gamma(B_s(t) \to \mu^+\mu^-)\rangle \, dt$$

$\to$ *time-integrated untagged rate*, involving

$$\langle \Gamma(B_s(t) \to \mu^+\mu^-)\rangle \equiv \Gamma(B_s^0(t) \to \mu^+\mu^-) + \Gamma(\bar{B}_s^0(t) \to \mu^+\mu^-)$$

$$\propto e^{-t/\tau_{B_s}}\left[\cosh(y_s t/\tau_{B_s}) + \mathcal{A}_{\Delta\Gamma}\sinh(y_s t/\tau_{B_s})\right]$$

- Conversion into the "theoretical" branching ratio: $\to$ *NP searches:*

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = \left[\frac{1-y_s^2}{1+\mathcal{A}_{\Delta\Gamma}\, y_s}\right]\mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm exp}$$

- $\mathcal{A}_{\Delta\Gamma}$ depends on NP and is hence unknown: $\in [-1,+1] \Rightarrow$ *two options:*
  - Add extra error: $\Delta\mathrm{BR}(B_s \to \mu^+\mu^-)|_{y_s} = \pm y_s \mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm exp}$.
  - $\mathcal{A}_{\Delta\Gamma}^{\rm SM} = 1$ gives *new SM reference value* [rescale $\mathrm{BR}_{\rm SM}$ by $1/(1-y_s)$]:

$$\mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm SM}|_{y_s} = (3.54 \pm 0.30) \times 10^{-9}.$$

Effective $B_s \to \mu^+\mu^-$ Lifetime:

$\diamond$ *Collecting more and more data* $\oplus$ *include decay time information* $\Rightarrow$

- Access to the effective $B_s \to \mu^+\mu^-$ lifetime:

$$\tau_{\mu^+\mu^-} \equiv \frac{\int_0^\infty t\, \langle \Gamma(B_s(t) \to \mu^+\mu^-)\rangle\, dt}{\int_0^\infty \langle \Gamma(B_s(t) \to \mu^+\mu^-)\rangle\, dt}$$

- $\mathcal{A}_{\Delta\Gamma}$ can then be extracted: $\mathcal{A}_{\Delta\Gamma} = \frac{1}{y_s}\left[\frac{(1-y_s^2)\tau_{\mu^+\mu^-} - (1+y_s^2)\tau_{B_s}}{2\tau_{B_s} - (1-y_s^2)\tau_{\mu^+\mu^-}}\right]$

- Finally, extraction of the "theoretical" BR: $\to$ *clean expression:*

$$\mathrm{BR}\left(B_s \to \mu^+\mu^-\right) = \underbrace{\left[2 - \left(1 - y_s^2\right)\frac{\tau_{\mu^+\mu^-}}{\tau_{B_s}}\right] \mathrm{BR}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{exp}}}_{\to \text{ only measurable quantities}}$$

  – It is *crucial* that $\mathcal{A}_{\Delta\Gamma}$ does *not* depend on the muon helicity.

  – *Important new measurement for the high-luminosity LHC upgrade:*
  $\Rightarrow$ precision of 5% or better appears feasible for $\tau_{\mu^+\mu^-}$ ...

Constraints on New Physics

- Information from the $B_s \to \mu^+\mu^-$ branching ratio:

$$R \equiv \frac{\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{exp}}}{\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}}} = \left[\frac{1+\mathcal{A}_{\Delta\Gamma}y_s}{1-y_s^2}\right]\left(|P|^2+|S|^2\right)$$

$$= \left[\frac{1+y_s\cos 2\varphi_P}{1-y_s^2}\right]|P|^2 + \left[\frac{1-y_s\cos 2\varphi_S}{1-y_s^2}\right]|S|^2 \overset{\mathrm{LHCb}}{=} 0.99^{+0.47}_{-0.38}$$

  – R does not allow a separation of the P and S contributions:

  $\Rightarrow$ *large NP could be present, even if the BR is close to the SM value.*

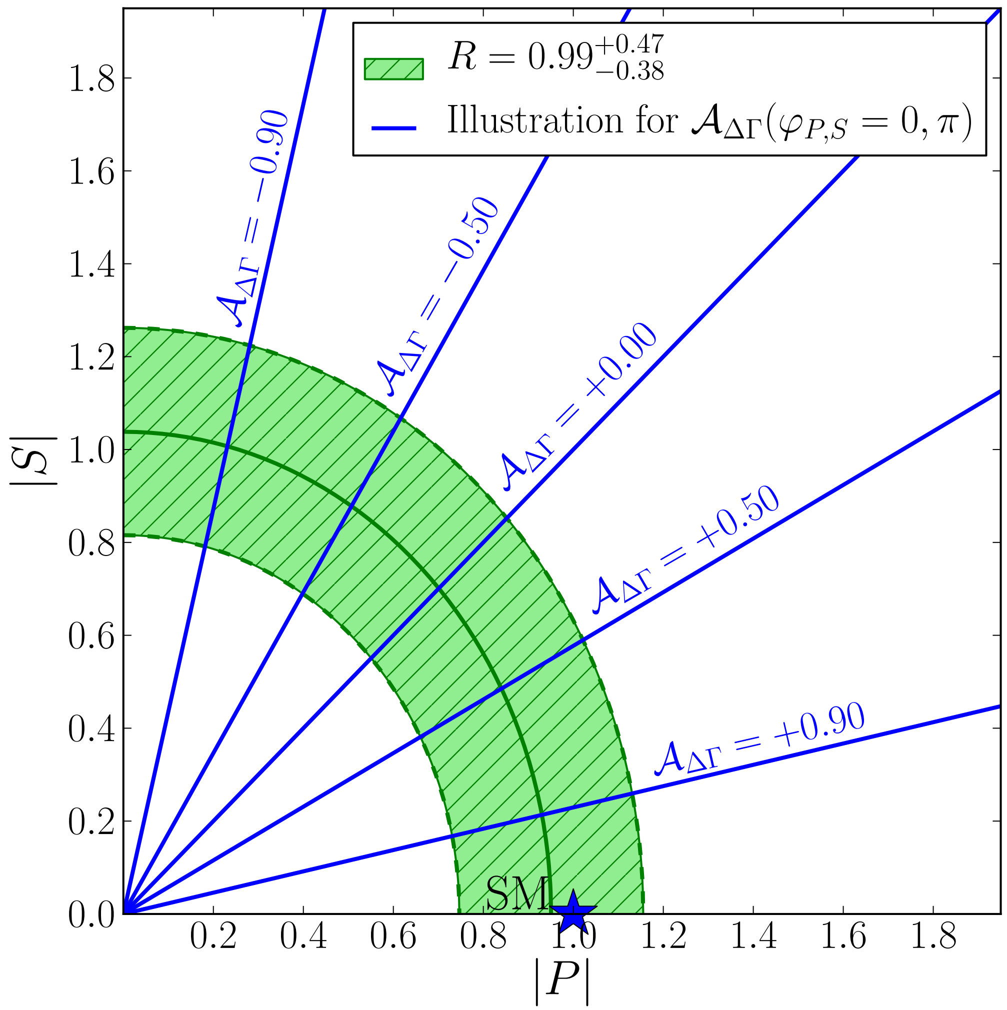
- Further information from the measurement of $\tau_{\mu^+\mu^-}$ yielding $\mathcal{A}_{\Delta\Gamma}$:

$$|S| = |P|\sqrt{\frac{\cos 2\varphi_P - \mathcal{A}_{\Delta\Gamma}}{\cos 2\varphi_S + \mathcal{A}_{\Delta\Gamma}}}$$

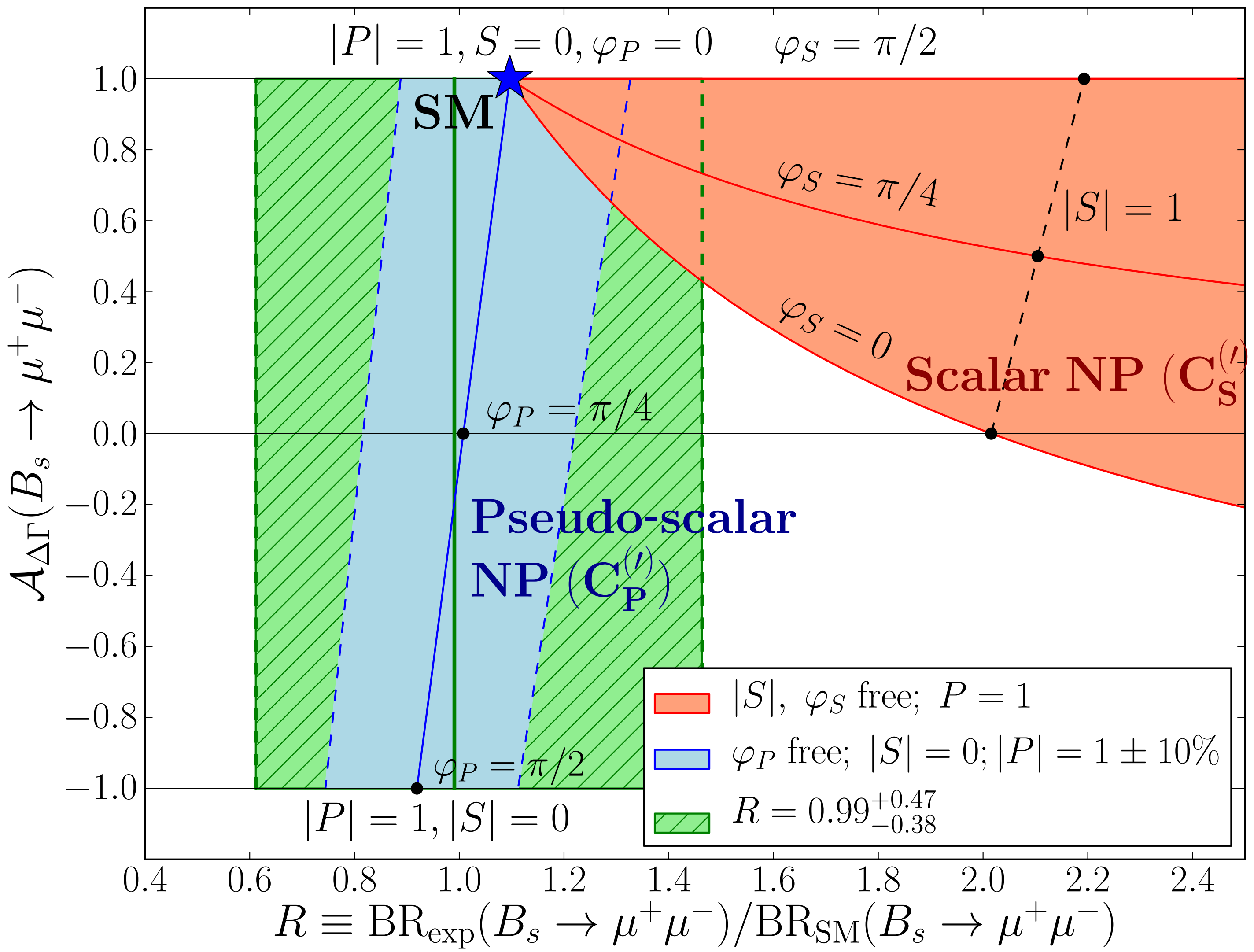
$\Rightarrow$ **offers a new window for New Physics in $B_s \to \mu^+\mu^-$**

How does the situation in NP parameter space look like?

- Current constraints in the $|P|$–$|S|$ plane and illustration of those following from a future measurement of the $B_s \to \mu^+\mu^-$ lifetime yielding $\mathcal{A}_{\Delta\Gamma}$:



- Illustration of the allowed regions in the R–$\mathcal{A}_{\Delta\Gamma}$ plane for scenarios with scalar or non-scalar NP contributions:



- Authors have started to include the effect of $\Delta\Gamma_s$ in analyses of the constraints on NP that are implied by BR$(B_s \to \mu^+\mu^-)_{\rm exp}$:

O. Buchmueller, R. Cavanaugh, M. Citron, A. De Roeck, M. J. Dolan, J. R. Ellis, H. Flächer and S. Heinemeyer *et al.*, "The CMSSM and NUHM1 in Light of 7 TeV LHC, $B_s \to \mu^+\mu^-$ and XENON100 Data," arXiv:1207.7315 [hep-ph]

T. Hurth and F. Mahmoudi, "The Minimal Flavour Violation benchmark in view of the latest LHCb data," arXiv:1207.0688 [hep-ph]

A. J. Buras and J. Girrbach, "On the Correlations between Flavour Observables in Minimal $U(2)^3$ Models," arXiv:1206.3878 [hep-ph]

W. Altmannshofer and D. M. Straub, "Cornering New Physics in $b \to s$ Transitions," arXiv:1206.0273 [hep-ph]

D. Becirevic, N. Kosnik, F. Mescia and E. Schneider, "Complementarity of the constraints on New Physics from $B_s \to \mu^+\mu^-$ and from $B \to K\ell^+\ell^-$ decays," arXiv:1205.5811 [hep-ph]

F. Mahmoudi, S. Neshatpour and J. Orloff, "Supersymmetric constraints from $B_s \to \mu^+\mu^-$ and $B \to K^*\mu^+\mu^-$ observables," arXiv:1205.1845 [hep-ph]

T. Li, D. V. Nanopoulos, W. Wang, X. -C. Wang and Z. -H. Xiong, "Rare B decays in the flip SU(5) Model," JHEP **1207** (2012) 190 arXiv:1204.5326 [hep-ph]

...

◇ Different Hot Topic:

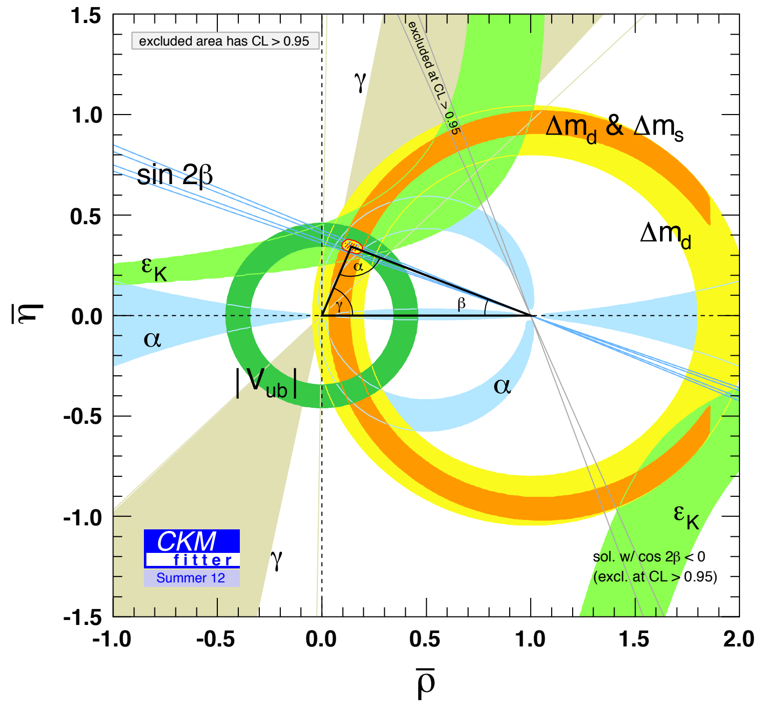
Precision Studies of CP Violation

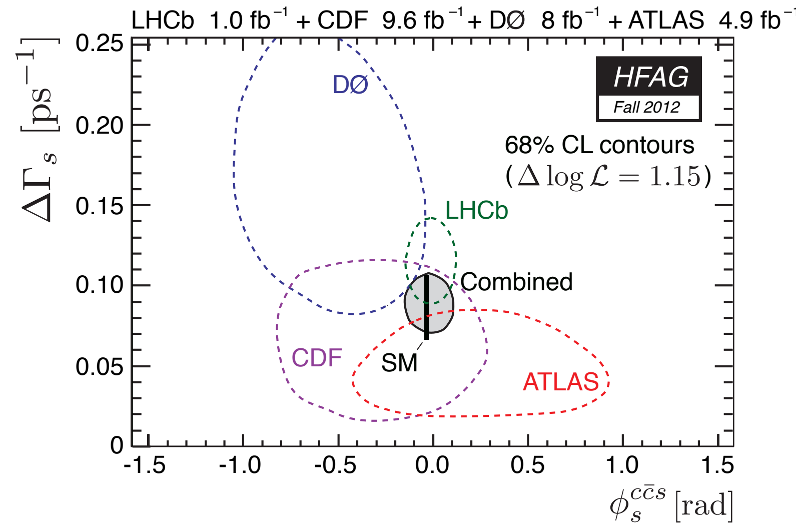
$$B_d \to J/\psi K_{\rm S},\ B_s \to J/\psi \phi,\ B_s \to J/\psi f_0(980)$$

- Allow measurements of the $B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing phases $\phi_{d,s}$.
- Uncertainties from doubly Cabibbo-suppressed penguin contributions.
- These effects are usually neglected, cannot be calculated reliably ...

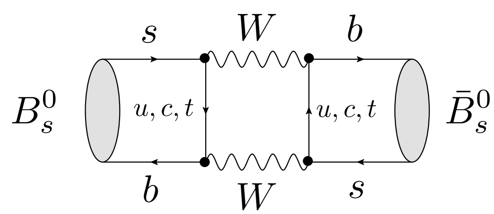
$\Rightarrow$ **How big are they & how can they be controlled?**

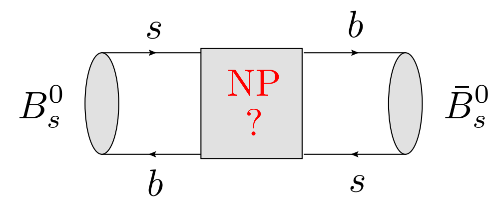
Experimental Situation





- $\underline{B_s^0\text{–}\bar{B}_s^0 \text{ mixing phase:}}$





$$\boxed{\phi_s^{c\bar{c}s} \equiv \phi_s = \phi_s^{\rm SM} + \phi_s^{\rm NP} = -2\lambda^2\eta + \phi_s^{\rm NP}}$$

- HFAG average:

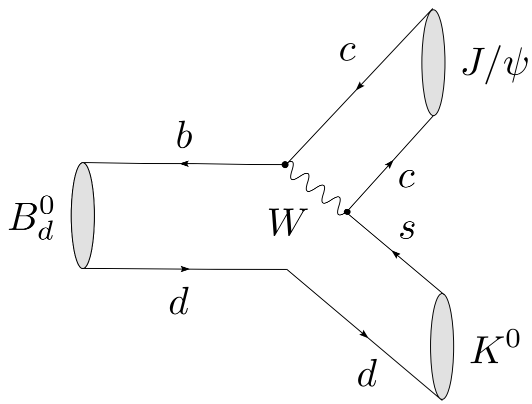
$$\phi_s = -(2.5^{+4.8}_{-5.2})^\circ \quad \text{vs.} \quad \phi_s^{\rm SM} = -(2.08 \pm 0.09)^\circ$$

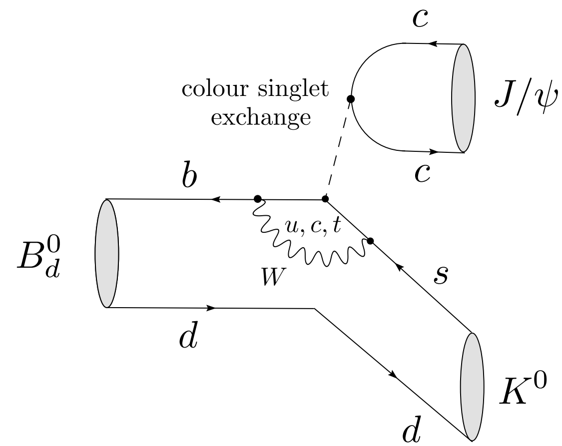
$$\boxed{B_d^0 \to J/\psi K_S \oplus B_s^0 \to J/\psi K_S}$$

Current picture of the penguin parameters?

[Thanks to Kristof De Bruyn for plots/numerics; work in progress.]

The Decay $B_d \to J/\psi K_S$





- Decay amplitude in the SM:

$$A(B_d^0 \to J/\psi\, K_S) = \lambda_c^{(s)} \left[A_T^{(c)'} + A_P^{(c)'} \right] + \lambda_u^{(s)} A_P^{(u)'} + \lambda_t^{(s)} A_P^{t'}$$

- Unitarity of the CKM matrix: $\Rightarrow\ \lambda_t^{(s)} = -\lambda_c^{(s)} - \lambda_u^{(s)}\quad [\lambda_q^{(s)} \equiv V_{qs}V_{qb}^*]$:

$$\Rightarrow \boxed{A(B_d^0 \to J/\psi\, K_S) = \left(1 - \lambda^2/2\right) \mathcal{A}' \left[1 + \epsilon\, a' e^{i\theta'} e^{i\gamma} \right]}$$

$$\mathcal{A}' \equiv \lambda^2 A \left[A_T^{(c)'} + A_P^{(c)'} - A_P^{(t)'} \right], \qquad a' e^{i\theta'} \equiv R_b \left[\frac{A_P^{(u)'} - A_P^{(t)'}}{A_T^{(c)'} + A_P^{(c)'} - A_P^{(t)'}} \right]$$

$$A \equiv |V_{cb}|/\lambda^2 \sim 0.8, \;\; R_b \equiv \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| \sim 0.5, \;\; \epsilon \equiv \frac{\lambda^2}{1-\lambda^2} = 0.053$$

- Time-dependent CP asymmetry (CP-odd final state):

$$\frac{\Gamma(B_d^0 \to J/\psi K_S) - \Gamma(\bar{B}_d^0 \to J/\psi K_S)}{\Gamma(B_d^0 \to J/\psi K_S) + \Gamma(\bar{B}_d^0 \to J/\psi K_S)}$$
$$= C(B_d \to J/\psi K_S) \cos(\Delta M_d t) - S(B_d \to J/\psi K_S) \sin(\Delta M_d t)$$

- CP-violating observables: $[\phi_d = 2\beta + \phi_d^{\rm NP} \to B_d^0$–$\bar{B}_d^0$ mixing phase$]$

$$C(B_d \to J/\psi K_S) = -\frac{2\epsilon a \sin\theta \sin\gamma}{1 + 2\epsilon a \cos\theta \cos\gamma + \epsilon^2 a^2}$$

$$\frac{S(B_d \to J/\psi K_S)}{\sqrt{1 - C(B_d \to J/\psi K_S)^2}} = \sin(\phi_d + \Delta\phi_d)$$

$$\sin\Delta\phi_d = \frac{2\epsilon a' \cos\theta' \sin\gamma + \epsilon^2 a'^2 \sin 2\gamma}{(1 + 2\epsilon a' \cos\theta' \cos\gamma + \epsilon^2 a'^2)\sqrt{1 - C(B_d \to J/\psi K_S)^2}}$$

$$\cos\Delta\phi_d = \frac{1 + 2\epsilon a' \cos\theta \cos\gamma + \epsilon^2 a'^2 \cos 2\gamma}{(1 + 2\epsilon a' \cos\theta' \cos\gamma + \epsilon^2 a'^2)\sqrt{1 - C(B_d \to J/\psi K_S)^2}}$$

[Faller, R.F., Jung & Mannel (2008)]

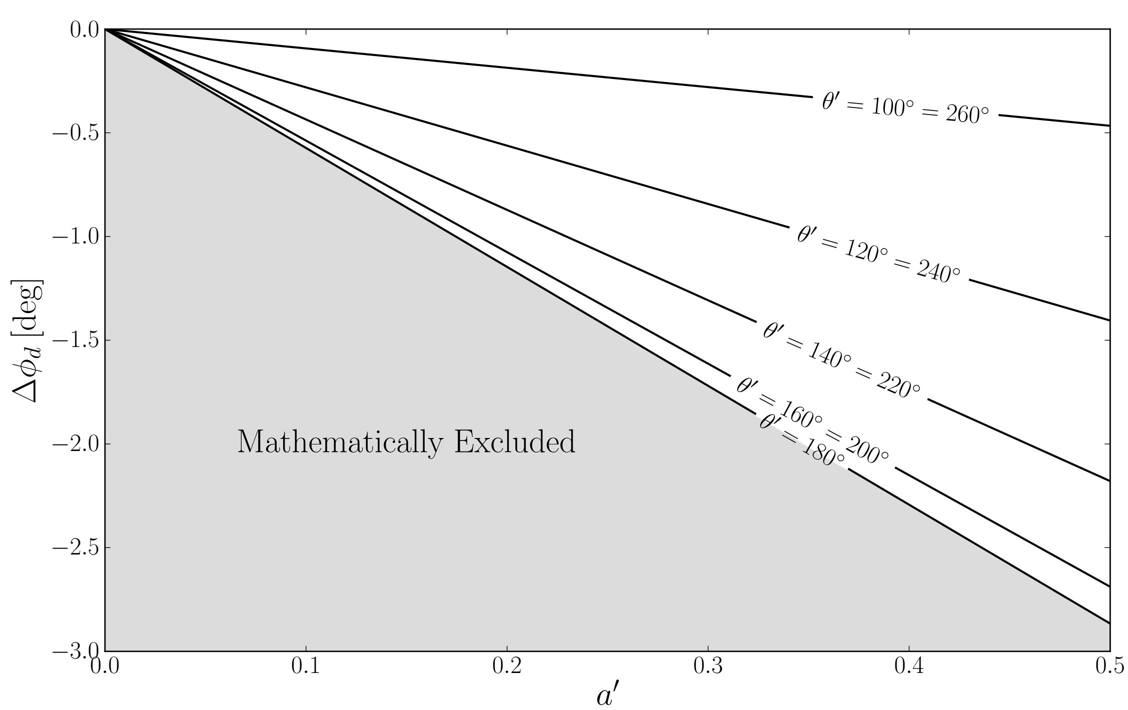
- Current experimental status: [HFAG]

$$S(B_d \to J/\psi K_S) = 0.665 \pm 0.024$$

$$C(J/\psi K_S) = 0.024 \pm 0.026 \quad \Rightarrow \quad \sqrt{1 - C(J/\psi K_S)^2} = 0.99971^{+0.00029}_{-0.00096}$$

$$\Rightarrow \boxed{S(B_d \to J/\psi K_S) = \sin(\phi_d + \Delta\phi_d) = 0.665 \pm 0.024}$$

- Illustration of the impact of the penguin topologies: $a' e^{i\theta'} \sim R_b \left[\frac{\text{“pen”}}{\text{“tree”}}\right]$



How can we control $\Delta\phi_d$?

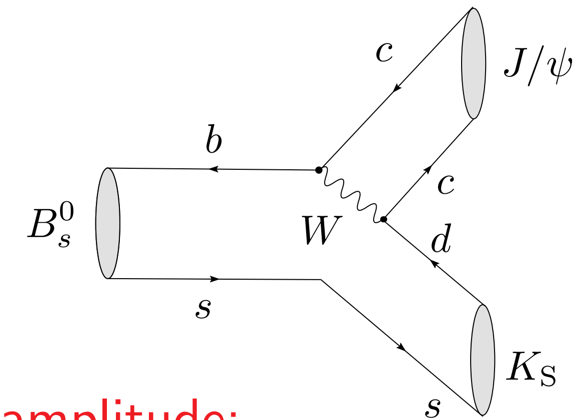
$$\tan\Delta\phi_d = \frac{2\epsilon a'\cos\theta'\sin\gamma + \epsilon^2 a'^2\sin 2\gamma}{1 + 2\epsilon a'\cos\theta\cos\gamma + \epsilon^2 a'^2\cos 2\gamma}$$

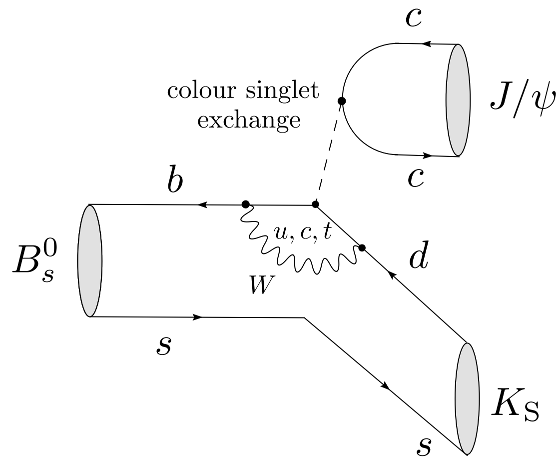
$\rightarrow$ hadronic parameters a', θ' cannot be calculated:

$\Rightarrow$ use control channel(s): $B_s^0 \rightarrow J\psi K_{\rm S} \oplus U$-spin symmetry

[R.F., Eur. Phys. J. C **10** (1999) 299 [hep-ph/9903455]]

The Decay $B_s \to J/\psi K_S$





- Decay amplitude:

$$A(B_s^0 \to J/\psi\, K_S) = \lambda_c^{(d)} \left[A_T^{(c)} + A_P^{(c)} \right] + \lambda_u^{(d)} A_P^{(u)} + \lambda_t^{(d)} A_P^t$$

- Unitarity of the CKM matrix: $\lambda_t^{(d)} = -\lambda_c^{(d)} - \lambda_u^{(d)}$

$$\Rightarrow \boxed{A(B_s^0 \to J/\psi\, K_S) = -\lambda\, \mathcal{A} \left[1 - a e^{i\theta} e^{i\gamma} \right]}$$

$$\mathcal{A} \equiv \lambda^2 A \left[A_T^{(c)} + A_P^{(c)} - A_P^{(t)} \right], \quad a e^{i\theta} \equiv R_b \left[\frac{A_P^{(u)} - A_P^{(t)}}{A_T^{(c)} + A_P^{(c)} - A_P^{(t)}} \right]$$

- In contrast to $B_d^0 \to J/\psi K_S$, $a e^{i\theta}$ is *not* suppressed by $\epsilon = 0.05$:

$\Rightarrow$ penguin effects are "magnified"!

- Untagged rate: $\langle\Gamma(B_s(t)\to f)\rangle \equiv \Gamma(B_s^0(t)\to f)+\Gamma(\bar{B}_s^0(t)\to f)$

$$\langle\Gamma(B_s(t)\to f)\rangle \propto \left[\cosh\left(\frac{y_s\,t}{\tau_{B_s}}\right)+\mathcal{A}_{\Delta\Gamma}^{f}\sinh\left(\frac{y_s\,t}{\tau_{B_s}}\right)\right]$$

  – "Experimental" branching ratio: $[y_s \equiv \Delta\Gamma_s/(2\,\Gamma_s)\sim 0.1]$

$$\mathrm{BR}\,(B_s\to f)_{\rm exp} \equiv \frac{1}{2}\int_0^\infty \langle\Gamma(B_s(t)\to f)\rangle\,dt$$

  – "Theoretical" branching ratio: $\to$ *will be used below ...*

$$\mathrm{BR}\,(B_s\to f)_{\rm theo}\equiv\frac{\tau_{B_s}}{2}\langle\Gamma(B_s^0(t)\to f)\rangle\Big|_{t=0}$$

- Conversion between both BRs: $\to$ effective decay lifetime τ_f useful:

$$\mathrm{BR}\,(B_s\to f)_{\rm theo}=\left[\frac{1-y_s^2}{1+\mathcal{A}_{\Delta\Gamma}^{f}\,y_s}\right]\mathrm{BR}\,(B_s\to f)_{\rm exp}$$

$$=\left[2-\left(1-y_s^2\right)\frac{\tau_f}{\tau_{B_s}}\right]\mathrm{BR}\,(B_s\to f)_{\rm exp}$$

[De Bruyn, R.F., Knegjens, Koppenburg, Merk & Tuning (2012); see above]

- Useful quantity: [$\Phi^s_{J/\psi K_S}$, $\Phi^d_{J/\psi K_S}$: phase-space factors]

$$H \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{A}'}{\mathcal{A}}\right|^2 \left[\frac{\tau_{B_d}\Phi^d_{J/\psi K_S}}{\tau_{B_s}\Phi^s_{J/\psi K_S}}\right] \frac{\mathrm{BR}\,(B_s \to J/\psi K_S)_{\rm theo}}{\mathrm{BR}(B_d \to J/\psi K_S)_{\rm theo}}$$

$$= \frac{1 - 2a\cos\theta\cos\gamma + a^2}{1 + 2\epsilon a'\cos\theta'\cos\gamma + \epsilon^2 a'^2}$$

- Further $B_s^0 \to J/\psi K_S$ observables from *tagged* time-dependent rates:

$$\frac{\Gamma(B_s^0 \to J/\psi K_S) - \Gamma(\bar{B}_s^0 \to J/\psi K_S)}{\Gamma(B_s^0 \to J/\psi K_S) + \Gamma(\bar{B}_s^0 \to J/\psi K_S)}$$

$$= \frac{C(B_s \to J/\psi K_S)\cos(\Delta M_s t) - S(B_s \to J/\psi K_S)\sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2) + \mathcal{A}_{\Delta\Gamma}(B_s \to J/\psi K_S)\sinh(\Delta\Gamma_s t/2)}$$

$$\Rightarrow \quad C, \quad S, \quad \mathcal{A}_{\Delta\Gamma}$$

– Note that these observables are not independent: $C^2 + S^2 + \mathcal{A}^2_{\Delta\Gamma} = 1$.

Extraction of γ and Penguin Parameters

- U-spin flavour symmetry:

$$a = a', \quad \theta = \theta'$$

$$\Rightarrow \qquad \mathcal{A}' = \mathcal{A}$$

- Observables:

$$\begin{aligned} H &= \text{function}(a, \theta, \gamma) \\ C(B_s \to J/\psi K_{\rm S}) &= \text{function}(a, \theta, \gamma) \\ S(B_s \to J/\psi K_{\rm S}) &= \text{function}(a, \theta, \gamma; \phi_s) \end{aligned}$$

$\Rightarrow$ **γ, a and θ can be extracted from the 3 observables**

[ϕ_s denotes the B_s^0–$\bar{B}_s^0$ mixing phase, with $\phi_s^{\rm SM} = -2\lambda^2\eta \sim -2^\circ$]

- Change of focus of interest since 1999:
  - Extraction of γ @ LHCb is feasible but probably not competitive ...
  - Assume that γ is know $\Rightarrow$ clean determination of the penguin parameters a, θ from C and S (further info from H).

[R.F. (1999); De Bruyn, R.F. & Koppenburg (2010)] through

$$\Gamma(B(t) \to f) + \Gamma(\overline{B}(t) \to f) = \mathrm{PhSp} \times |\mathcal{N}|^2 \times \left[R_H e^{-\Gamma_H t} + R_L e^{-\Gamma_L t}\right], \quad (28)$$

where PhSp denotes an appropriate, straightforwardly calculable phase-space factor. Consequently, the overall normalization $|\mathcal{N}|^2$ is required in order to determine R. In the case of the decay $B_s \to J/\psi K_S$, this normalization can be fixed through the CP-averaged $B_d \to J/\psi K_S$ rate with the help of the U-spin symmetry.

In the case of $B_d \to J/\psi K_S$, we have

$$\mathcal{N} = \left(1 - \frac{\lambda^2}{2}\right)\mathcal{A}', \quad b = \epsilon a',$$
$$\rho = \theta' + 180^\circ, \quad \text{with} \quad \epsilon \equiv \frac{\lambda^2}{1-\lambda^2}, \quad (29)$$

whereas we have in the $B_s \to J/\psi K_S$ case

$$\mathcal{N} = -\lambda \mathcal{A}, \quad b = a, \quad \rho = \theta. \quad (30)$$

Consequently, we obtain

$$H \equiv \frac{1}{\epsilon}\left(\frac{|\mathcal{A}'|}{|\mathcal{A}|}\right)^2 \left[\frac{M_{B_d}\Phi(M_{J/\psi}/M_{B_d}, M_K/M_{B_d})}{M_{B_s}\Phi(M_{J/\psi}/M_{B_s}, M_K/M_{B_s})}\right]^3 \frac{\langle\Gamma\rangle}{\langle\Gamma'\rangle}$$
$$= \frac{1 - 2a\cos\theta\cos\gamma + a^2}{1 + 2\epsilon a'\cos\theta'\cos\gamma + \epsilon^2 a'^2}, \quad (31)$$

where

in the case of $B_d \to J/\psi K_S$. Since the value of the CP-violating parameter ε_K of the neutral kaon system is small, ϕ_K can only be affected by very contrived models of new physics [14].

An important by-product of the strategy described above is that the quantities a' and θ' allow us to take into account the penguin contributions in the determination of β from $B_d \to J/\psi K_S$, which are presumably very small because of the Cabibbo suppression of $\lambda^2/(1-\lambda^2)$ in (3). Moreover, using (34), we obtain an interesting relation between the direct CP asymmetries arising in the modes $B_d \to J/\psi K_S$ and $B_s \to J/\psi K_S$ and their CP-averaged rates:

$$\frac{\mathcal{A}_{CP}^{\mathrm{dir}}(B_d \to J/\psi K_S)}{\mathcal{A}_{CP}^{\mathrm{dir}}(B_s \to J/\psi K_S)} = -\epsilon H \quad (35)$$
$$= -\left(\frac{|\mathcal{A}'|}{|\mathcal{A}|}\right)^2 \left[\frac{M_{B_d}\Phi(M_{J/\psi}/M_{B_d}, M_K/M_{B_d})}{M_{B_s}\Phi(M_{J/\psi}/M_{B_s}, M_K/M_{B_s})}\right]^3 \frac{\langle\Gamma\rangle}{\langle\Gamma'\rangle}.$$

An analogous relation holds also between the $B^\pm \to \pi^\pm K$ and $B^\pm \to K^\pm K$ CP-violating asymmetries [11,12]. At "second-generation" B-physics experiments at hadron machines, for instance at LHCb, the sensitivity may be good enough to resolve a direct CP asymmetry in $B_d \to J/\psi K_S$. In view of the impressive accuracy that can be achieved in the era of such experiments, it is also an important issue to think about the theoretical accuracy of the determination of β from $B_d \to J/\psi K_S$. The approach discussed above allows us to control these – presumably very small – hadronic uncertainties with the help of $B_s \to J/\psi K_S$.

Current information on the penguin parameters?

- $B_s^0 \to J/\psi K_{\rm S}$ has been observed by CDF and LHCb, but only the BR.
- Use data for decays with a CKM structure similar to $B_s^0 \to J/\psi K_{\rm S}$:

$$B_d^0 \to J/\psi \pi^0,\ B^+ \to J/\psi \pi^+$$

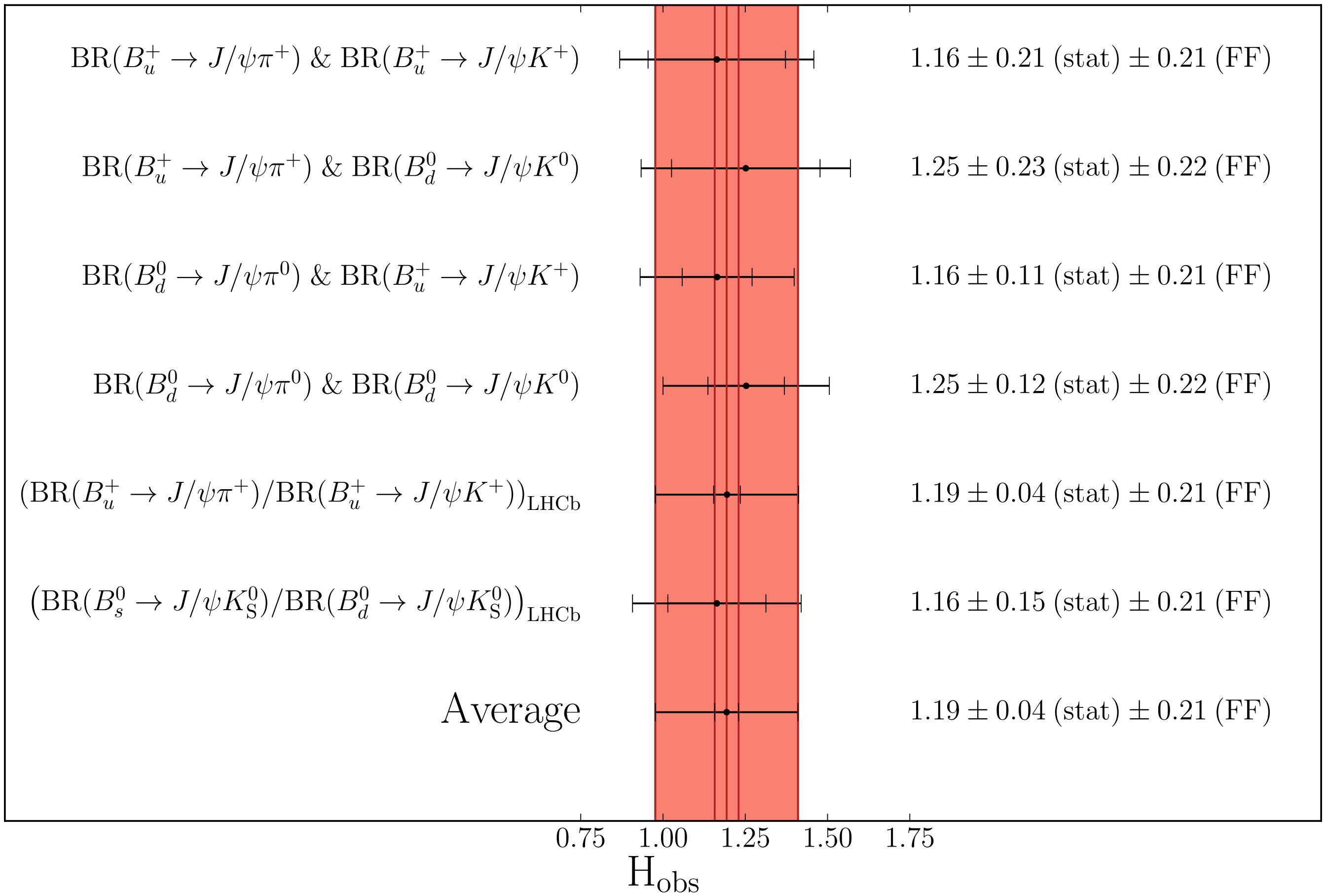
... and complement them with data for $B_d^0 \to J/\psi K^0$, $B^+ \to J/\psi K^+$.

Work in progress with K. De Bruyn & P. Koppenburg
see also Ciuchini, Pierini & Silvestrini (2005);
Faller, R.F., Jung & Mannel (2008);
Jung (2012)

Compilation of H Observables

- BR ratios, including factorizable $SU(3)$-breaking corrections:

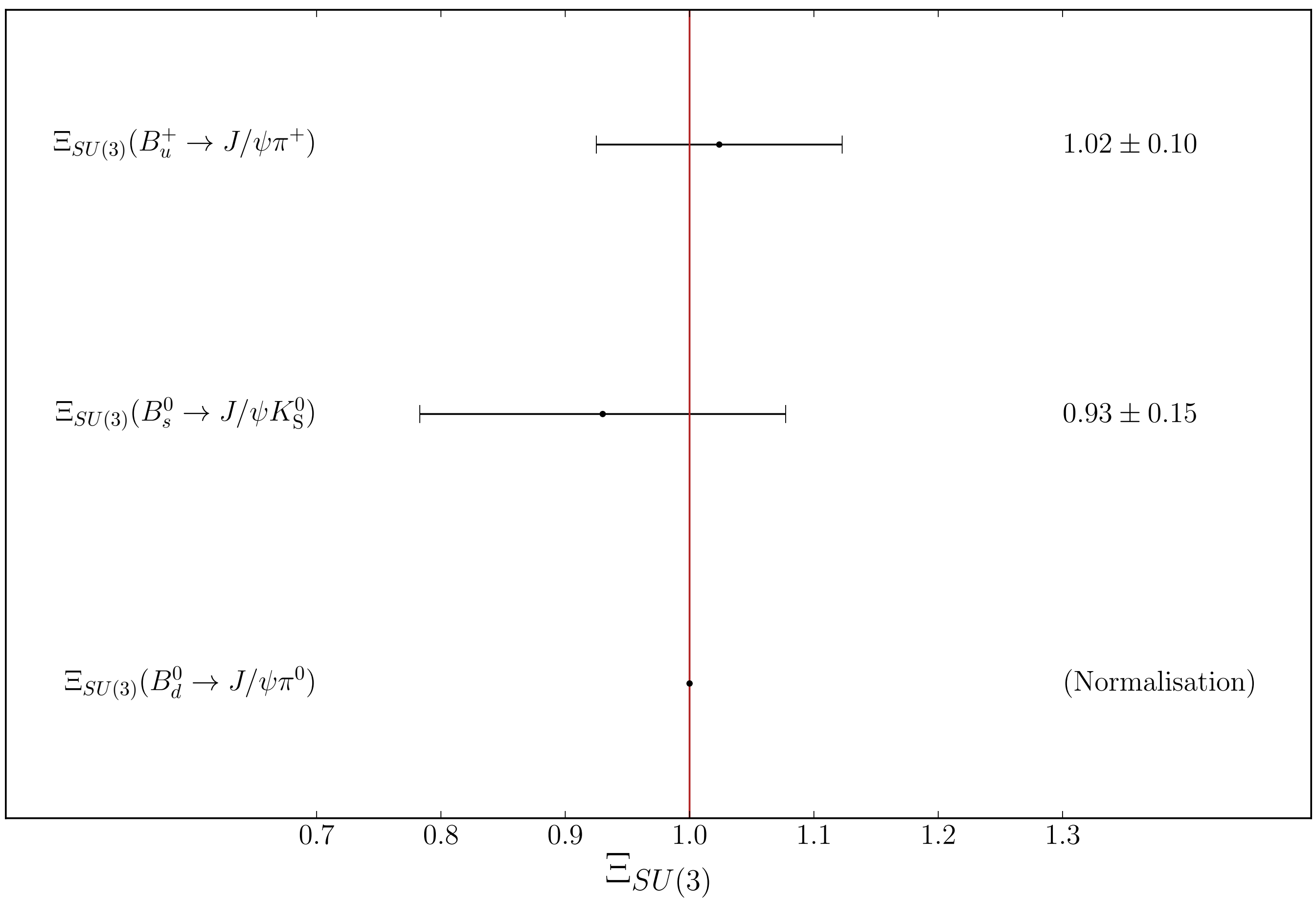
$$H \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{A}'}{\mathcal{A}}\right|^2 \left[\frac{\tau_{B_d}\Phi^d_{J/\psi K_S}}{\tau_{B_s}\Phi^s_{J/\psi K_S}}\right] \frac{\mathrm{BR}\,(B_s \to J/\psi K_S)_{\rm theo}}{\mathrm{BR}(B_d \to J/\psi K_S)_{\rm theo}}$$



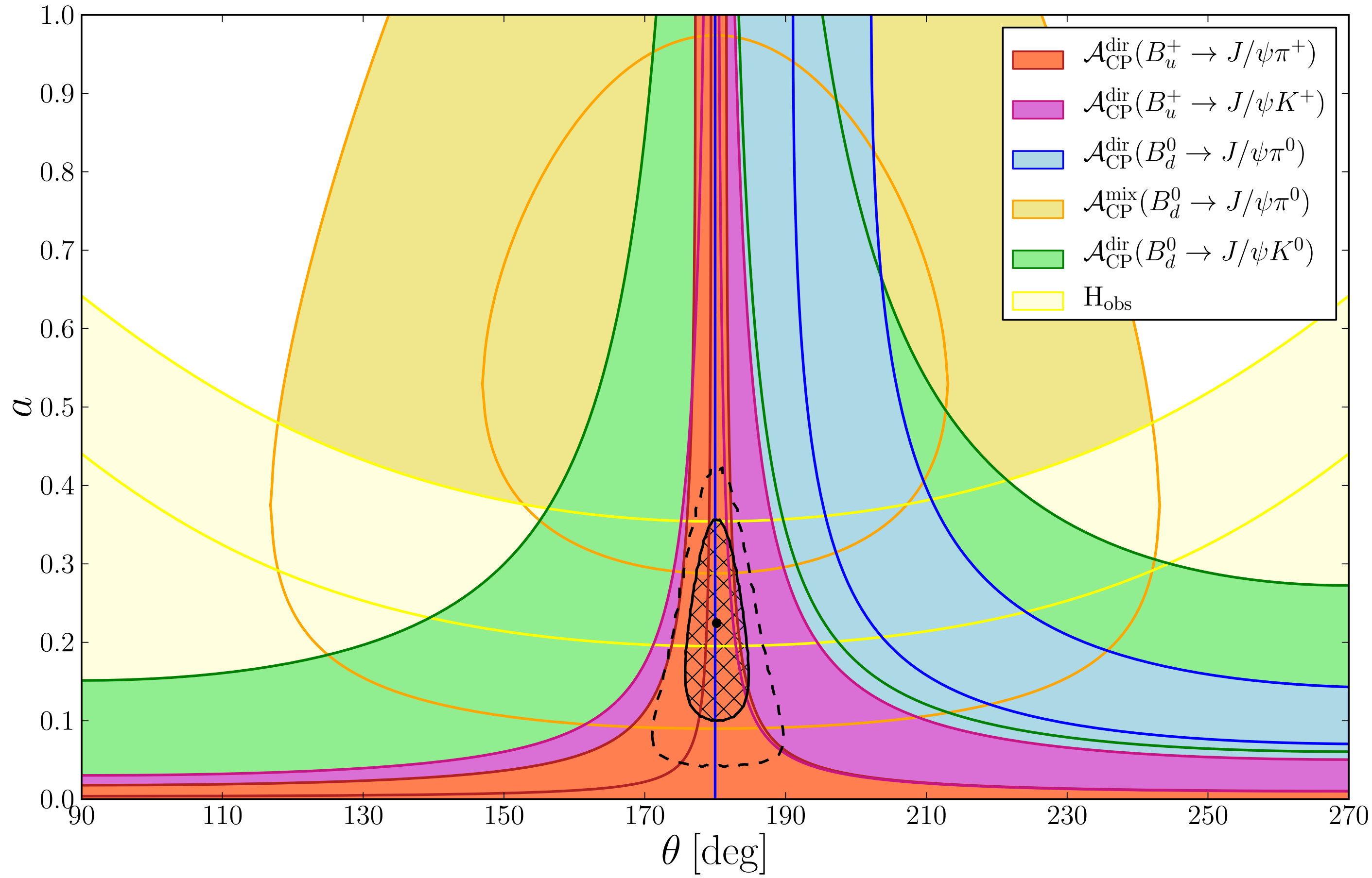
$SU(3)$ Tests

- Neglecting penguin annihilation & exchange topologies:

$$\Xi_{SU(3)} \equiv \frac{\mathrm{BR}(B_s^0 \to J/\psi \bar{K}^0)_{\mathrm{theo}}}{2\mathrm{BR}(B_d^0 \to J/\psi \pi^0)_{\mathrm{theo}}} \frac{\tau_{B_d}}{\tau_{B_s}} \frac{\Phi^d_{J/\psi \pi^0}}{\Phi^s_{J/\psi K_\mathrm{S}}} \xrightarrow{SU(3)} 1$$



Constraints on Penguin Parameters

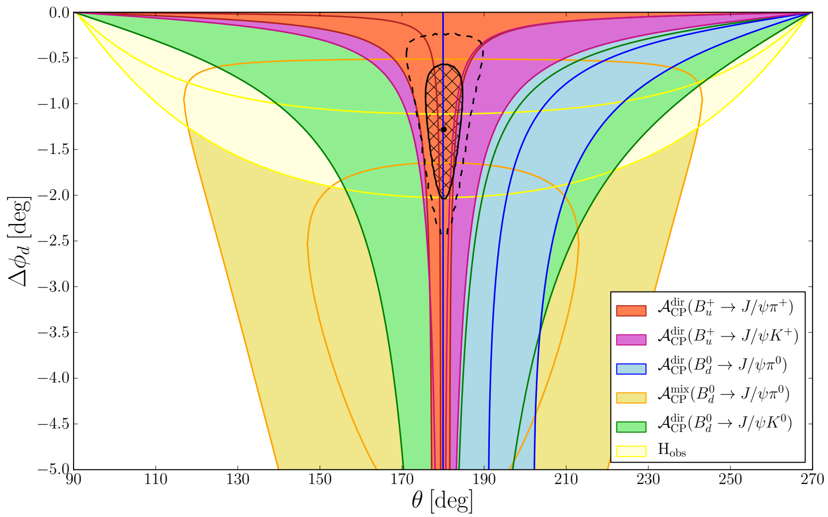


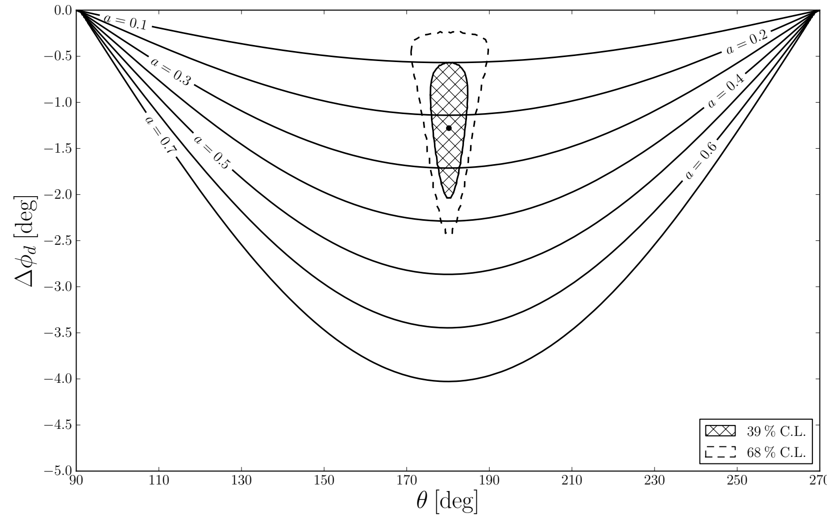
[39% and 68% C.L.]

$$a = 0.22 \pm 0.13, \quad \theta = (180.2 \pm 4.5)^\circ$$ [1σ ranges]

[Comparison with Faller, R.F., Jung & Mannel ('08): $a \in [0.15, 0.67]$, $\theta \in [174, 213]^\circ$]

Constraints on $\Delta\phi_d$





$$\boxed{\Delta\phi_d = -(1.28 \pm 0.74)^\circ}$$

$$S(B_d \to J/\psi K_{\rm S}) = \sin(\phi_d + \Delta\phi_d) = 0.665 \pm 0.024 \Rightarrow$$

$$\phi_d + \Delta\phi_d = (41.7 \pm 1.7)^\circ \Rightarrow$$

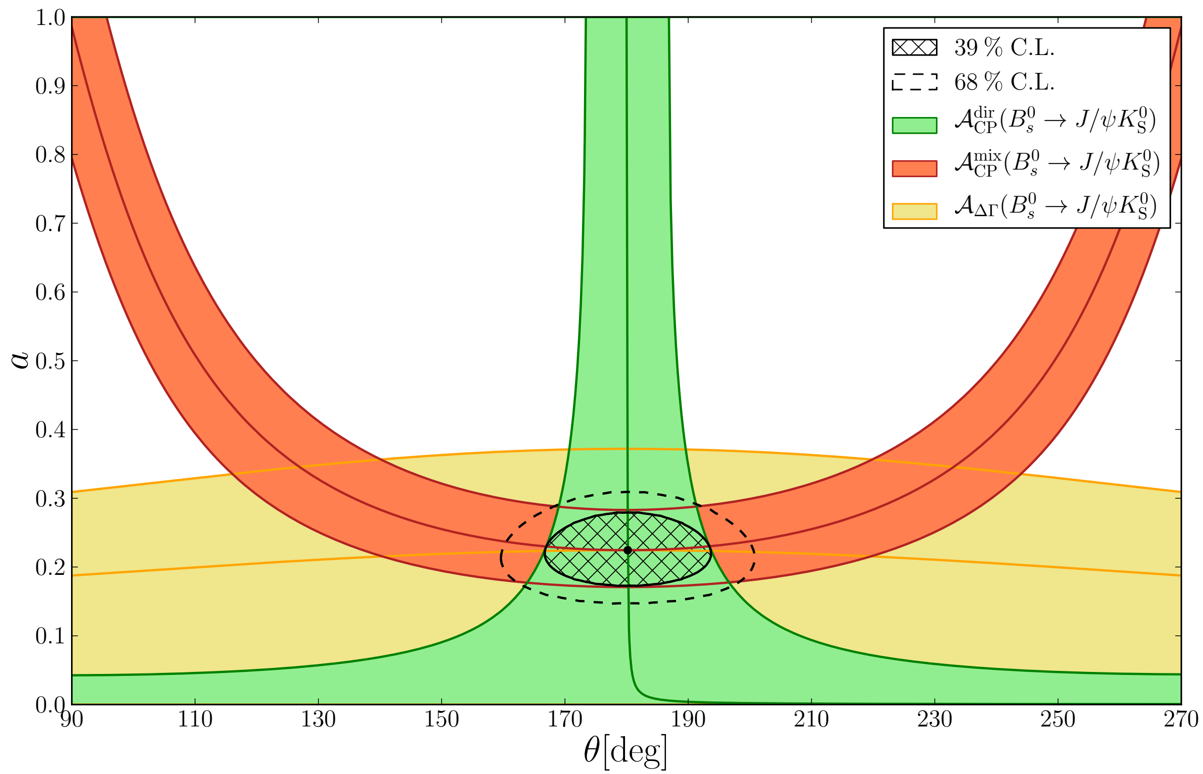
$$\phi_d = (43.0 \pm 1.7|_S \pm 0.7|_{\Delta\phi_d})^\circ = (43.0 \pm 1.8)^\circ$$

- Situation is similar in the extraction of ϕ_s from $B_s \to J/\psi\phi$ ...
- LHCb strategy document [arXiv:1208.3355]:

  $\to$ theory uncertainty of ϕ_s measurement quoted as $\sim 0.003 = 0.17^\circ$!?

Prospects for LHCb Upgrade

- Extrapolation from toy study (i.e. not official LHCb):



- Comments:
  - This determination of a and θ is theoretically clean.
  - Relation to a', θ' (enter $B_d \to J/\psi K_{\rm S}$) through U-spin symmetry.

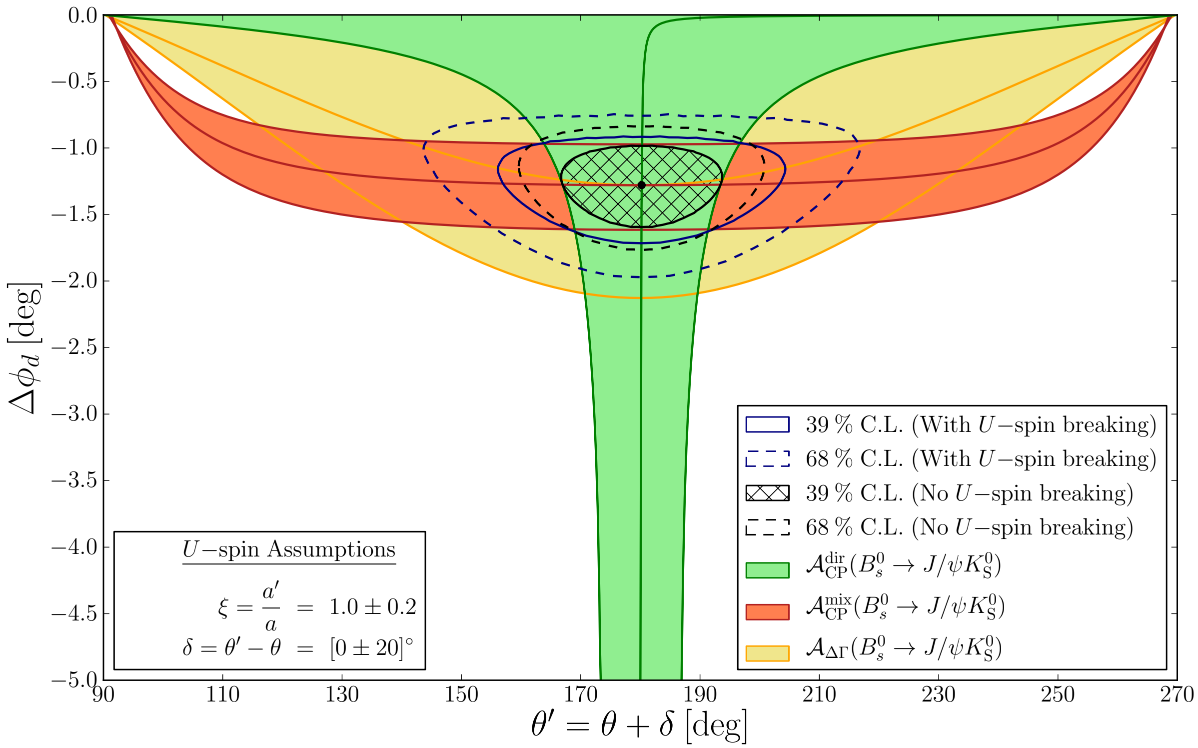
[Update of De Bruyn, R.F. & Koppenburg (2010)]

... Conversion into $\Delta\phi_d$

- Use U-spin symmetry between $B_s^0 \to J/\psi K_S$ and $B_d^0 \to J/\psi K_S$:

$$a' = a, \quad \theta' = \theta$$

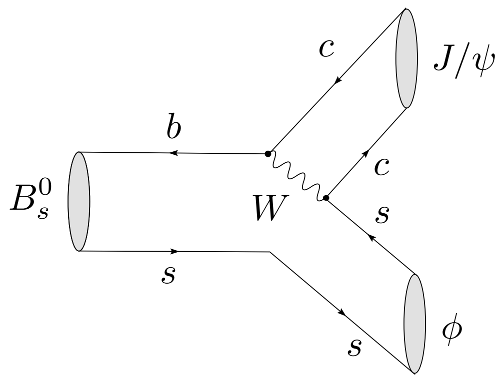
$$\Rightarrow \tan\Delta\phi_d = \frac{2\epsilon a' \cos\theta' \sin\gamma + \epsilon^2 a'^2 \sin 2\gamma}{1 + 2\epsilon a' \cos\theta \cos\gamma + \epsilon^2 a'^2 \cos 2\gamma}$$

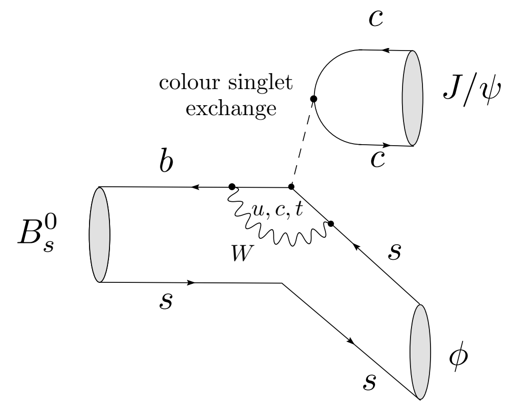


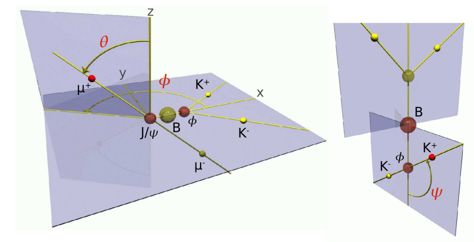
$$\boxed{B_s \to J/\psi\phi\text{:}}$$

$\Rightarrow$ B_s counterpart of $B_d \to J/\psi K_{\rm S}$

CP Violation in $B_s \to J/\psi\phi$







- Final state is mixture of CP-odd and CP-even states:

  $\to$ disentangle through $J/\psi[\to \mu^+\mu^-]\phi[\to K^+K^-]$ angular distribution

- Impact of SM penguin contributions (which are usually neglected):

$$A(B_s^0 \to (J/\psi\phi)_f) \propto \mathcal{A}_f \left[1 + \lambda^2(a_f e^{i\theta_f})e^{i\gamma}\right]$$

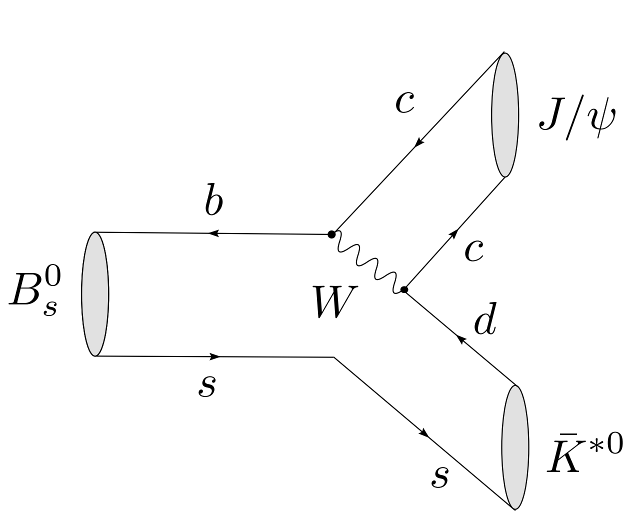
$$\mathcal{A}_{\mathrm{CP},f}^{\mathrm{mix}} = \sin\phi_s \to \sin(\phi_s + \Delta\phi_s^f)$$

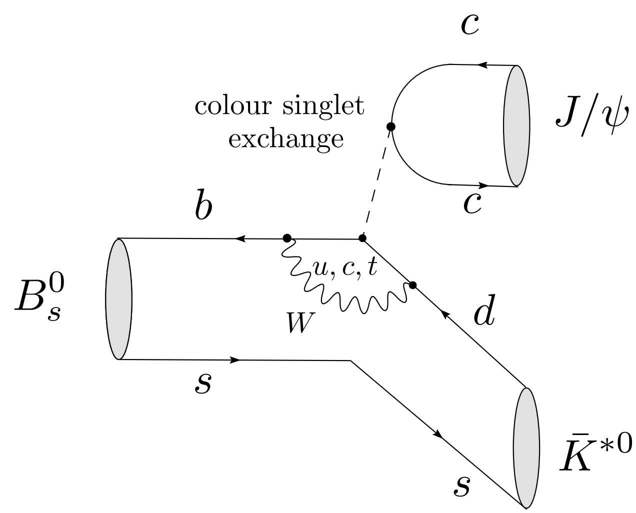
- Smallish B_s^0–$\bar{B}_s^0$ mixing phase ϕ_s (indicated by data ...):

  $\Rightarrow$ $\Delta\phi_s^f$ at the 1° level would have a significant impact ...

[Faller, R.F. & Mannel (2008)]

Control Channel: $B_s^0 \to J/\psi \bar{K}^{*0}$





- Decay amplitude: $$A(B_s^0 \to (J/\psi \bar{K}^{*0})_f) = \lambda \mathcal{A}'_f \left[1 - a'_f e^{i\theta'_f} e^{i\gamma}\right]$$
  - Neglect PA and E topologies [upper bound on $\mathrm{BR}(B_d^0 \to J/\psi \phi) \Rightarrow |E + PA|/|T| \lesssim 0.1$] and use the $SU(3)$ flavour symmetry:

$$\Rightarrow \quad |\mathcal{A}_f| = |\mathcal{A}'_f| \quad \text{and} \quad a_f = a'_f, \quad \theta_f = \theta'_f.$$

- Implementation: $\to$ no mixing-induced CP in $B_s^0 \to J/\psi \bar{K}^{*0}$, but ...
  - Untagged rate measurement $\oplus$ direct CP violation.
  - Angular analysis is required to disentangle final states $f \in \{0, \parallel, \perp\}$

Comments

- $B_s^0 \to J/\psi \bar{K}^{*0}$ was observed by CDF and LHCb [arXiv:1208.0738]:
  - Branching ratio $(4.4^{+0.5}_{-0.4} \pm 0.8) \times 10^{-5}$ agrees well with the prediction $(4.6 \pm 0.4) \times 10^{-5}$ from $B_d \to J/\psi \rho^0$ [Faller, R.F. & Mannel (2008)].
  - Polarization fractions agree well with those of $B_d^0 \to J/\psi K^{*0}$.

$\Rightarrow$ look forward to future data...

- Sensitivity at the LHCb upgrade ($50\,\mathrm{fb}^{-1}$) [arXiv:1208.3355]:

$$\Delta\phi_s|_{\rm exp} \sim 0.008 = 0.46^\circ$$

  - Theoretical uncertainty quoted as $\Delta\phi_s|_{\rm theo} \sim 0.003 = 0.17^\circ$ (!), ...
  - Data for $B \to J/\psi\pi, J/\psi K$ decays with a similar dynamics:

$$\Delta\phi_d = -(1.28 \pm 0.74)^\circ$$

  - Such phase shifts may mimic New Physics: $\mathcal{A}_{\rm CP,f}^{\rm mix} = \sin(\phi_s + \Delta\phi_s^f)$

$\Rightarrow$ we have to get a handle on the penguin effects ...

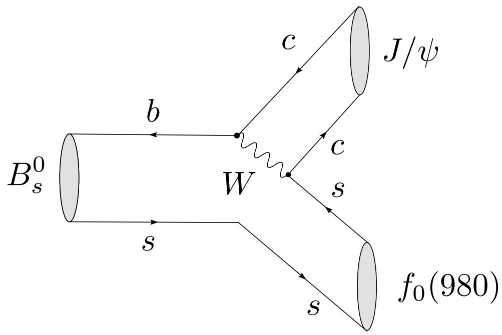
$B_s \to J/\psi f_0(980)$:

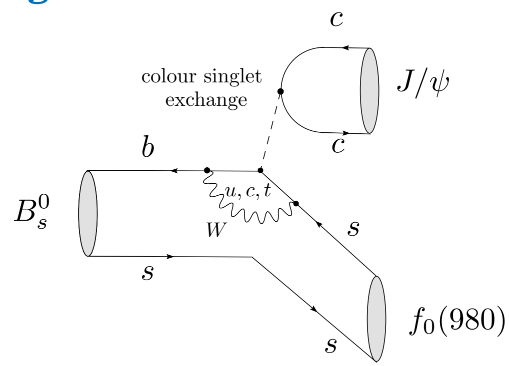
$\to$ interesting new decay

Detailed analysis: R.F., R. Knegjens & G. Ricciardi, arXiv:1109.1112 [hep-ph];

see also arXiv:1110.5490 [hep-ph], giving a discussion of $B_{s,d} \to J/\psi \eta^{(\prime)}$

General Features of $B_s^0 \to J/\psi f_0(980)$





- $f_0(980)$ is a scalar $J^{PC} = 0^{++}$ state: $\Rightarrow$ no angular analysis is required!

- Dominant mode: $B_s^0 \to J/\psi f_0$ with $f_0 \to \pi^+\pi^-$.

- Observation of $B_s^0 \to J/\psi f_0$ at LHCb, Belle, DØ and CDF:

$$R_{f_0/\phi} \equiv \frac{\mathrm{BR}(B_s^0 \to J/\psi f_0; f_0 \to \pi^+\pi^-)}{\mathrm{BR}(B_s^0 \to J/\psi\phi; \phi \to K^+K^-)} \sim 0.25$$

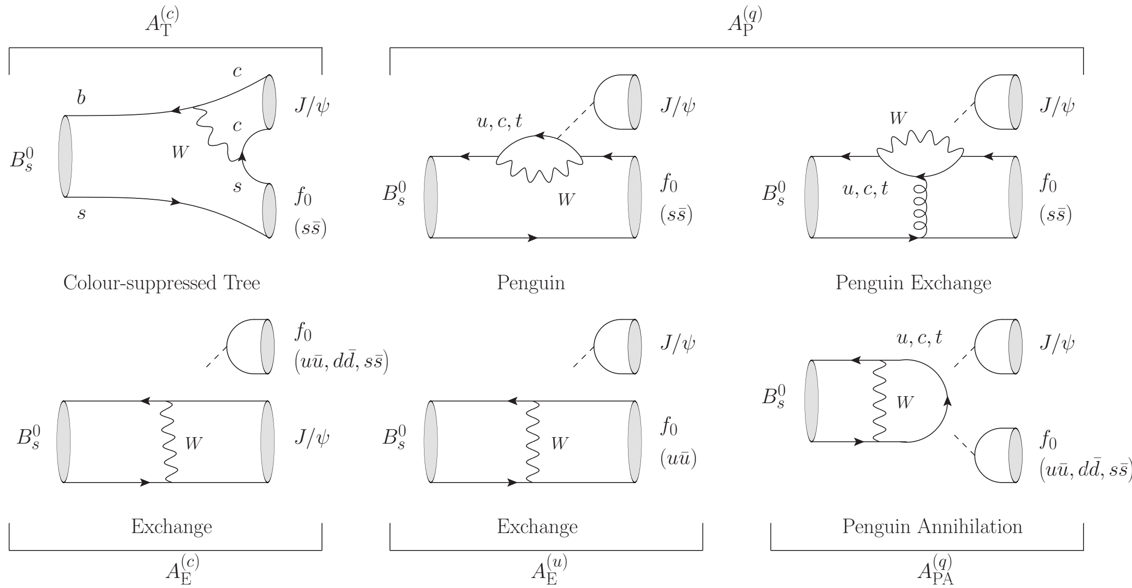
... but as no angular analysis is required:

$\Rightarrow$ **$B_s^0 \to J/\psi f_0$ offers an interesting alternative to $B_s^0 \to J/\psi\phi$**

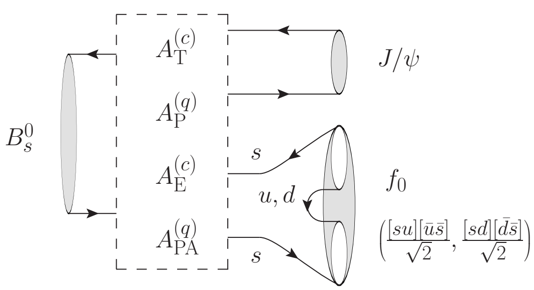
[S. Stone & L. Zhang (2009)]

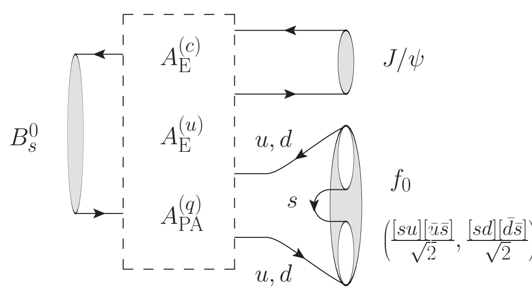
Theoretical Uncertainties?

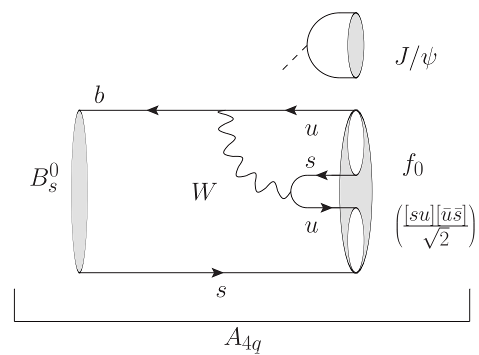
- Decay topologies:



- The composition of the $f_0(980$ is still poorly known: $\rightarrow$ 2 benchmarks:
  - Quark–antiquark: $|f_0(980)\rangle = \cos\varphi_M|s\bar{s}\rangle + \sin\varphi_M \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right)$
  - Tetraquark: $|f_0(980)\rangle = \frac{1}{\sqrt{2}}\left([su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]\right) \rightarrow$







no counterpart in $q\bar{q}$!

[R.F., R. Knegjens & G. Ricciardi, arXiv:1109.1112 [hep-ph]]

Amplitude Structure of $B_s^0 \to J/\psi f_0$

- General SM parametrization:

$$A(B_s^0 \to J/\psi f_0) \propto \left[1 + \epsilon\, b e^{i\vartheta} e^{i\gamma}\right] \quad \text{with} \quad \epsilon \equiv \lambda^2/(1-\lambda^2)$$

- Here we have introduced a CP-conserving hadronic parameter:

$$b e^{i\vartheta} \equiv R_b \left[\frac{A_{\rm P}^{(ut)} + A_{\rm E}^{(u)} + A_{\rm PA}^{(ut)}}{A_{\rm T}^{(c)} + A_{\rm P}^{(ct)} + A_{\rm E}^{(c)} + A_{\rm PA}^{(ct)}} \right]$$

$\to$ hadron dynamics (?), *but enters in a doubly Cabibbo-suppressed way*

- Characteristic hadronic phase shift:

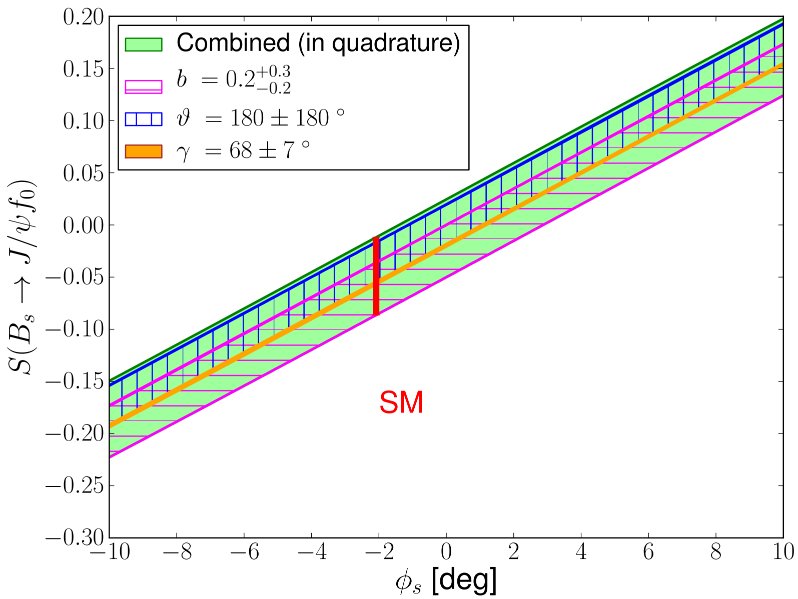
$$\tan \Delta\phi_{J/\psi f_0} = \frac{2\epsilon b \cos\vartheta \sin\gamma + \epsilon^2 b^2 \sin 2\gamma}{1 + 2\epsilon b \cos\vartheta \cos\gamma + \epsilon^2 b^2 \cos 2\gamma}$$

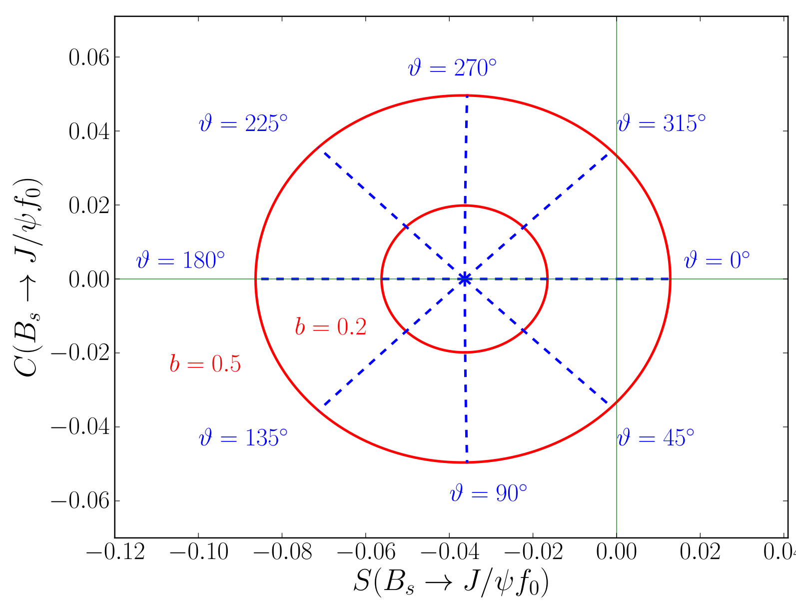
– Conservative range for $b e^{i\theta}$: $0 \leq b \leq 0.5$, $0^\circ \leq \vartheta \leq 360^\circ \quad \Rightarrow$

$$\Delta\phi_{J/\psi f_0} \in [-2.9^\circ, 2.8^\circ]$$

CP Violation in $B_s^0 \to J/\psi f_0$

$$\frac{\Gamma(B_s(t)\to J/\psi f_0)-\Gamma(\bar{B}_s(t)\to J/\psi f_0)}{\Gamma(B_s(t)\to J/\psi f_0)+\Gamma(\bar{B}_s(t)\to J/\psi f_0)}=\frac{C\cos(\Delta M_s t)-S\sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2)+\mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)}$$





- Mixing-induced CP asymmetry: $\boxed{S=\sqrt{1-C^2}\sin(\phi_s+\Delta\phi)}$
  - Naïve SM value: $(\sin\phi_s)|_{\rm SM}=-0.036\pm0.002$;
  - Allowing for hadronic effects: $S(B_s^0\to J/\psi f_0)\big|_{\rm SM}\in[-0.086,-0.012]$

Comments

- Should smallish CPV $-0.1 \lesssim S \lesssim 0$ be found:

$\Rightarrow$ crucial to constrain hadronic corrections to disentangle NP from SM

- LHCb result for ϕ_s from $B_s^0 \to J/\psi f_0$:

$$\phi_s = -(25 \pm 25 \pm 1)^\circ, \text{ corrsponds to } S = -0.43^{+0.43}_{-0.34}.$$

  - Hadronic corrections were not taken into account; still some way to go until we may eventually enter the limiting range $-0.1 \lesssim S \lesssim 0$:

$$S = \sqrt{1 - C^2}\sin(\phi_s + \Delta\phi); \quad \Delta\phi_{J/\psi f_0} \in [-2.9^\circ, 2.8^\circ]$$

  - LHCb [arXiv:1208.3355]: theory uncertainty of $\sim 0.01 = 0.57^\circ$!?

- Average with $B_s^0 \to J/\psi\phi$:

  - Increase of exp. precision: average is problematic because of hadronic effects and their different impact on $B_s^0 \to J/\psi f_0$ and $B_s^0 \to J/\psi\phi$.
  - It will actually be interesting to compare the individual measurements.

[Remember discussions about averages for CP asymmetries in $b \to s$ penguin modes]

Control Channel: $B_d^0 \to J/\psi f_0(980)$

- Leading contributions emerge from the $d\bar{d}$ component of the $f_0(980)$:

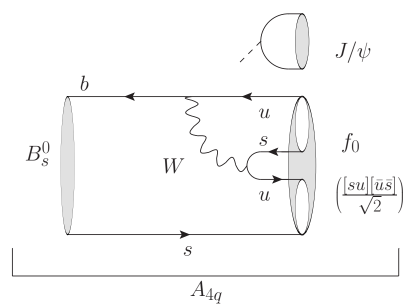
$$A(B_d^0 \to J/\psi f_0) = -\lambda \mathcal{A}' \left[1 - b' e^{i\vartheta'} e^{i\gamma} \right]$$

- Measurement of branching ratio and CP-violating asymmetries:

$\Rightarrow$ **b' and ϑ' can be (cleanly) determined**

- Relation to the b and ϑ hadronic parameters of $B_s^0 \to J/\psi f_0$:
  - $q\bar{q}$ interpretation of the $f_0(980)$: $\to b \approx b'$, $\vartheta \approx \vartheta'$ through $SU(3)$ if mixing angle is significantly different from $0°$ or $180°$.
  - Tetraquark description: topology contributing to $B_s^0 \to J/\psi f_0$ does not have a counterpart in $B_s^0 \to J/\psi f_0 \to$ how important is it!?

$\to$ **hadronic f_0 structure !?**



- Branching ratio:
  - 4q estimate: BR$(B_d^0 \to J/\psi f_0; f_0 \to \pi^+\pi^-) \sim (1–3) \times 10^{-6}$
  - 1st LHCb analysis [arXiv:1301.5347 [hep-ex]]: $< 1.1 \times 10^{-6}$ (90% C.L.)

[Details: R.F., R. Knegjens & G. Ricciardi, arXiv:1109.1112 [hep-ph]]

Effective B_s Decay Lifetimes:

$\rightarrow \left\{ \begin{array}{l} \text{constraints on the } B_s^0\text{–}\bar{B}_s^0 \text{ mixing parameters} \\ \text{that are very robust w.r.t. hadronic parameters!} \end{array} \right.$

[R.F. & Rob Knegjens, arXiv:1109.5115 [hep-ph]]

General Formalism (See also above)

- $B_s \to f$ with a final state f into which both a B_s^0 and a $\bar{B}_s^0$ can decay:

$$\langle \Gamma(B_s(t) \to f) \rangle \equiv \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)$$

$$= R_{\rm H}^f \, e^{-\Gamma_{\rm H}^{(s)} t} + R_{\rm L}^f \, e^{-\Gamma_{\rm L}^{(s)} t} \propto \; e^{-\Gamma_s t} \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + \mathcal{A}_{\Delta\Gamma}^f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right)\right]$$

$$\left[2\Gamma_s \equiv \Gamma_{\rm L}^{(s)} + \Gamma_{\rm H}^{(s)}, \quad \Delta\Gamma_s \equiv \Gamma_{\rm L}^{(s)} - \Gamma_{\rm H}^{(s)}\right]$$

- Effective lifetime of the $B_s \to f$ decay: $[y_s \equiv \Delta\Gamma_s/(2\Gamma_s), \; \tau_{B_s} = 1/\Gamma_s]$

$$\tau_f \equiv \frac{\int_0^\infty t \, \langle \Gamma(B_s(t) \to f) \rangle \, dt}{\int_0^\infty \langle \Gamma(B_s(t) \to f) \rangle \, dt} = \frac{R_{\rm L}^f/\Gamma_{\rm L}^{(s)2} + R_{\rm H}^f/\Gamma_{\rm H}^{(s)2}}{R_{\rm L}^f/\Gamma_{\rm L}^{(s)} + R_{\rm H}^f/\Gamma_{\rm H}^{(s)}}$$

$$\frac{\tau_f}{\tau_{B_s}} = \frac{1}{1-y_s^2}\left(\frac{1 + 2\mathcal{A}_{\Delta\Gamma}^f \, y_s + y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^f \, y_s}\right) = 1 + \mathcal{A}_{\Delta\Gamma}^f \, y_s + \left[2 - (\mathcal{A}_{\Delta\Gamma}^f)^2\right] y_s^2 + \mathcal{O}(y_s^3)$$

- Decay dynamics: $\to$ encoded in the observable $\mathcal{A}_{\Delta\Gamma}^f$ $\to$?

- Consider the case where f is a CP eigenstate with eigenvalue η_f:

$$A(B_s^0 \to f) = A_1^f e^{i\delta_1^f} e^{i\varphi_1^f} + A_2^f e^{i\delta_2^f} e^{i\varphi_2^f}$$

  - $A_{1,2}^f$: real parameters (chosen to be ≥ 0)
  - $\delta_{1,2}^f$: CP-conserving strong phases
  - $\varphi_{1,2}^f$: CP-violating weak phases (enter through CKM matrix elements)

  $\to$ general SM expression, using the unitarity of the CKM matrix

- B_s^0–$\bar{B}_s^0$ mixing formalism:

$$\mathcal{A}_{\Delta\Gamma}^f = \frac{2\,\mathrm{Re}\,\xi_f^{(s)}}{1 + \left|\xi_f^{(s)}\right|^2}$$

$$\xi_f^{(s)} = -\eta_f e^{-i\phi_s}\left[\frac{e^{-i\varphi_1^f} + h_f e^{i\delta_f} e^{-i\varphi_2^f}}{e^{i\varphi_1^f} + h_f e^{i\delta_f} e^{i\varphi_2^f}}\right], \qquad h_f e^{i\delta_f} \equiv \frac{A_2^f}{A_1^f} e^{i(\delta_2^f - \delta_1^f)}$$

$\to$ can derive compact expressions: $\Rightarrow$

$$[\phi_s \equiv \phi_s^{\mathrm{SM}} + \phi_s^{\mathrm{NP}} \quad \text{with} \quad \phi_s^{\mathrm{SM}} \equiv -2\beta_s = -(2.08 \pm 0.09)^\circ]$$

- Relevant combination for the calculation of the observable(s):

$$\frac{2\,\xi_f^{(s)}}{1+\left|\xi_f^{(s)}\right|^2} = -\eta_f\sqrt{1-C_f^2}\,e^{-i(\phi_s+\Delta\phi_f)}$$

  - Direct CP asymmetry C_f of the $B_s \to f$ decay:

$$C_f \equiv \frac{1-|\xi_f|^2}{1+|\xi_f|^2} = \frac{2\,h_f \sin\delta_f \sin(\varphi_1^f - \varphi_2^f)}{N_f}$$

$$N_f \equiv 1 + 2h_f\cos\delta_f\cos(\varphi_1^f - \varphi_2^f) + h_f^2$$

  - Hadronic phase shift [also expressions for $\sin\Delta\phi_f$ and $\cos\Delta\phi_f$]:

$$\tan\Delta\phi_f = \frac{\sin 2\varphi_1^f + 2h_f\cos\delta_f\sin(\varphi_1^f+\varphi_2^f) + h_f^2\sin 2\varphi_2^f}{\cos 2\varphi_1^f + 2h_f\cos\delta_f\cos(\varphi_1^f+\varphi_2^f) + h_f^2\cos 2\varphi_2^f}.$$

- Final expression for $\mathcal{A}_{\Delta\Gamma}^f$:

$$\boxed{\mathcal{A}_{\Delta\Gamma}^f = -\eta_f\sqrt{1-C_f^2}\,\cos(\phi_s+\Delta\phi_f)}$$

Lifetime Contours in the ϕ_s–$\Delta\Gamma_s$ Plane

$$\frac{\tau_f}{\tau_{B_s}} = \frac{1}{1-y_s^2}\left(\frac{1+2\mathcal{A}_{\Delta\Gamma}^f\, y_s + y_s^2}{1+\mathcal{A}_{\Delta\Gamma}^f\, y_s}\right) \Rightarrow$$ cubic equation for y_s:

$$\boxed{y_s^3 + a_2 y_s^2 + a_1 y_s + a_0 = 0}$$

$$a_0 \equiv \frac{\tau_{B_s}-\tau_f}{\tau_f \mathcal{A}_{\Delta\Gamma}^f}, \quad a_1 \equiv \frac{2\,\tau_{B_s}-\tau_f}{\tau_f}, \quad a_2 \equiv \frac{\tau_{B_s}+\tau_f}{\tau_f \mathcal{A}_{\Delta\Gamma}^f}.$$

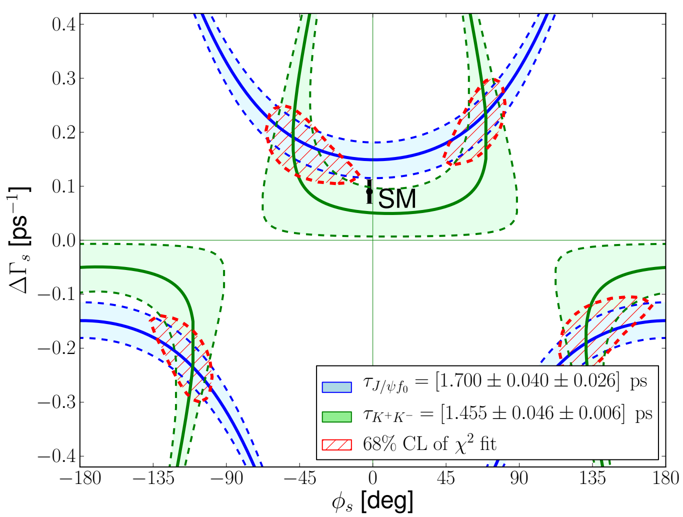
- Analytic solution: formula by Girolamo Cardano [1501–1576]

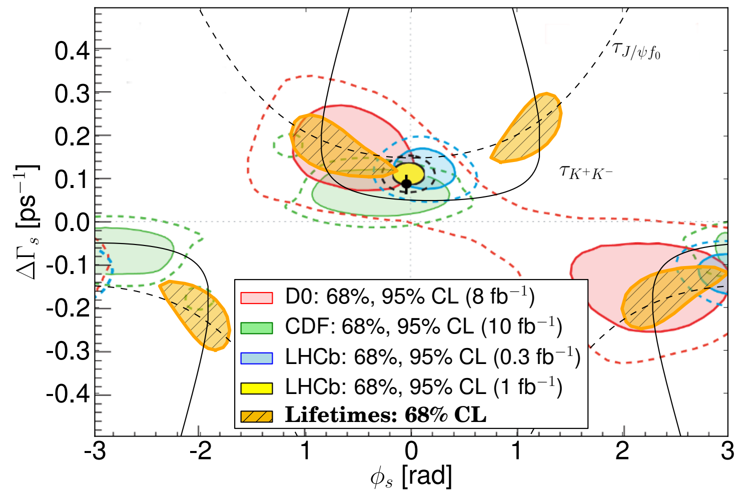
  $\to$ details in arXiv:1109.5115 [hep-ph]

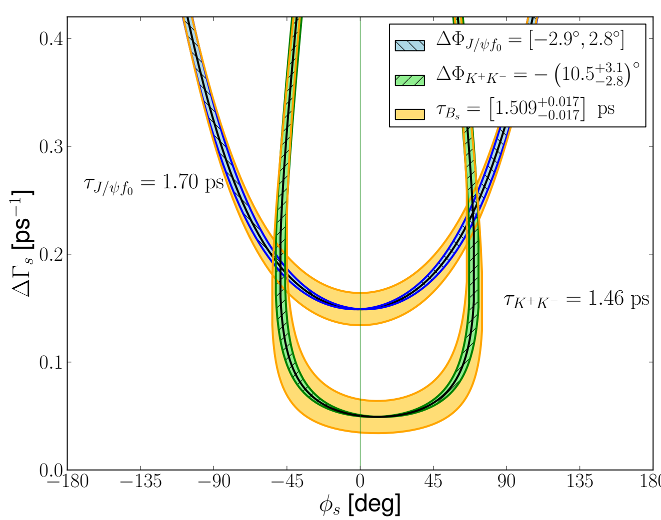
- Approximate solution: $\to$ excellent agreement with the exact solution:

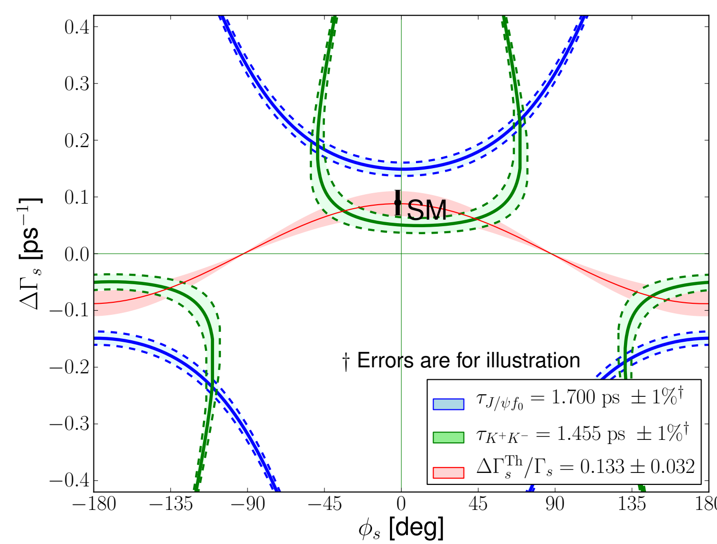
$$y_s \equiv \frac{\Delta\Gamma_s}{2\Gamma_s} \approx -\frac{1}{2}\left[\frac{\mathcal{A}_{\Delta\Gamma}^f}{2-(\mathcal{A}_{\Delta\Gamma}^f)^2}\right] \pm \frac{1}{2}\sqrt{\left[\frac{\mathcal{A}_{\Delta\Gamma}^f}{2-(\mathcal{A}_{\Delta\Gamma}^f)^2}\right]^2 + \frac{4}{\tau_{B_s}}\left[\frac{\tau_f-\tau_{B_s}}{2-(\mathcal{A}_{\Delta\Gamma}^f)^2}\right]}$$

Constraints from the $B_s^0 \to K^+K^-, J/\psi f_0$ Lifetimes









R.F. & R. Knegjens, arXiv:1109.5115 [hep-ph]; update: R. Knegjens, arXiv:1209.3206 [hep-ph]

Experimental overview: F. Dordei, arXiv:1212.3797 [hep-ex]

Comments

- The lifetime contours are very robust with respect to the hadronic uncertainties, which are described by the $\Delta\phi_{J/\psi f_0}$ and $\Delta\phi_{K^+K^-}$:

$$\to \text{ enter through } \boxed{\mathcal{A}^f_{\Delta\Gamma} \propto \cos(\phi_s + \Delta\phi_f)}$$

... while the CP asymmetries are given by $S_f \propto \sin(\phi_s + \Delta\phi_f)$.

[$\Delta\phi_f$: $B_s \to J/\psi f_0$ see discussion above, and "backup slides" for $B_s \to K^+K^-$]

- Improved measurements of the effective $B_s \to J/\psi f_0$ and $B_s \to K^+K^-$ lifetimes with 1% uncertainty will be very interesting.

- It would also be interesting to make such an analysis for the effective lifetimes of the $f \in \{0, \|, \perp\}$ final-state configurations of $B_s \to (J/\psi\phi)_f$.

Conclusions

◇ New Frontiers in Precision Physics:

- Still no signals for New Physics @ LHC:
  - Impressive (also frustrating ...), but more is yet to come!
  - Prepare to deal with "smallish" NP effects:

$\Rightarrow$ Match experimental with theoretical precision!

Subtleties for B_s Branching Ratios

- LHCb has recently established $\Delta\Gamma_s \neq 0$ at the 6σ level: $\Rightarrow$
  - Care has to be taken when dealing with B_s decay branching ratios.
  - Some confusion in the (experimental) literature ...
- Discussed how the measured "experimental" $B_s \to f$ branching ratios can be converted into the "theoretical" $B_s \to f$ branching ratios:
  - Use theoretical input to determine $\mathcal{A}_{\Delta\Gamma}^{f}$, depending on final state f:
    $\to$ hadronic parameters [use, e.g., $SU(3)_{\rm F}$ $\oplus$ assumptions about NP].
  - Use the measured effective $B_s \to f$ decay lifetime:
    $\to$ preferred avenue using *only data:* $\Rightarrow$ **BRs for particle listings**
- Examples of specific B_s decays:

$B_s^0 \to J/\psi f_0(980)$, $B_s^0 \to J/\psi K_{\rm S}$, $B_s^0 \to D_s^-\pi^+$, $B_s^0 \to K^+K^-$, $B_s^0 \to D_s^+D_s^-$, $B_s^0 \to J/\psi\phi$, $B_s^0 \to K^{(*)0}\bar{K}^{(*)0}$, $B_s^0 \to D_s^{*+}D_s^{*-}$, ...

What about $B_s^0 \to \mu^+\mu^-$ in the presence of $\Delta\Gamma_s \neq 0$?

- The theoretical $B_s \to \mu^+\mu^-$ SM branching ratio has to be rescaled by $1/(1-y_s)$ for the comparison with the experimental branching ratio:

$\Rightarrow$ *new SM reference:* $\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}}|_{y_s} = (3.54 \pm 0.30) \times 10^{-9}$

- $B_s \to \mu^+\mu^-$ is a sensitive probe for physics beyond the SM:
  - $\Delta\Gamma_s$ can be included in the NP constrains from $\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{exp}}$.

- The effective lifetime $\tau_{\mu^+\mu^-}$ offers a new observable (yielding $\mathcal{A}_{\Delta\Gamma}$):
  - Allows the extraction of the "theoretical" $B_s \to \mu^+\mu^-$ branching ratio.
  - New *theoretically clean* observable to search for NP: $\mathcal{A}_{\Delta\Gamma}^{\mathrm{SM}} = +1$
    * In contrast to the BR no dependence on the B_s-decay constant f_{B_s}.
    * May reveal NP effects even if the BR is close to the SM prediction: still largely unconstrained (pseudo-)scalar operators $O_{(P)S}$, $O'_{(P)S}$.

$\Rightarrow$ *exciting study the LHC upgrade physics programme!*

Towards Controlling Penguins

- Penguin parameters following from the current $B \to J/\psi\pi, J/\psi K$ data:

$$a = 0.22 \pm 0.13,\ \theta = (180.2 \pm 4.5)^\circ \ \Rightarrow\ \Delta\phi_d = -(1.28 \pm 0.74)^\circ$$

- Interesting penguin probe for the LHCb upgrade era: $\boxed{B_s^0 \to J/\psi K_{\rm S}}$
  - CP asymmetries allow clean extraction of a and θ.
  - Relation to $B_d^0 \to J/\psi K_{\rm S}$ through U-spin symmetry.

- Penguin uncertainties in $B_s^0 \to J/\psi\phi$:
  - $\Delta\phi_d = -(1.28 \pm 0.74)^\circ \sim \Delta\phi_s^f$ vs. $\phi_s^{\rm SM} = -2^\circ$ and $\Delta\phi_s|_{\rm exp} \sim 0.46^\circ$.
  - Control channels: $B_s^0 \to J/\psi \bar{K}^{*0}$ (and $B_d^0 \to J/\psi\rho^0$, not in this lecture).

- Penguin uncertainties in $B_s^0 \to J/\psi f_0(980)$:
  - Hadronic structure of $f_0(980)$ matters here!?
  - Conservative range: $S(B_s^0 \to J/\psi f_0)\big|_{\rm SM} \in [-0.086, -0.012]$.
  - Interesting future channel: $B_d^0 \to J/\psi f_0(980)$.

- Effective B_s decay lifetimes: $\to$ contours in the ϕ_s–$\Delta\Gamma_s$ plane
  - Analysis is very robust with respect to hadronic uncertainties!

Backup Slides

$$B_s \to K^+K^-$$

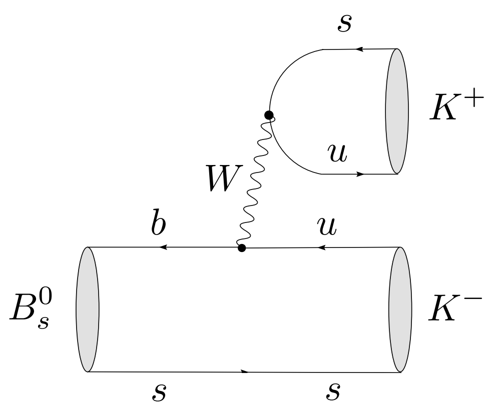
$\oplus$ U-Spin Partner

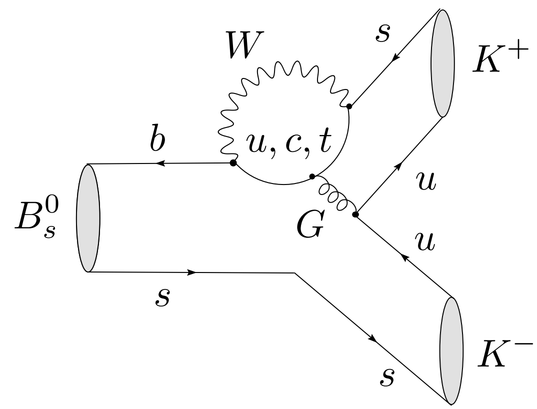
$$B_d \to \pi^+\pi^-$$

Decay Topologies & Amplitudes

- $B_s^0 \to K^+K^-$:

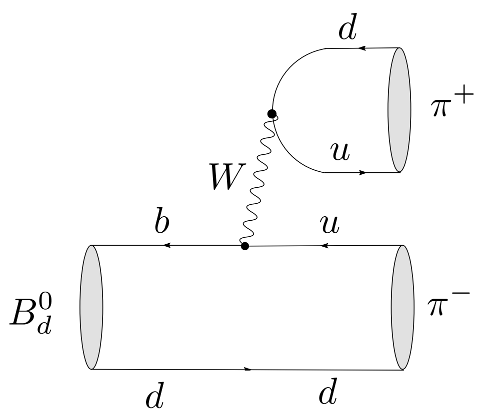
$$A(B_s^0 \to K^+K^-) \propto \mathcal{C}\left[e^{i\gamma} + \left(\frac{1-\lambda^2}{\lambda^2}\right) d e^{i\theta}\right]$$

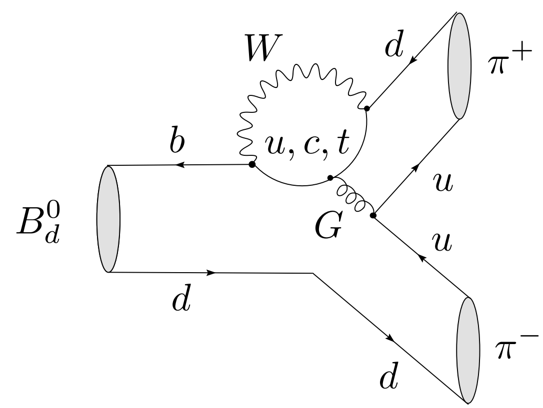




- $B_d^0 \to \pi^+\pi^-$:

$$A(B_d^0 \to \pi^+\pi^-) \propto \mathcal{C}'\left[e^{i\gamma} - d'\, e^{i\theta'}\right]$$





$$\Rightarrow \quad \boxed{s \leftrightarrow d}$$

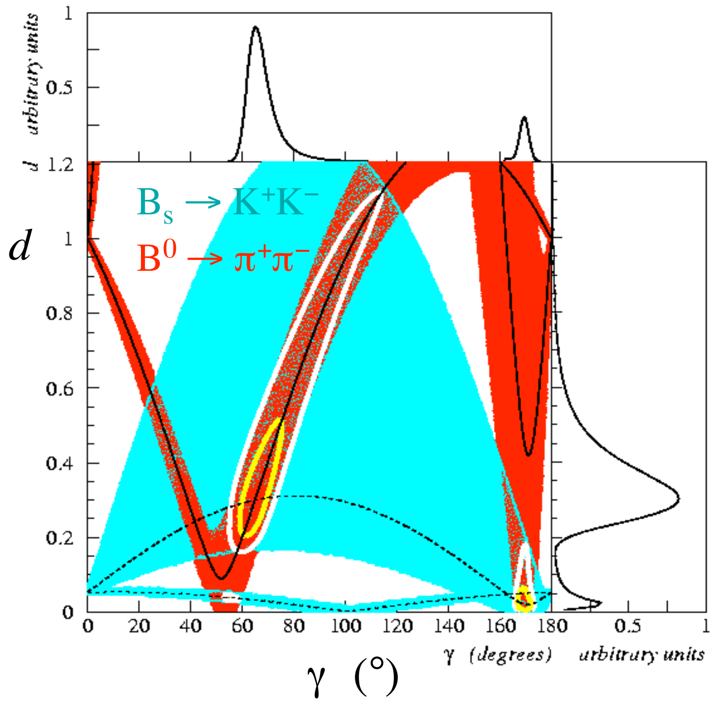
- The decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other through the interchange of all down and strange quarks:

$$\boxed{U\text{-spin symmetry}} \quad \Rightarrow \quad d' = d,\ \theta' = \theta$$

  – Determination of γ and hadronic parameters $d(=d')$, θ and θ'.
  – Internal consistency check of the U-spin symmetry: $\theta \stackrel{?}{=} \theta'$.

[R.F. (1999)]

- Detailed studies show that this strategy is very promising for LHCb:



$\rightarrow$ experimental accuracy for γ of a few degrees!

[LHCb Collaboration (B. Adeva *et al.*) LHCb-PUB-2009-029, arXiv:0912.4179v2]

Getting ready for LHCb data:

- Use B-factory data as input, as well as ...

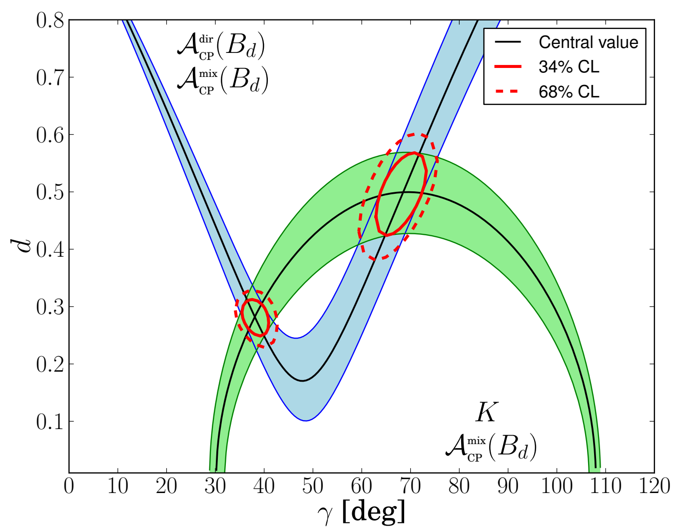
- $\mathrm{BR}(B_s \to K^+K^-)$ [CDF and Belle @ $\Upsilon(5S)$]

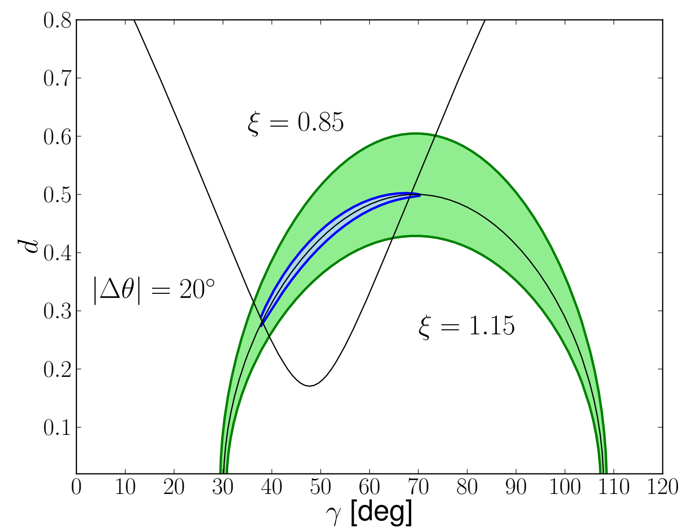
- Updated information of U-spin-breaking form-factor ratios.

[R.F. & R. Knegjens, arXiv:1011.1096 [hep-ph]]

Current Picture for γ

- Input data:
  - Information on $K \propto \text{BR}(B_s \to K^+K^-)/\text{BR}(B_d \to \pi^+\pi^-)$;
  - CP violation in $B_d^0 \to \pi^+\pi^-$ and $B_d^0 \to \pi^\mp K^\pm$;
  - U-spin-breaking corrections: $\xi \equiv d'/d = 1\pm 0.15$, $\Delta\theta \equiv \theta' - \theta = \pm 20°$:





$$\Rightarrow \boxed{\gamma = \left(68.3^{+4.9}_{-5.7}\big|_{\rm input}{}^{+5.0}_{-3.7}\big|_{\xi}{}^{+0.1}_{-0.2}\big|_{\Delta\theta}\right)^\circ}$$

(2-fold ambiguity can be resolved [R.F. ('07)])

- Fits of the UT: $\gamma = (67.2^{+3.9}_{-3.9})^\circ$ (CKMfitter), $(69.6 \pm 3.1)^\circ$ (UTfit).

Current Picture for the Hadronic Parameters

- Parameters of the general lifetime discussion: $[\epsilon \equiv \lambda^2/(1-\lambda^2)]$

$$A(B_s^0 \to K^+K^-) = \lambda \mathcal{C}\left[e^{i\gamma} + \frac{1}{\epsilon} d e^{i\theta}\right] \quad \Rightarrow$$

$$h_{K^+K^-} = d/\epsilon, \quad \delta_{K^+K^-} = \theta, \quad \varphi_1^{K^+K^-} = \gamma, \quad \varphi_2^{K^+K^-} = 0 \quad \Rightarrow$$

$$\tan \Delta\phi_{K^+K^-} = 2\epsilon \left[\frac{d\cos\theta + \epsilon\cos\gamma}{d^2 + 2\epsilon\, d\cos\theta\cos\gamma + \epsilon^2\cos 2\gamma}\right]\sin\gamma$$

- K, $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)$ and $\gamma = (68 \pm 7)^\circ$ [$\oplus$ U-spin-breaking]: $\Rightarrow$

$$d = 0.50^{+0.12}_{-0.11}, \quad \theta = (154^{+11}_{-14})^\circ \quad \Rightarrow$$

– Hadronic phase shift:

$$\Delta\phi_{K^+K^-} = -\left(10.5^{+0.3}_{-0.5}\Big|_\gamma{}^{+2.9}_{-2.1}\Big|_d{}^{+0.9}_{-1.7}\Big|_\theta\right)^\circ = -\left(10.5^{+3.1}_{-2.8}\right)^\circ$$

– Direct CP asymmetry: $C_{K^+K^-} = 0.09^{+0.05}_{-0.04}$

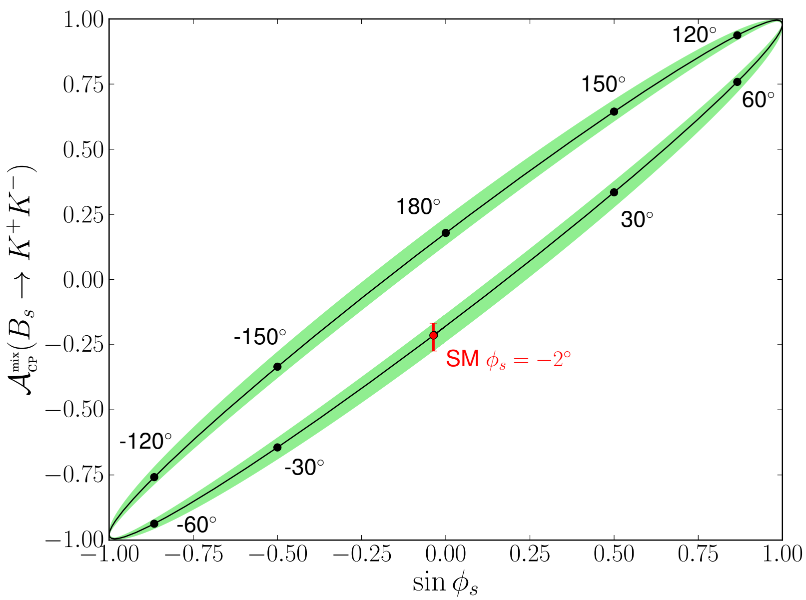
Mixing-Induced $B_s^0 \to K^+K^-$ CP Asymmetry

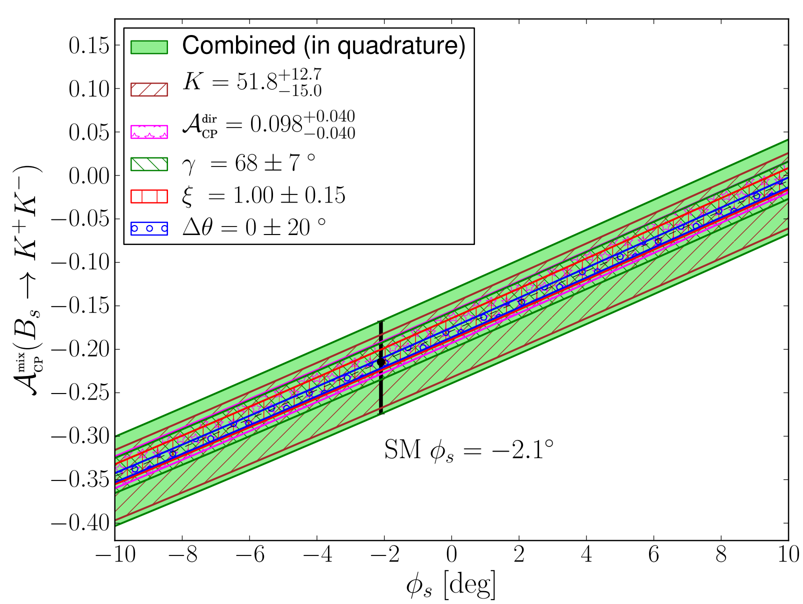
$$a_{\rm CP}(t) = \frac{C\cos(\Delta M_s t) + \mathcal{A}_{\rm CP}^{\rm mix}\sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2) + \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)}$$

- Compact expression:

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-) = \sqrt{1 - C_{K^+K^-}^2}\,\sin(\phi_s + \Delta\phi_{K^+K^-})$$

- K, $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)$, $\gamma \oplus U$-spin-breaking effects: $\Rightarrow$



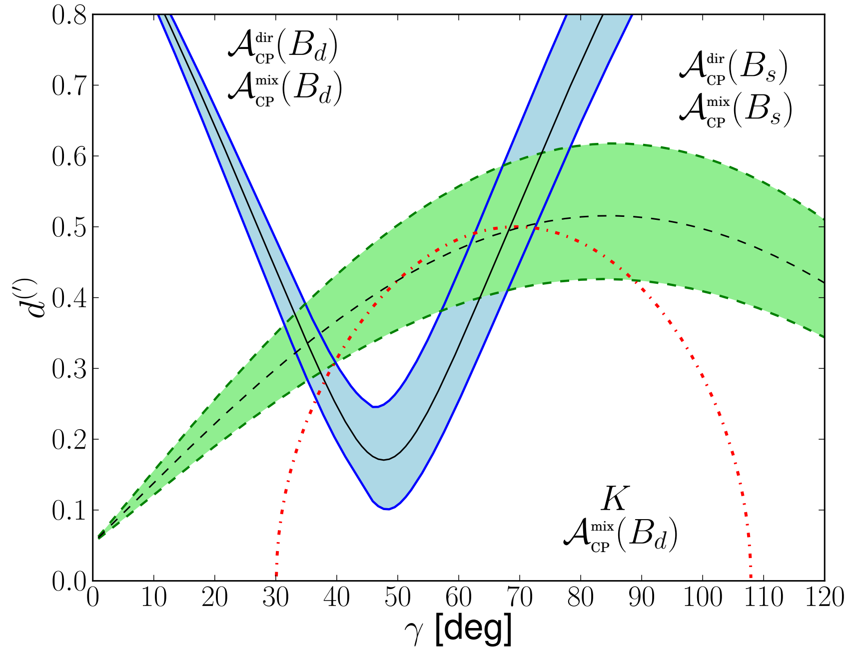


- SM prediction: $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)|_{\rm SM} = -0.215^{+0.047}_{-0.060}$

Final Goal: Optimal Determination of γ

- <u>Measurement of the CP asymmetries of $B_s^0 \to K^+K^-$:</u>

$\Rightarrow$ **theoretically clean contour in the γ–d plane:**



[Green band represents the 1σ errors of the current SM projection.]

- Intersection with the γ–d contour fixed through the CP asymmetries of $B_s^0 \to \pi^+\pi^-$ allows us to determine γ, $d = d'$ and θ, θ' [$\to$ U-spin test].

- Expect a stable situation with respect to U-spin-breaking corrections.