Quark-Gluon Plasma Physics

open charm hadrons, deconfinement, and universal hadronization

charm hadrons in the statistical hadronization model

P. Braun-Munzinger, J. Stachel
Phys.Lett.B 490 (2000) 196-202
nucl-th/0007059 [nucl-th]

A. Andronic, P. Braun-Munzinger, J. Stachel, M. Koehler,
A. Mazeliauskas, K. Redlich, V. Vislavicius,
JHEP 07 (2021) 035
arXiv:2104.12754

focus on production of open (multi)-charm hadrons at LHC energy
production yields, rapidity and transverse momentum distributions

the production of hidden charm hadrons such a charmonia will be dealt with in the next lecture

Production of hadrons and (anti-)nuclei at LHC

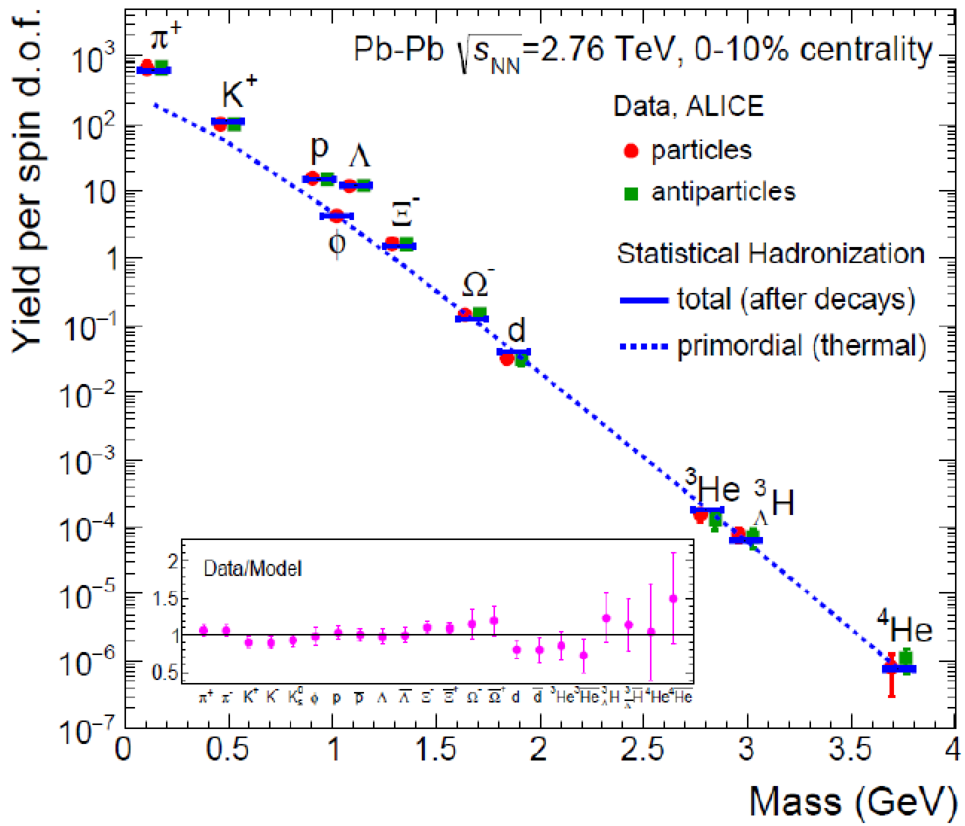
1 free parameter: temperature T

T = 156.5 ± 1.5 MeV

agreement over 9 orders of magnitude with QCD statistical operator prediction
(- strong decays need to be added)

- matter and antimatter are formed in equal portions at LHC
- even large very fragile hypernuclei follow the same systematics

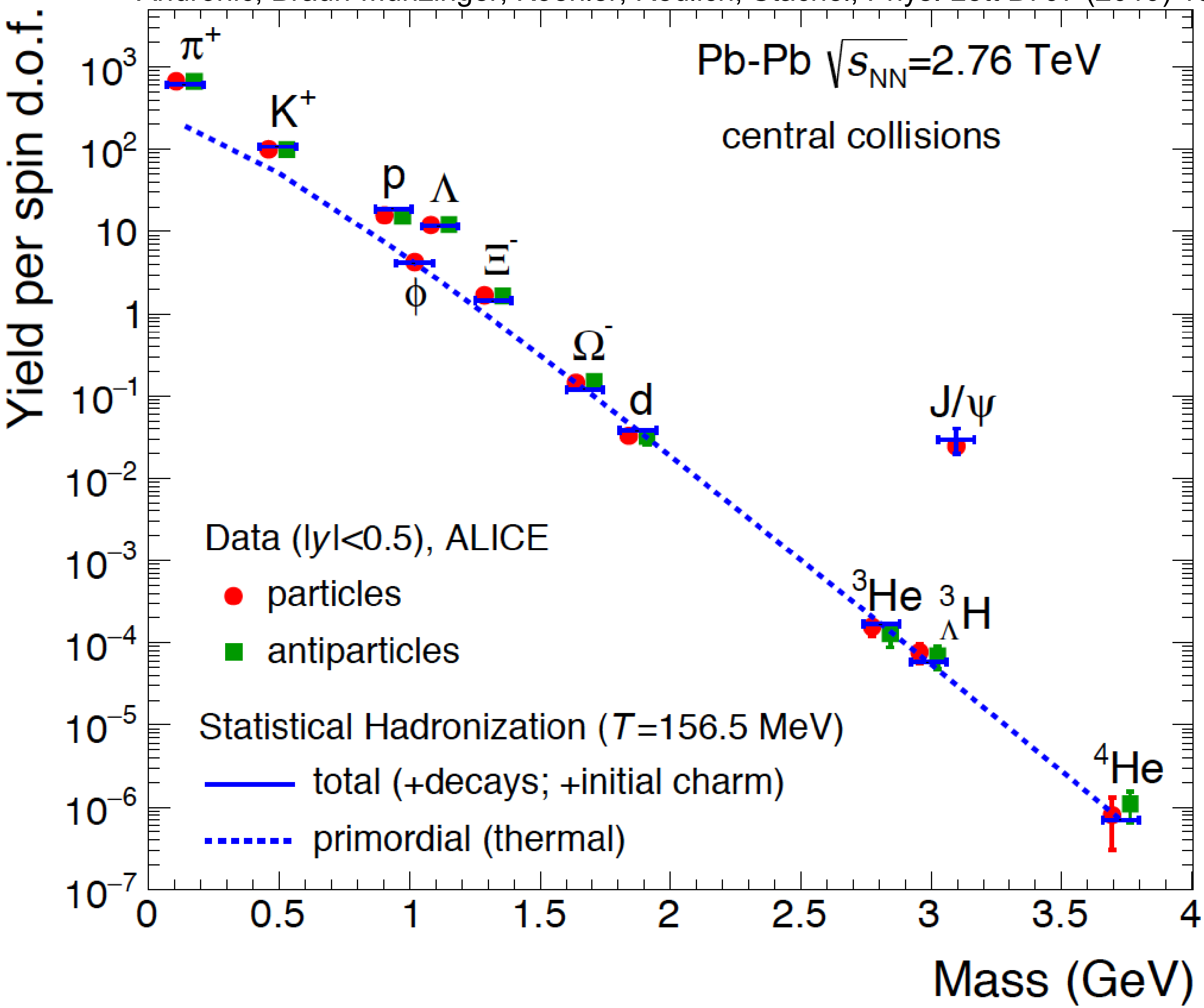
A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321



at LHC energy, all chemical potentials vanish, so strangeness is immaterial for particle production, particle yields ~ $M^{3/2}$ exp(-M/T)
(no 'strangeness enhancement')

statistical hadronization - reminder

Andronic, Braun-Munzinger, Koehler, Redlich, Stachel, Phys. Lett B797 (2019) 134836



outline

- production of hadrons with charm in relativistic nuclear collisions
- brief review of quark model of baryons and mesons
- focus on baryons containing charm quarks
- reminder of the statistical hadronization model for (u,d,s) hadrons
- adding charm: the charm balance equation and canonical thermodynamics
- yields and spectra of open charm hadrons
- the multiple charm hierarchy
- deconfinement and hadronization of a fireball containing charm quarks

quarks and their quantum numbers

$$Q = I_z + \frac{\mathcal{B} + S + C + B + T}{2}$$

$\mathcal{B}$ is the baryon number

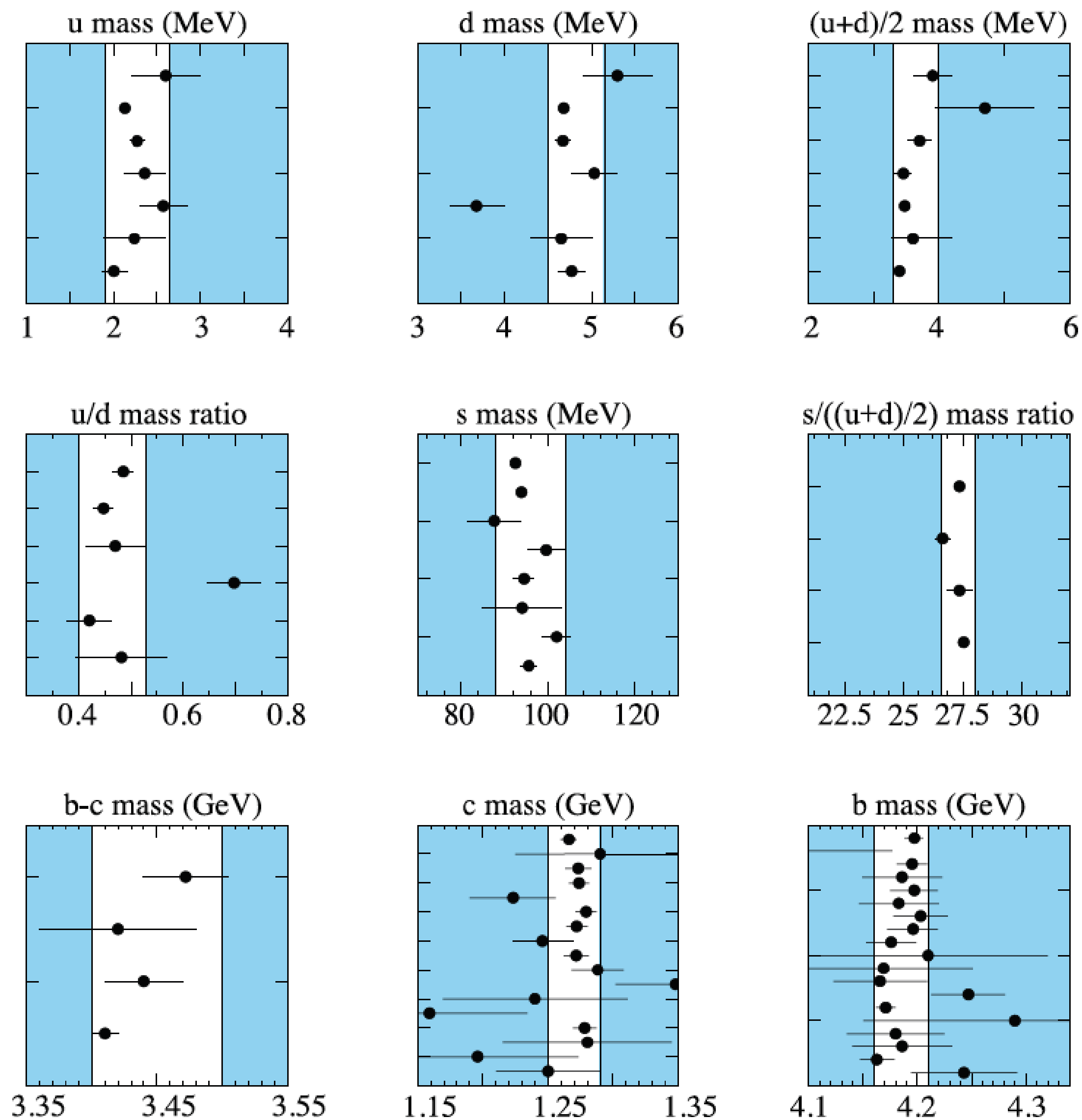
	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

hyper-charge Y

$$Y = \mathcal{B} + S - \frac{C - B + T}{3}$$

all plots on this and the following 6 slides are from the PDG, Particle Data Group, https://pdg.lbl.gov

the masses of the quarks, data points are in chronological order
the latest entry is on top

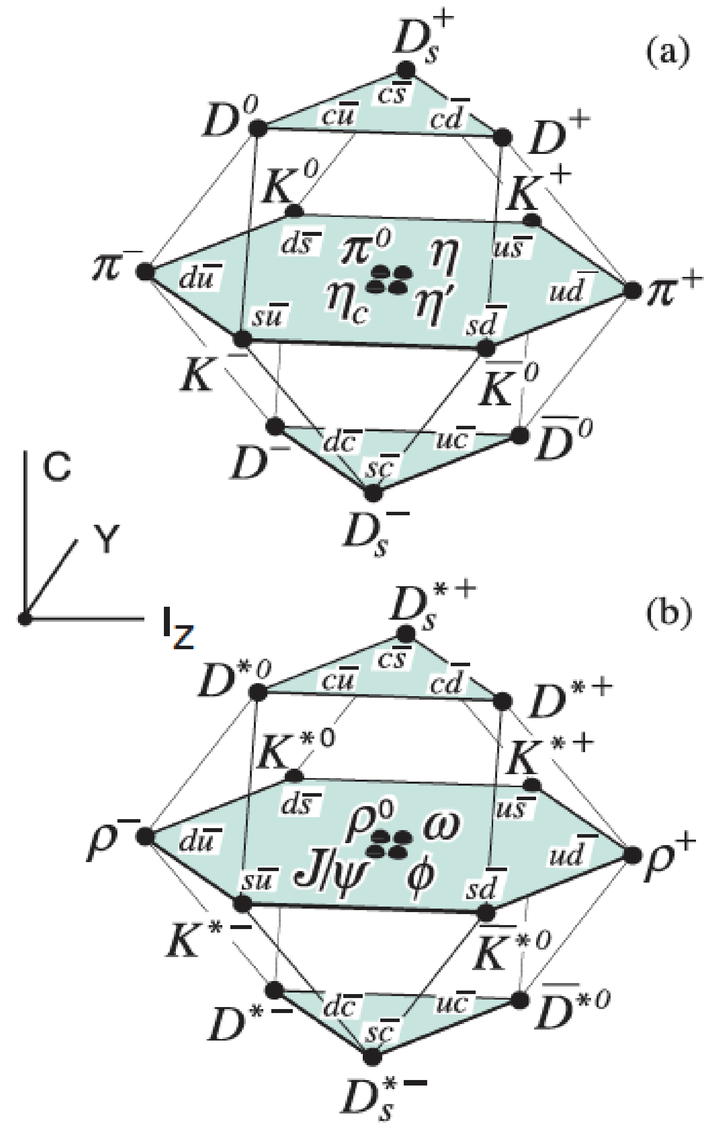


n.b.: quarks are confined and cannot be isolated → a model or theory is needed for mass determination for details see PDG entry

(u,d,s) mesons and the quark model

$n^{2s+1}\ell_J$	J^{PC}	$I=1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$I=\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	$I=0$ f'	$I=0$ f	θ_{quad} [°]	θ_{lin} [°]
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$	−11.3	−24.5
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
1^1P_1	1^{+-}	$b_1(1235)$	$K_{1B}{}^{\dagger}$	$h_1(1415)$	$h_1(1170)$		
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
1^3P_1	1^{++}	$a_1(1260)$	$K_{1A}{}^{\dagger}$	$f_1(1420)$	$f_1(1285)$		
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)^{\dagger}$	$\eta_2(1870)$	$\eta_2(1645)$		
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)^{\ddagger}$		$\omega(1650)$		
1^3D_2	2^{--}		$K_2(1820)^{\dagger}$				
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$		
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)^{\ddagger}$	$\phi(1680)$	$\omega(1420)$		
2^3P_1	1^{++}	$a_1(1640)$					
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$		

the quark model and (u,d,s,c) mesons

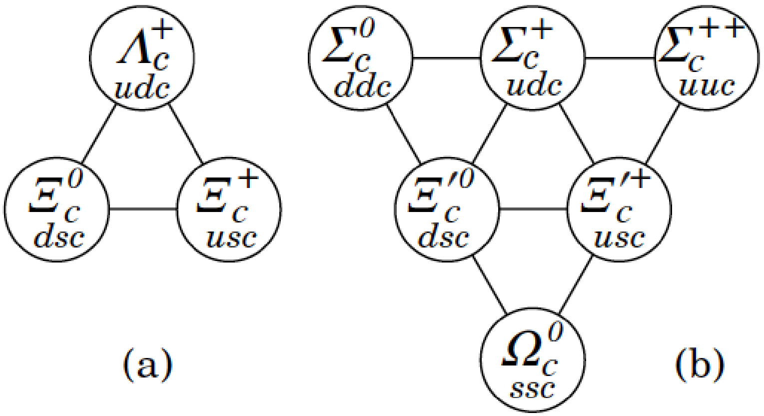


pseudo-scalar, $J = 0^-$

pseudo-vector, $J = 1^-$

$n^{2s+1}\ell_J$	J^{PC}	I = 0 $c\bar{c}$	I = $\frac{1}{2}$ $c\bar{u}$, $c\bar{d}$; $\bar{c}u$, $\bar{c}d$	I = 0 $c\bar{s}$; $\bar{c}s$
$1\,^1S_0$	0^{-+}	$\eta_c(1S)$	D	$D_s^{\pm}$
$1\,^3S_1$	1^{--}	$J/\psi(1S)$	D^*	$D_s^{*\pm}$
$1\,^3P_0$	0^{++}	$\chi_{c0}(1P)$	$D_0^*(2300)$	$D_{s0}^*(2317)^{\pm\dagger}$
$1\,^3P_1$	1^{++}	$\chi_{c1}(1P)$	$D_1(2430)$	$D_{s1}(2460)^{\pm\dagger}$
$1\,^1P_1$	1^{+-}	$h_c(1P)$	$D_1(2420)$	$D_{s1}(2536)^{\pm}$
$1\,^3P_2$	2^{++}	$\chi_{c2}(1P)$	$D_2^*(2460)$	$D_{s2}^*(2573)$
$2\,^1S_0$	0^{-+}	$\eta_c(2S)$		
$2\,^3S_1$	1^{--}	$\psi(2S)$		$D_{s1}^*(2700)^{\pm\ddagger}$
$1\,^3D_1$	1^{--}	$\psi(3770)$		$D_{s1}^*(2860)^{\pm\ddagger}$
$1\,^3D_2$	2^{--}	$\psi_2(3823)$		
$2\,^3P_J$	$0,1^{++}$	$\chi_{c0}(3860)$		
	2^{++}	$\chi_{c2}(3930)$		
$3\,^3S_1$	1^{--}	$\psi(4040)$		
$2\,^3D_1$	1^{--}	$\psi(4160)$		
$4\,^3S_1$	1^{--}	$\psi(4415)$		
$1\,^3D_3$	3^{--}		$D_3^*(2750)$	$D_{s3}^*(2860)^{\pm}$

charm baryons with C = 1

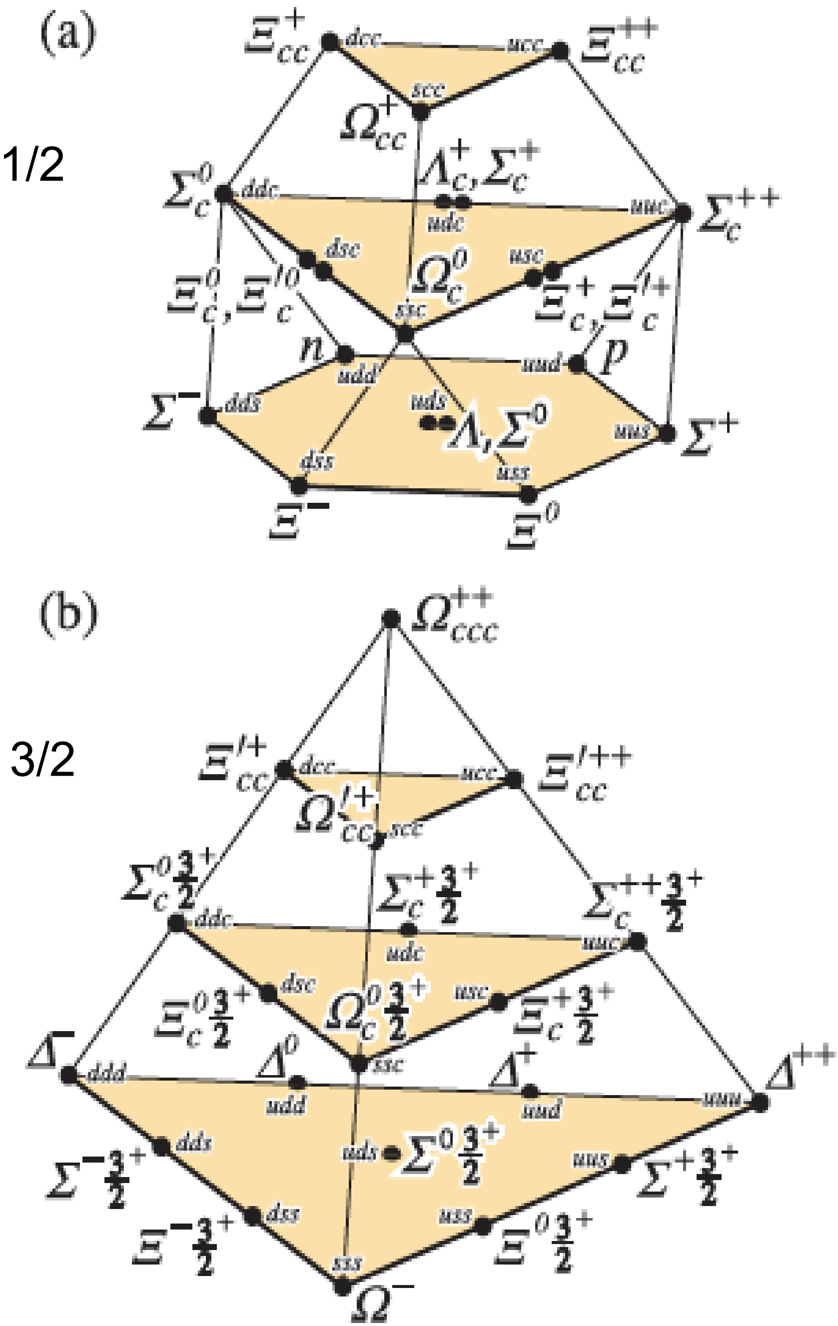


The SU(3) $\bar{\mathbf{3}}$ (a) and $\mathbf{6}$ (b) ground-state $J^P = 1/2^+$ representations. The $\mathbf{6}$ ground-state with $J^P = 3/2^+$ is identical in structure to the right-hand figure.

the charm baryons in the quark model

J = 1/2

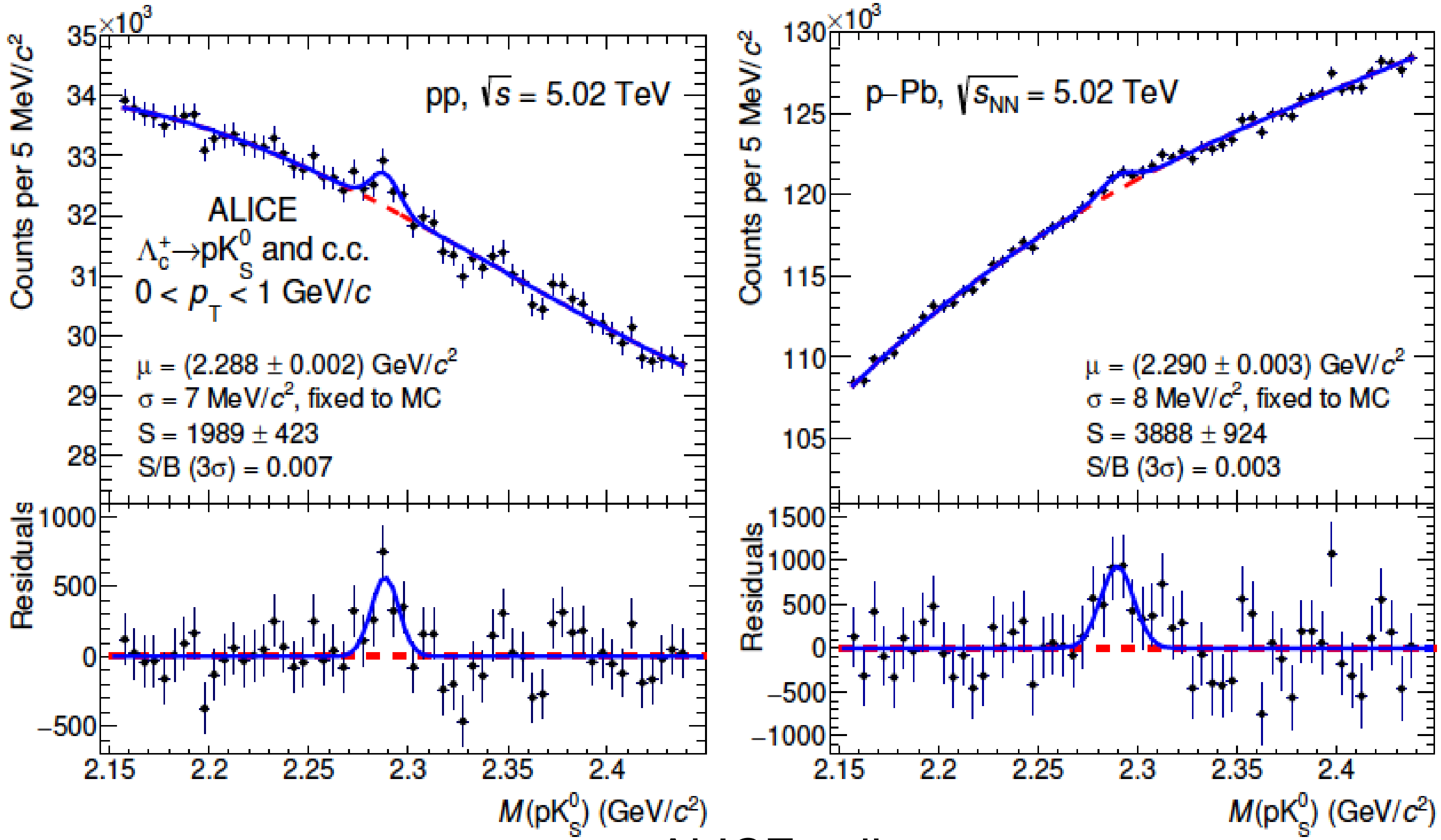
J = 3/2



how can baryons with charm be identified?

one reconstructs their invariant mass from the weak decay products:

$$\Lambda_c^+ \to pK_S^0$$



ALICE coll.

Phys. Rev.C 107 (2023) 6, 064901

2211.14032 [nucl-ex]

why are multi-charm baryons important to measure?

these complex baryons are assembled at the QCD phase transition from the quarks in the fireball

in the SHMc the production probability scales as $g_c n^c$ if charm quarks are deconfined over the volume of the fireball formed in the Pb-Pb collision, see slide 15 below

it follows that the yield of the doubly charmed Ξ_{cc}^{++} should be strongly (by a factor 900, see below) enhanced compared to min. bias pp collisions

measurement of this enhancement is hence a proof of deconfinement of charm quarks over distances determined by the volume of the fireball

in central Pb-Pb collisions this volume is of order 4000 fm^3

this implies deconfinement over linear dimensions of order 10 fm
much larger than the size of a (confined) nucleon (size of order 0.8 fm)

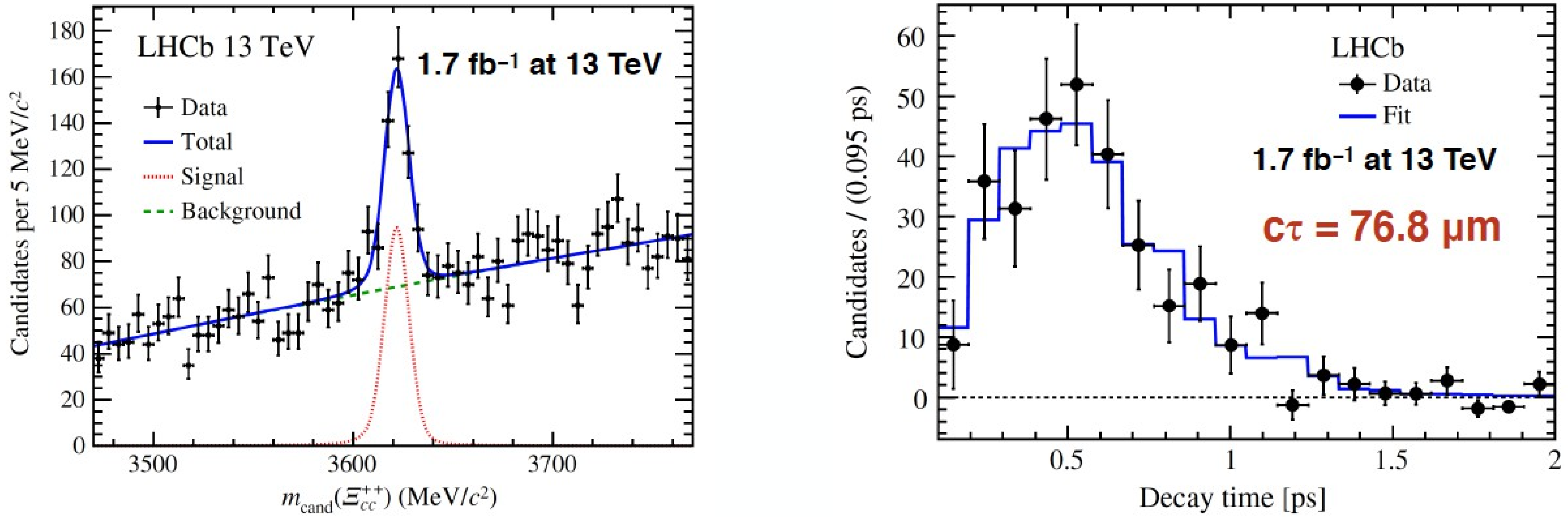
how to measure multi-charm baryons?

measurements are generally done via invariant mass analysis

but: such measurements need very sophisticated detectors since the decay chains can be very complicated

example: $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$

$$\Lambda_c^+ \rightarrow p K^- \pi^+$$



LHCb collaboration, arXiv:1910.11316

the mechanism for statistical hadronization with charm (SHMc)

[Braun-Munzinger and Stachel, PLB 490 (2000) 196]
[Andronic, Braun-Munzinger and Stachel, NPA 789 (2007) 334]

- Charm quarks are produced in initial hard scatterings ($m_{c\bar{c}} \gg T_c$) and production can be described by pQCD ($m_{c\bar{c}} \gg \Lambda_{QCD}$)
- Charm quarks survive and *thermalise* in the QGP
- Full screening before T_{CF}
- Charmonium is formed at phase boundary (together with other hadrons)
- Thermal model input ($T_{CF}, \mu_b \rightarrow n_X^{th}$)

$$N_{c\bar{c}}^{dir} = \underbrace{\frac{1}{2} g_c V \left(\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th} + \cdots \right)}_{\text{Open charm}} + \underbrace{g_c^2 V \left(\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th} + \cdots \right)}_{\text{Charmonia}}$$

- Canonical correction is applied to n_{oc}^{th}
- Outcome $N_{J/\psi}, N_D, \ldots$

core-corona picture: treat low density part of nuclear overlap region, where a nucleon undergoes 1 or less collisions as pp collisions, use measured pp cross section scaled by $T_{AA} = N_{coll}/\sigma_{inel}^{pp}$ with N_{coll} the number of (hard) collisions as obtained in the Glauber approach

statistical hadronization model for charm (SHMC) including canonical thermodynamics

- selected early references:

1. P. Braun-Munzinger, J. Stachel: Phys. Lett. B 490 (2000) 196-202, nucl-th/0007059 — the beginning
2. M. Gorenstein, A.P. Kostyuk, H. Stoecker, W. Greiner, Phys.Lett.B 524 (2002) 265-272, hep-ph/0104071 — SPS/RHIC
3. A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Phys. Lett. B 571 (2003) 36-44, nucl-th/0303036 — open/hidden charm
4. F. Becattini, Phys.Rev.Lett. 95 (2005) 022301, hep-ph/0503239 — multi-charm baryons
5. A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nucl.Phys.A 789 (2007) 334-356, nucl-th/0611023 — detailing the model
6. P. Braun-Munzinger, J. Stachel: Nature 448 (2007) 302-309 — LHC predictions
7. A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Phys.Lett.B 652 (2007) 259-261, nucl-th/0701079 — rapidity dependence
8. P. Braun-Munzinger, J. Stachel: Landolt-Bornstein 23 (2010) 424, 0901.2500 — deconfined c quarks

- the charm balance eq. developed in 1., 2., and 3. determines the fugacity g_c

$$N_{c\bar{c}} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th}$$

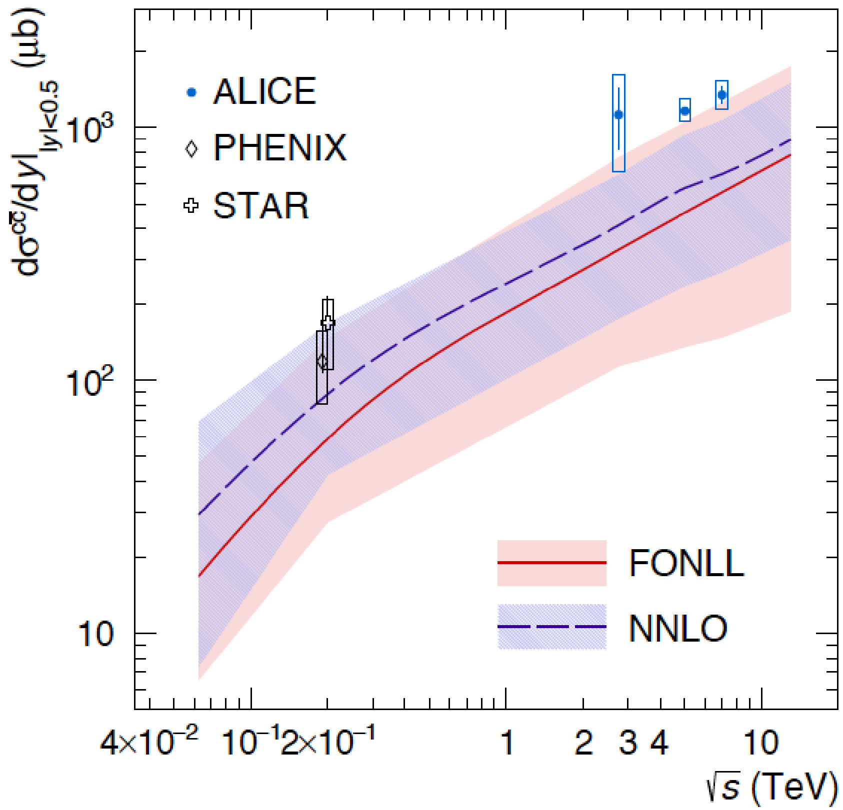
$N_{c\bar{c}}$: obtained from measured open charm cross section

N^{th}_{oc}: # of thermal open charm hadrons

- balance equation with canonical suppression needs to be solved numerically to obtain g_c

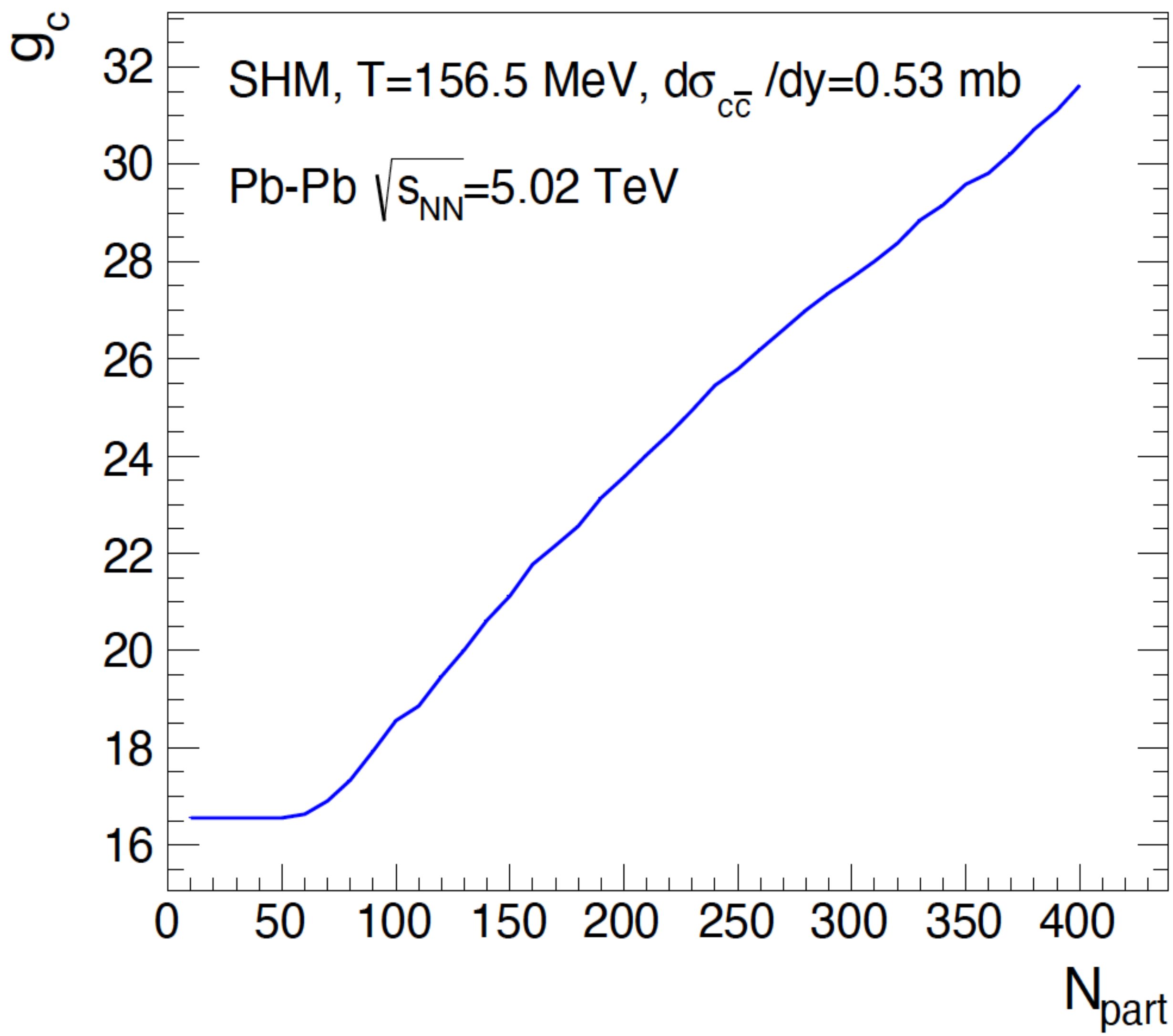
- for yields of charm hadron i with n_c charm quarks $N_{n_c}(i) = g_c^{n_c} N_{n_c}(i)^{th} \frac{I_{n_c}(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})}$

energy dependence of charm production cross section at mid-rapidity



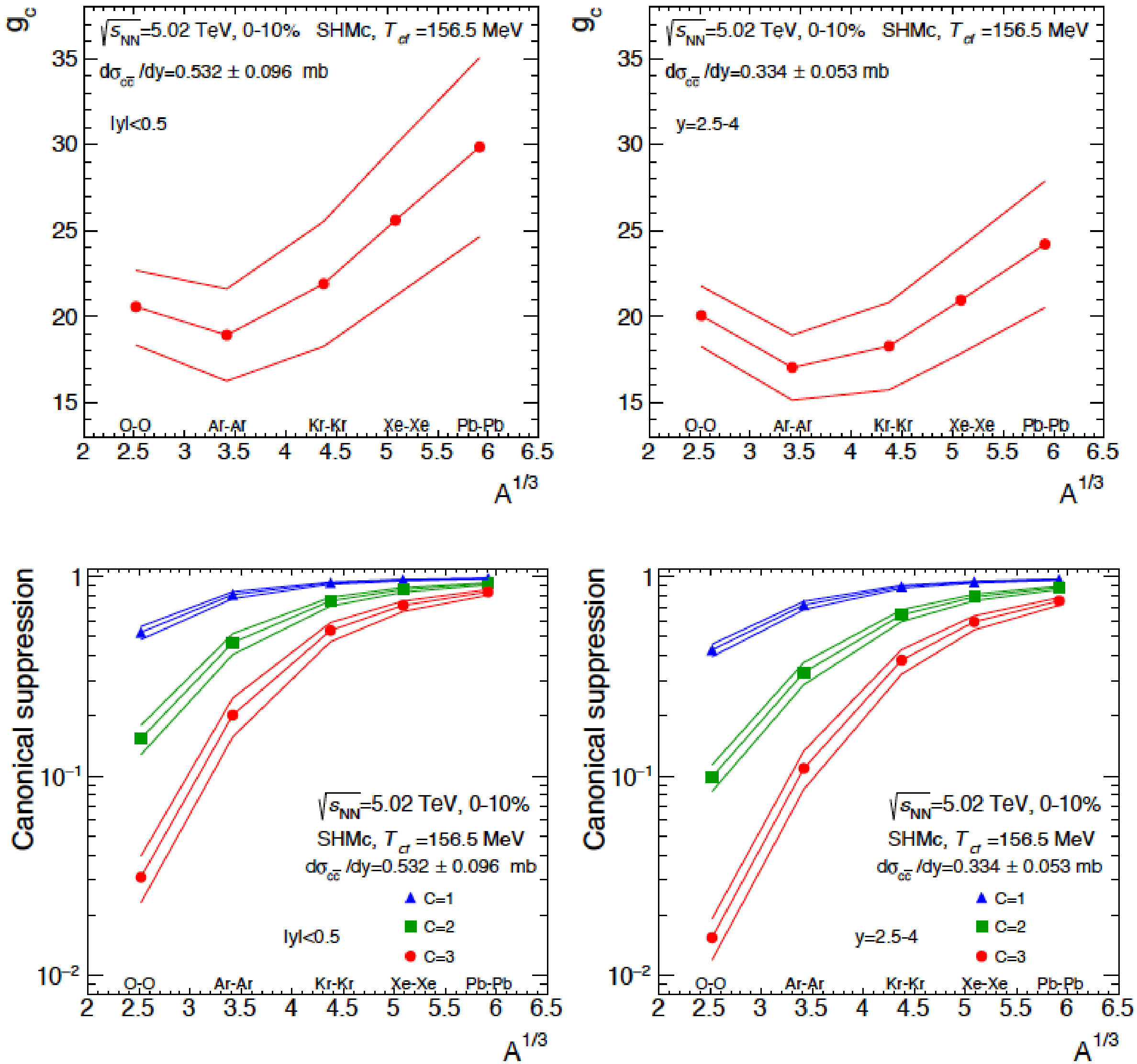
ALICE collaboration, arXiv:2105.06335

centrality dependence of charm fugacity g_c at LHC energy



charm fugacities and canonical suppression factors

different collision systems:



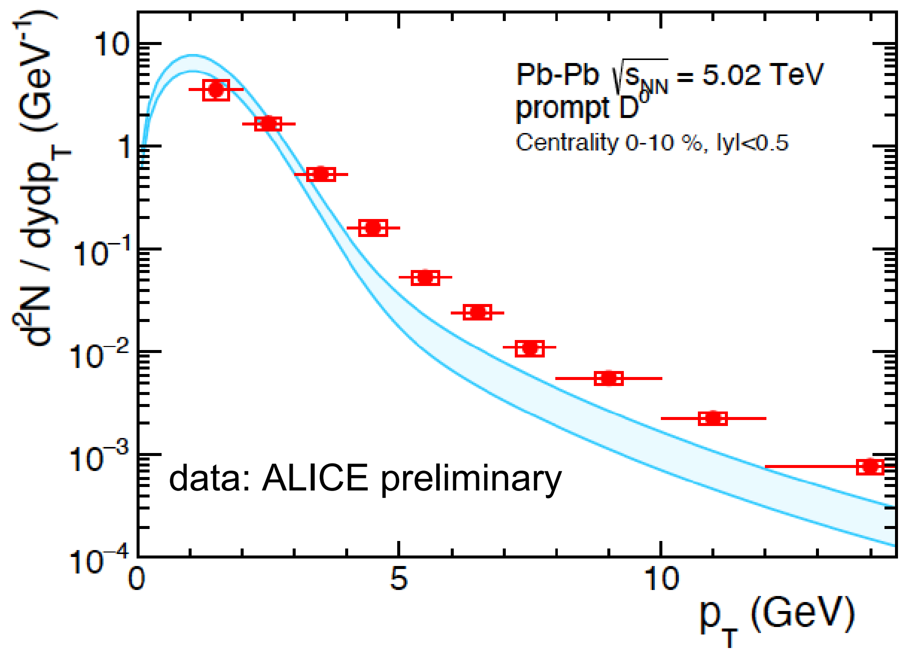
statistical hadronization for hidden and open charm

J/ψ enhanced compared to other M = 3 GeV hadrons since number of c-quarks is about 30 times larger than expected for pure thermal production at T = 156 MeV due to production in initial hard collisions and subsequent thermalization in the fireball.

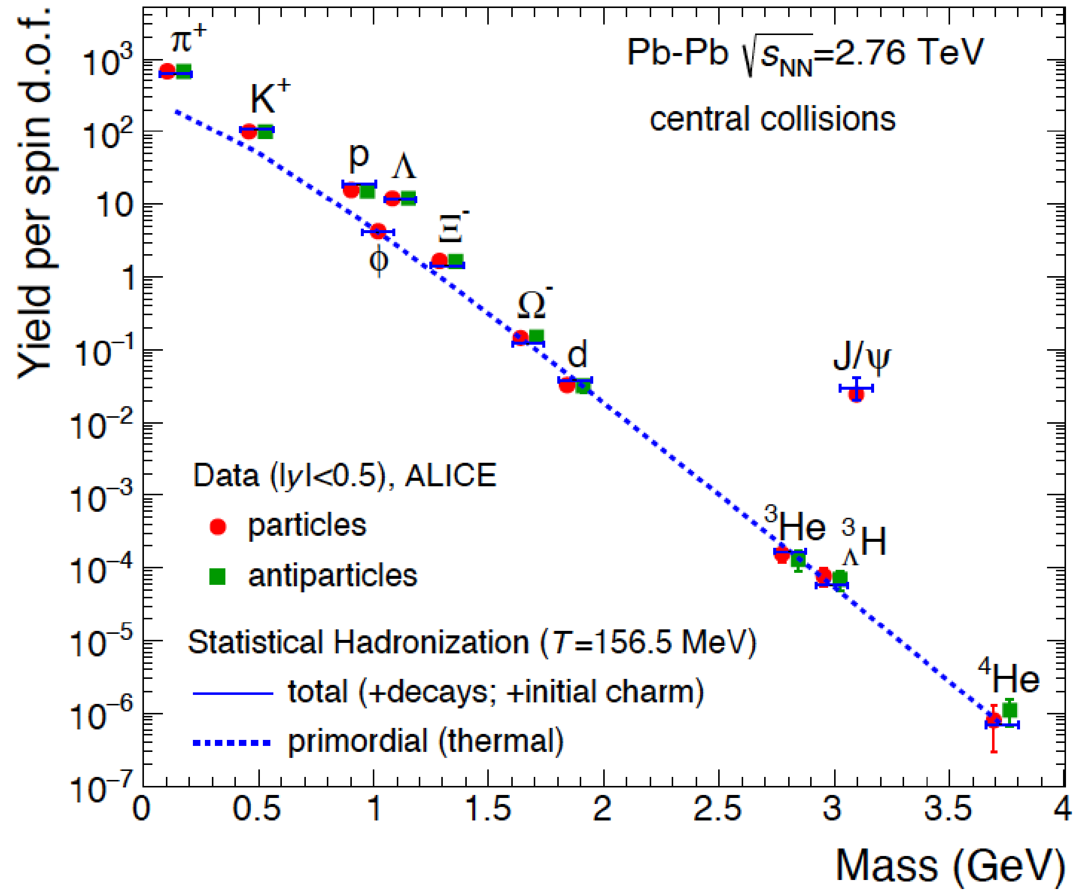
production probability scales with $N_{c\bar{c}}^2$

enhancement factor is 900 for J/ψ

enhancement factor is 30 for D^0



Andronic, Braun-Munzinger, Koehler, Redlich, JS PLB 792 (2019) 304
Andronic, Braun-Munzinger, Koehler, Mazeliauskas, Redlich, Stachel, Vislavicius arXiv:2104.12754

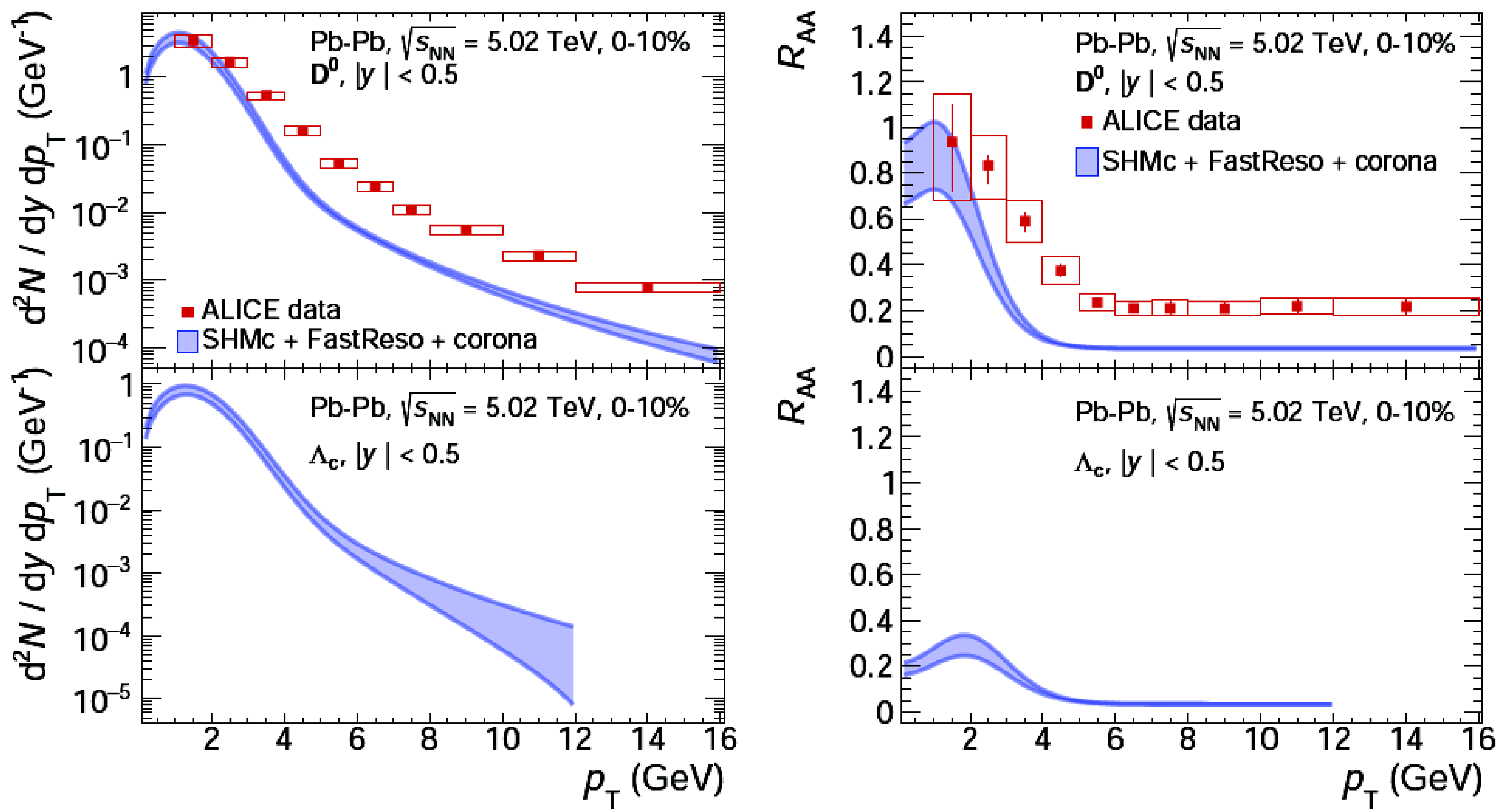


quantitative agreement for open and hidden charm hadrons, same mechanism should work for all open and hidden charm hadrons,
even for exotica such as Ω_{ccc} where enhancement factor is nearly 30000
quantitative tests in LHC Run3/Run4

enhancement is defined relative to purely thermal value, not to pp yield

spectra and R_{AA} of D^0 mesons and Λ_c baryons

for open heavy flavor hadrons strong contribution from resonance decays
- include all known charm hadron states as of PDG2020 in SHMc
- compute decay spectra with FastReso: 76 2-body and 10 3-body decays
  (A. Mazeliauskas, S. Floerchinger, E. Grossi, D. Teaney, EPJ C79 (2019) 284 arXiv: 1809.11049)

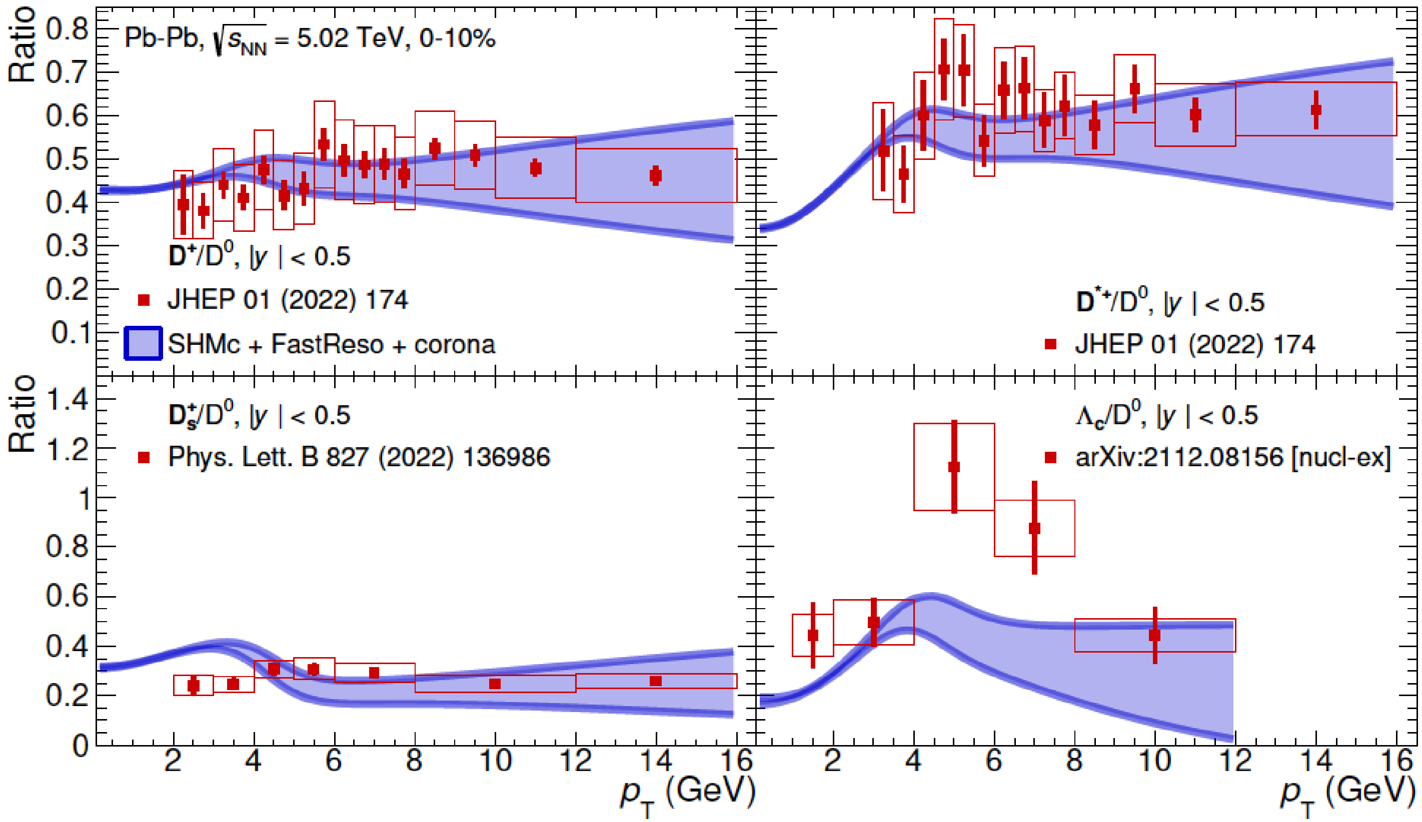


Λ_c data next

Ratios of charm hadron to D^0 spectra

A. Andronic, P. Braun-Munzinger, J. Stachel, M. Koehler, A. Mazeliauskas,

K. Redlich, V. Vislavicius, JHEP07 (2021) 035, arXiv:2104.12754

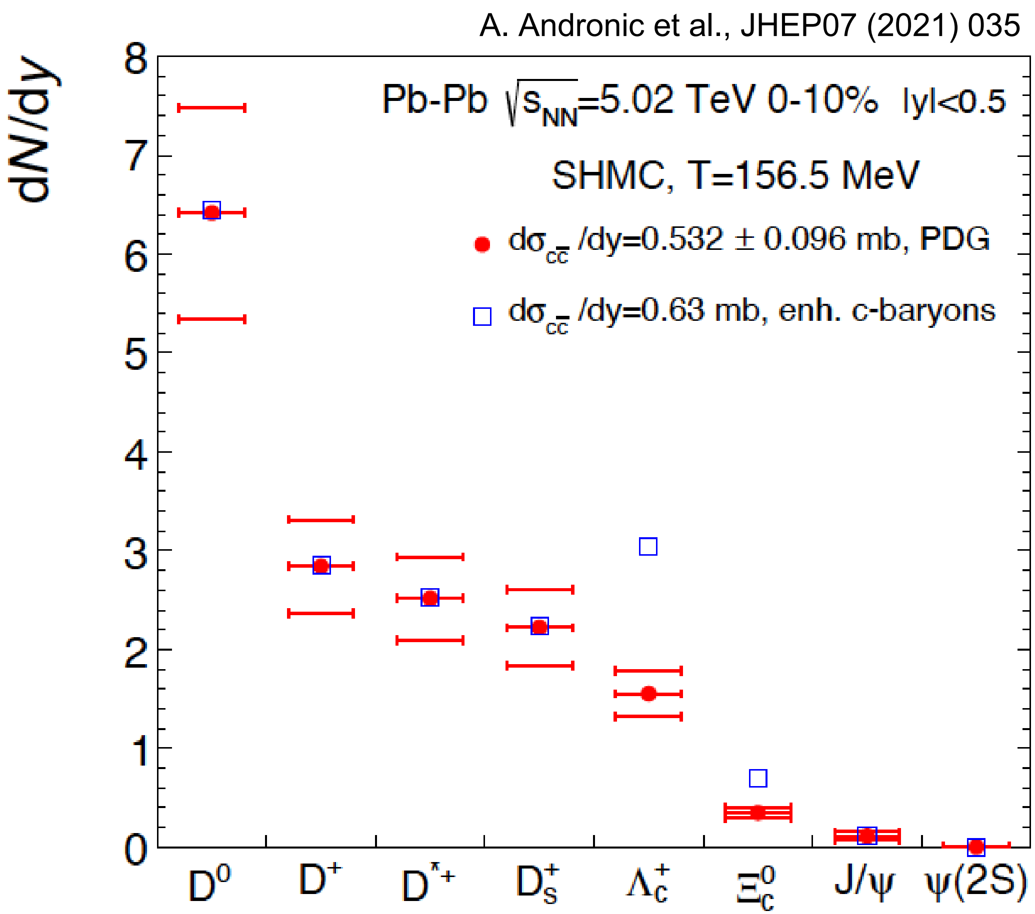


Charm-hadron spectrum: PDG

excellent agreement for D mesons considering there are no free parameters, but too low for Λ_c

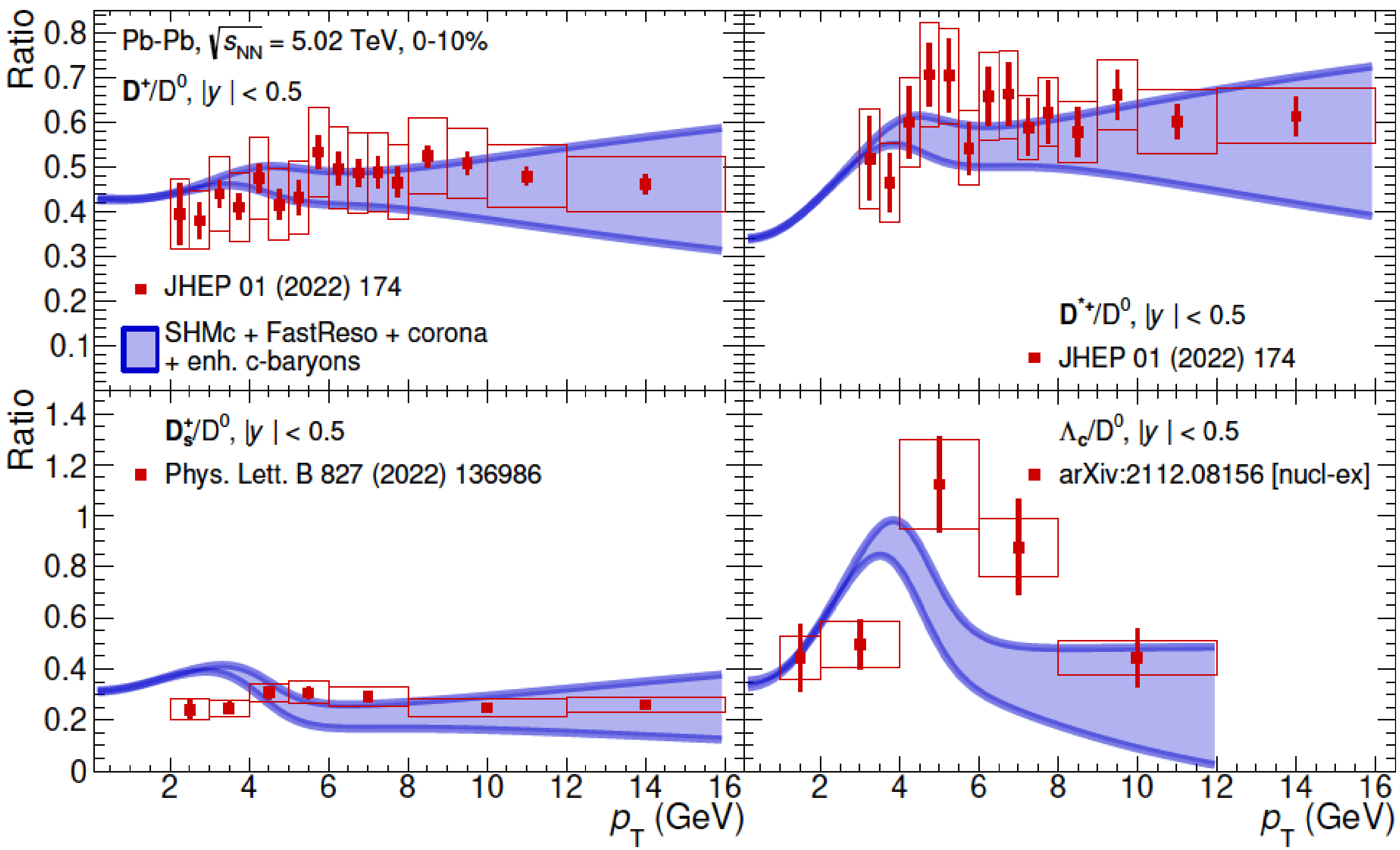
Charm hadron yields with modified charm resonance spectrum

recently a lot of speculation about possibly incomplete charm baryon spectrum
to test impact, tripled statistical weights of excited charm baryons



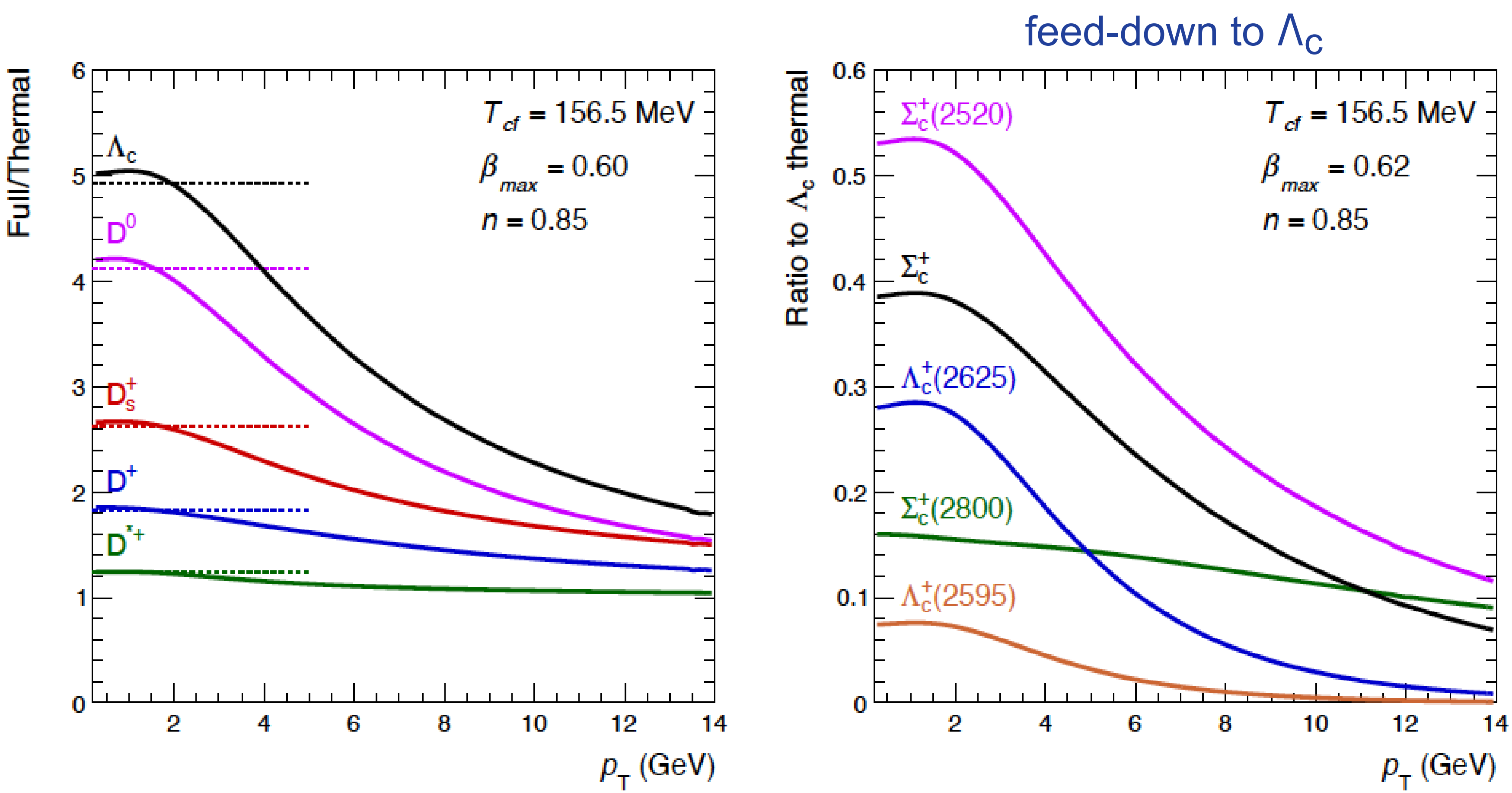
charm cross section increases 20%
yield of charm baryons nearly doubles
mesons practically unaffected

Ratios of charm hadron to D^0 spectra



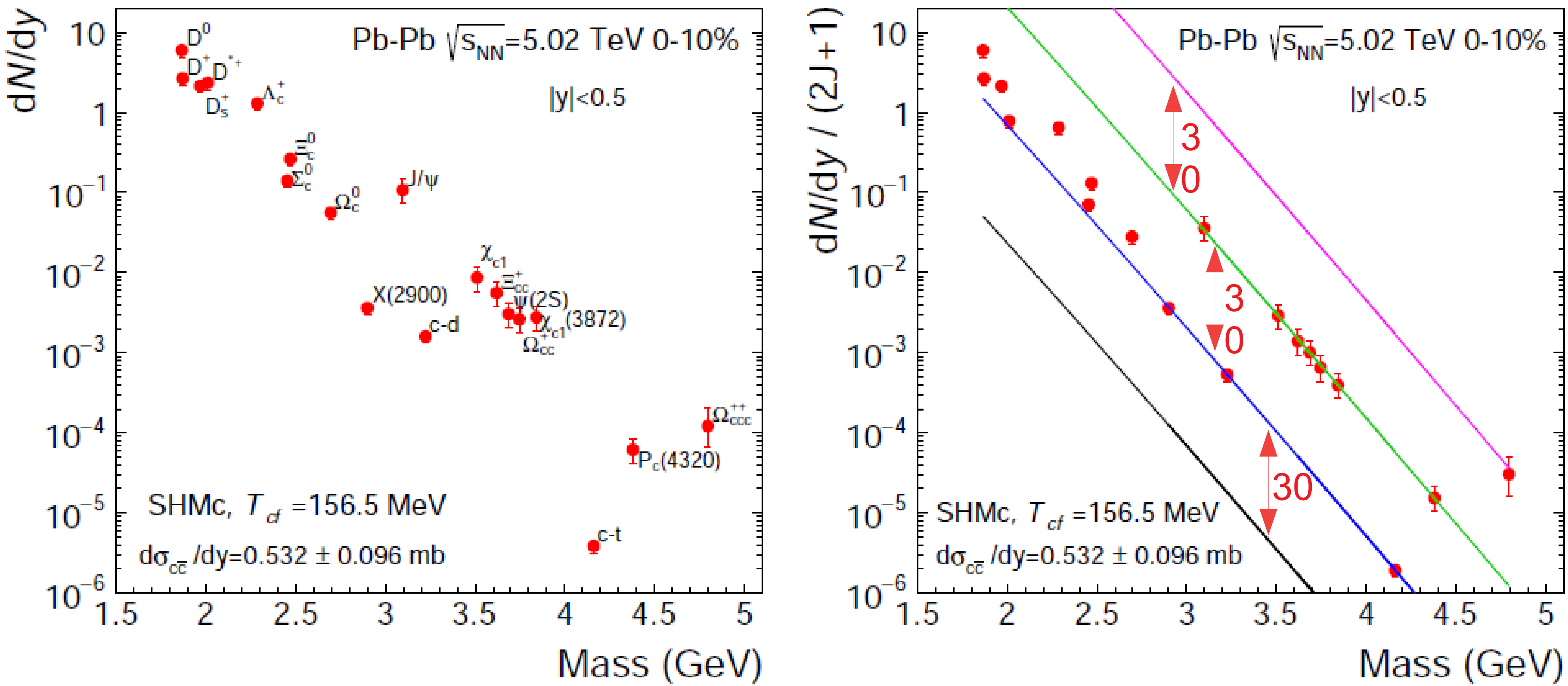
Charm-hadron spectrum: enhanced c-baryons (tripled excited states)

impact of resonance decays



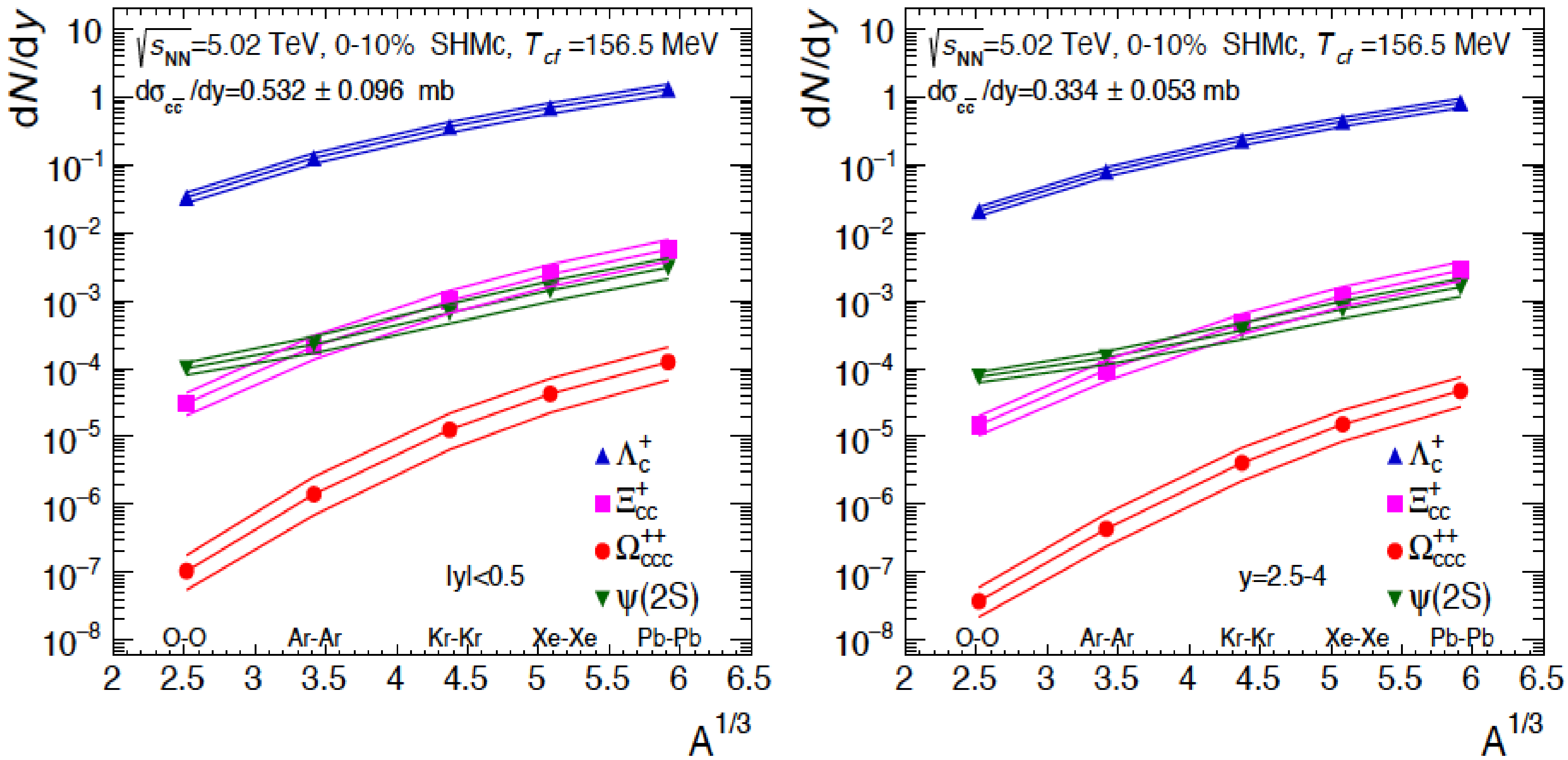
the multi-charm hierarchy

open and hidden charm hadrons, including exotic objects, such as X-states, c-deuteron, pentaquark, Ω_{ccc}



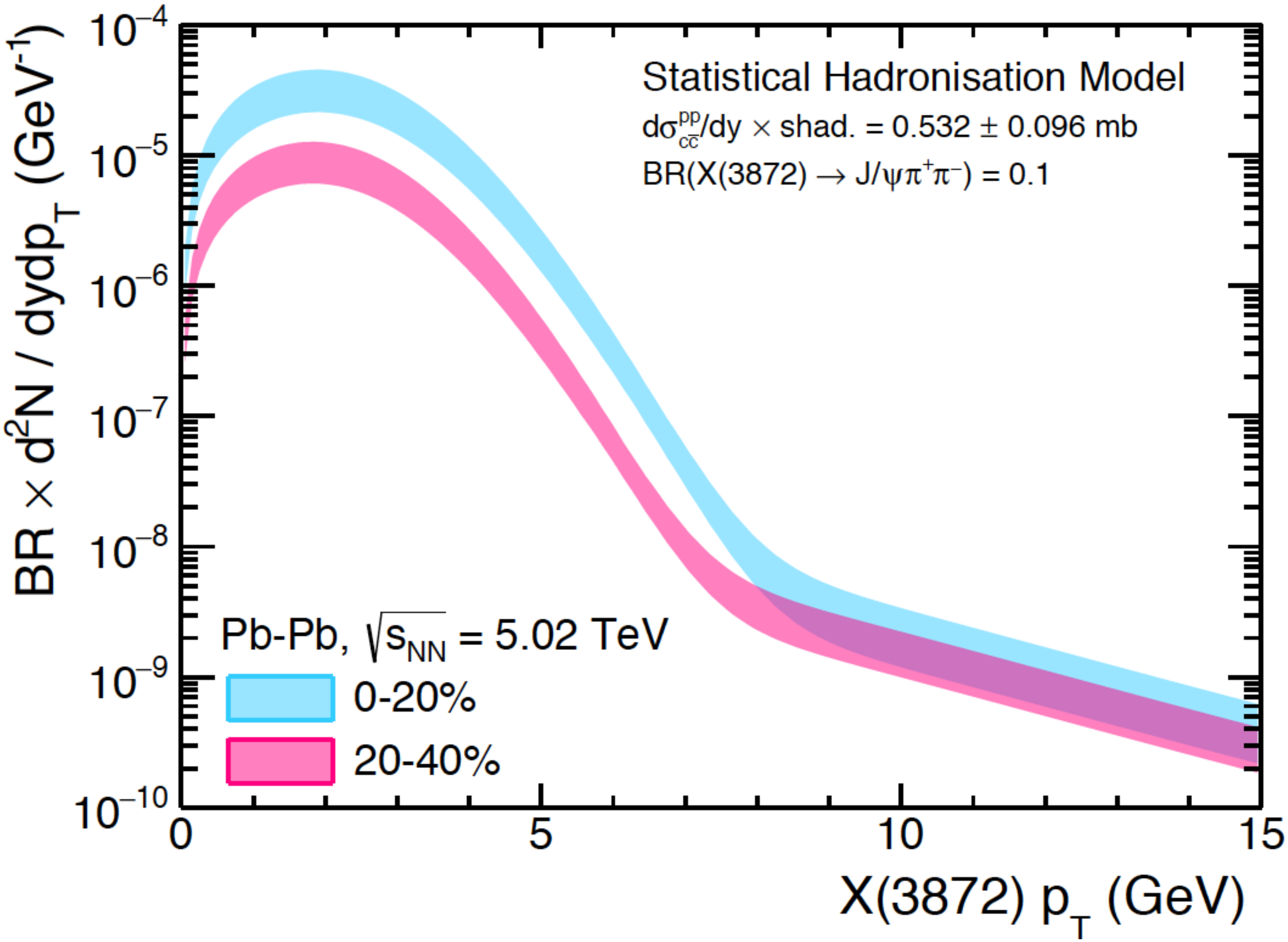
emergence of a unique pattern, due to g_c^n and mass hierarchy
perfect testing ground for deconfinement for LHC Run 3 and beyond

system size dependence of yields



due to different charm quark content different canonical suppression
for multicharm very light collision systems not favored

transverse momentum spectrum for $\chi_{c1}(3872)$ in the SHMc



note: dramatic enhancement at low p_t predicted

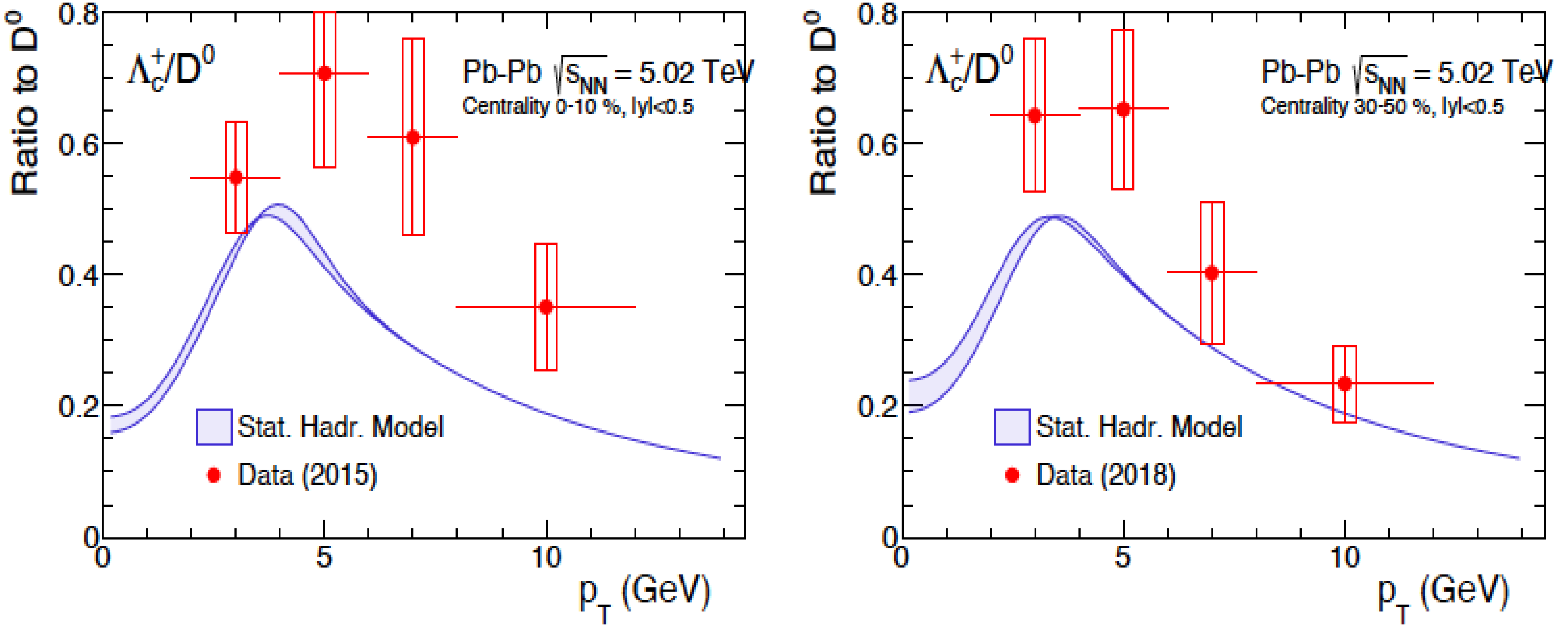
summary – charm production
probing the QCD phase boundary with heavy quarks

- statistical hadronization works quantitatively for hadrons with charm quarks
- charm quarks are not thermally produced but in initial hard collisions and subsequently thermalize in the hot and dense fireball
- predicted charmonium enhancement at low p_T established at LHC energies
- charmonium enhancement implies that charm quarks are deconfined over distances > 5 - 10 fm

- the study of open charm hadron production has just begun
- predict dN/dy for hierarchy of multi-charm states, very large (> 5000) enhancement expected
- precision study of such hadrons → further insight into deconfinement and hadronization
- universal hadronization for hadrons with (u,d,s,c) quarks

backup

Λ_c/D^0 ratio



data: ALICE preliminary
SHMc predictions: A. Andronic, P. Braun-Munzinger, J. Stachel, M. Koehler, A. Mazeliauskas, K. Redlich, V. Vislavicius, arXiv:2104.12754

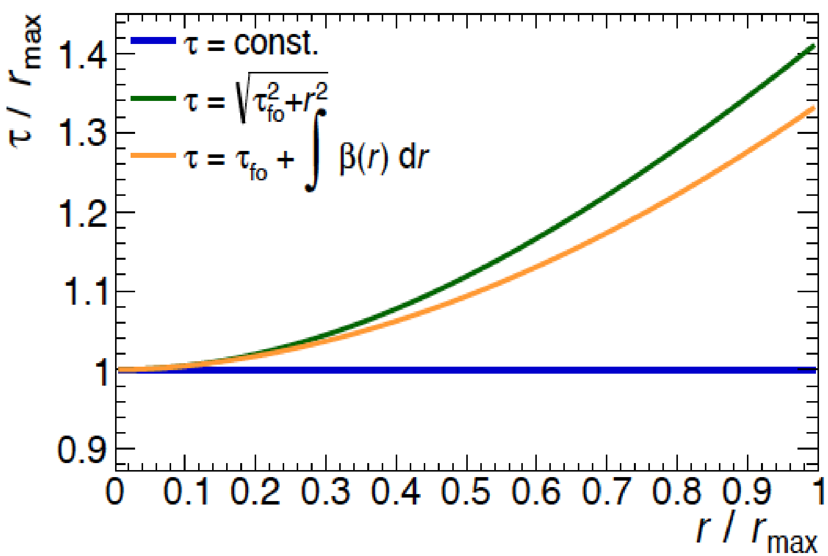
blast wave parametrization of transverse momentum spectrum

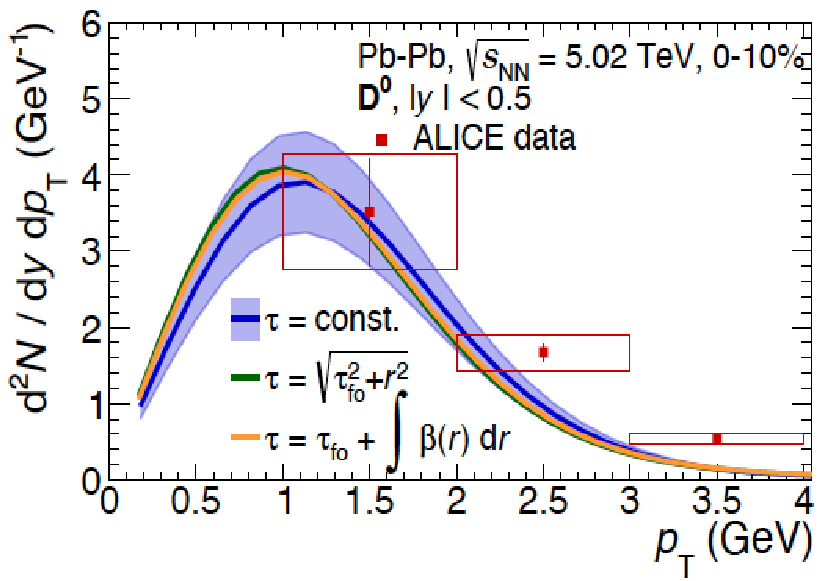
$$\frac{\mathrm{d}^2 N}{2\pi p_\mathrm{T} dp_\mathrm{T} dy} = \frac{2J+1}{(2\pi)^3} \int \mathrm{d}\sigma_\mu p^\mu f(p)$$

$$= \frac{2J+1}{(2\pi)^3} \int_0^{r_\mathrm{max}} \mathrm{d}r\ \tau(r) r \left[K_1^\mathrm{eq}(p_\mathrm{T}, u^r) - \frac{\partial \tau}{\partial r} K_2^\mathrm{eq}(p_\mathrm{T}, u^r) \right]$$

$$K_1^\mathrm{eq}(p_\mathrm{T}, u^r) = 4\pi m_\mathrm{T} I_0 \left(\frac{p_\mathrm{T} u^r}{T} \right) K_1 \left(\frac{m_\mathrm{T} u^\tau}{T} \right)$$

$$K_2^\mathrm{eq}(p_\mathrm{T}, u^r) = 4\pi p_\mathrm{T} I_1 \left(\frac{p_\mathrm{T} u^r}{T} \right) K_0 \left(\frac{m_\mathrm{T} u^\tau}{T} \right)$$





mid-rapidity yields for Pb-Pb collisions

Particle	dN/dy core (SHMc)	dN/dy corona	dN/dy total
		0-10%	
D^0	6.02 ± 1.07	0.396 ± 0.032	6.42 ± 1.07
D^+	2.67 ± 0.47	0.175 ± 0.026	2.84 ± 0.47
D^{*+}	2.36 ± 0.42	$0.160 +0.048-0.022$	2.52 ± 0.42
D_s^+	2.15 ± 0.38	$0.074 +0.024-0.015$	2.22 ± 0.38
Λ_c^+	1.30 ± 0.23	0.250 ± 0.028	1.55 ± 0.23
Ξ_c^0	0.263 ± 0.047	0.090 ± 0.035	0.353 ± 0.058
J/ψ	$0.108 +0.041-0.035$	$(5.08 \pm 0.37) \cdot 10^{-3}$	$0.113 +0.041-0.035$
$\psi(2S)$	$(3.04 +1.2-1.0) \cdot 10^{-3}$	$(7.61 \pm 0.55) \cdot 10^{-4}$	$(3.80 +1.2-1.0) \cdot 10^{-3}$
		30-50%	
D^0	0.857 ± 0.153	0.207 ± 0.017	1.06 ± 0.154
D^+	0.379 ± 0.068	0.092 ± 0.014	0.471 ± 0.069
D^{*+}	0.335 ± 0.060	$0.084 +0.025-0.011$	$0.419 +0.065-0.061$
D_s^+	0.306 ± 0.055	$0.039 +0.013-0.008$	0.344 ± 0.056
Λ_c^+	0.185 ± 0.033	0.131 ± 0.015	0.316 ± 0.036
Ξ_c^0	0.038 ± 0.007	0.047 ± 0.018	0.084 ± 0.020
J/ψ	$(1.12 +0.37-0.32) \cdot 10^{-2}$	$(2.65 \pm 0.19) \cdot 10^{-3}$	$(1.39 +0.37-0.32) \cdot 10^{-2}$
$\psi(2S)$	$(3.16 +1.04-0.89) \cdot 10^{-4}$	$(3.98 \pm 0.29) \cdot 10^{-4}$	$(7.14 +1.08-0.94) \cdot 10^{-4}$

dependence of Ω_{ccc} production yields on system size for a run time of 10^6 s

	O-O	Ar-Ar	Kr-Kr	Xe-Xe	Pb-Pb
$\sigma_{\mathrm{inel}}(10\%)$ mb	140	260	420	580	800
$T_{\mathrm{AA}}(0-10\%)$ mb^{-1}	0.63	2.36	6.80	13.0	24.3
$\mathcal{L}(\mathrm{cm}^{-2}\mathrm{s}^{-1})$	$4.5 \cdot 10^{31}$	$2.4 \cdot 10^{30}$	$1.7 \cdot 10^{29}$	$3.0 \cdot 10^{28}$	$3.8 \cdot 10^{27}$
	$\mathrm{d}\sigma_{c\bar{c}}/\mathrm{d}y = 0.53$ mb				
$\mathrm{d}N_{\Omega_{ccc}}/\mathrm{d}y$	$8.38 \cdot 10^{-8}$	$1.29 \cdot 10^{-6}$	$1.23 \cdot 10^{-5}$	$4.17 \cdot 10^{-5}$	$1.25 \cdot 10^{-4}$
Ω_{ccc} Yield	$5.3 \cdot 10^{5}$	$8.05 \cdot 10^{5}$	$8.78 \cdot 10^{5}$	$7.26 \cdot 10^{5}$	$3.80 \cdot 10^{5}$
	$\mathrm{d}\sigma_{c\bar{c}}/\mathrm{d}y = 0.63$ mb				
$\mathrm{d}N_{\Omega_{ccc}}/\mathrm{d}y$	$1.44 \cdot 10^{-7}$	$2.33 \cdot 10^{-6}$	$2.14 \cdot 10^{-5}$	$7.03 \cdot 10^{-5}$	$2.07 \cdot 10^{-4}$
Ω_{ccc} Yield	$9.2 \cdot 10^{5}$	$1.45 \cdot 10^{6}$	$1.53 \cdot 10^{6}$	$1.22 \cdot 10^{6}$	$6.29 \cdot 10^{5}$

example: X(3872)

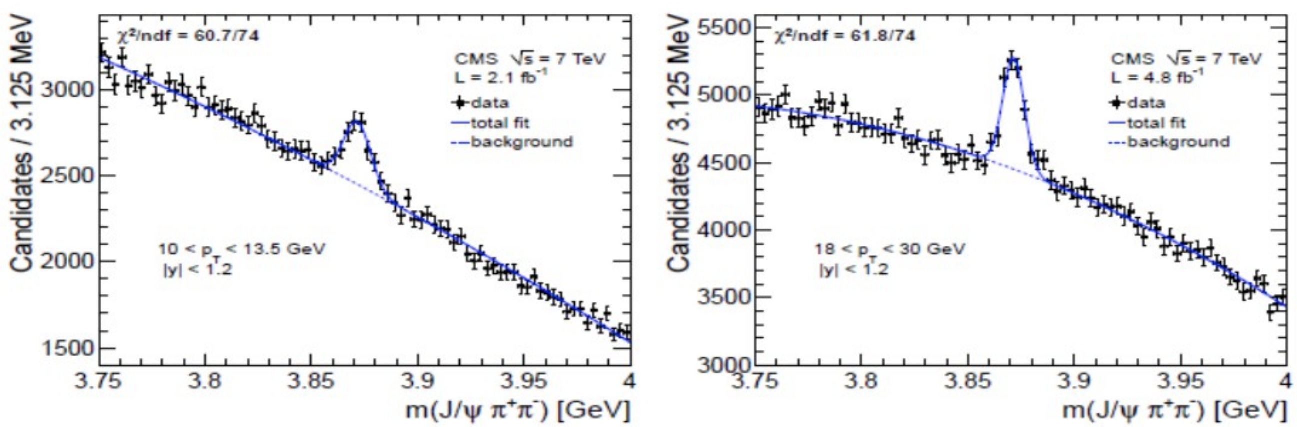
X(3872)

- 2003 -

$$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$$

$$M = 3872.0 \pm 0.6 \pm 0.5\,\text{MeV}$$

- 2013 -



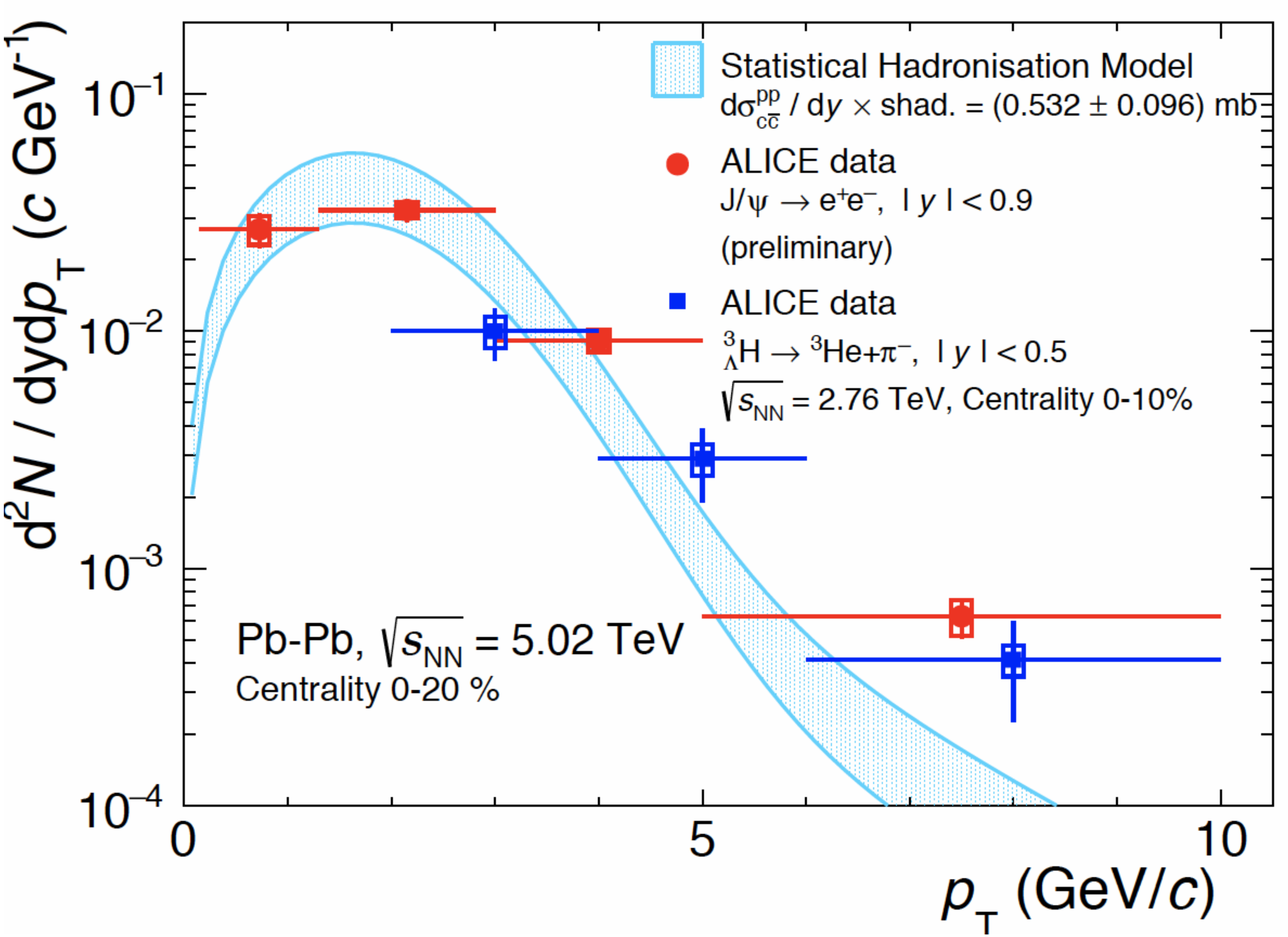
X(3872)

$$I^G(J^{PC}) = 0^+(1^{++})$$

Mass $m = 3871.69 \pm 0.17$ MeV
$m_{X(3872)} - m_{J/\psi} = 775 \pm 4$ MeV
$m_{X(3872)} - m_{\psi(2S)}$
Full width $\Gamma < 1.2$ MeV, CL = 90%

22

J/psi and hyper-triton described with the same flow parameters in the statistical hadronization model

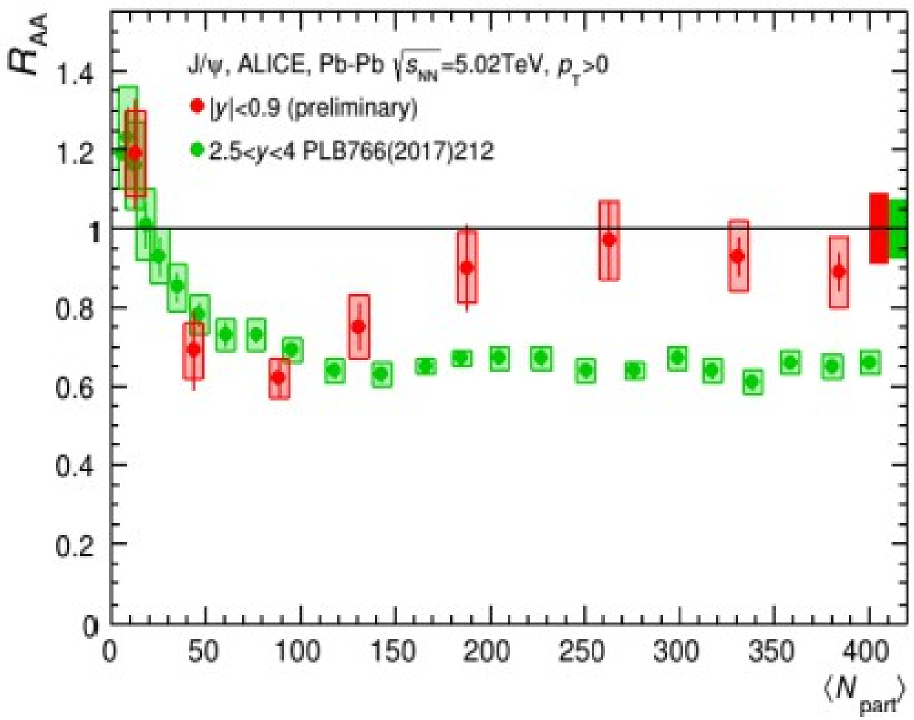


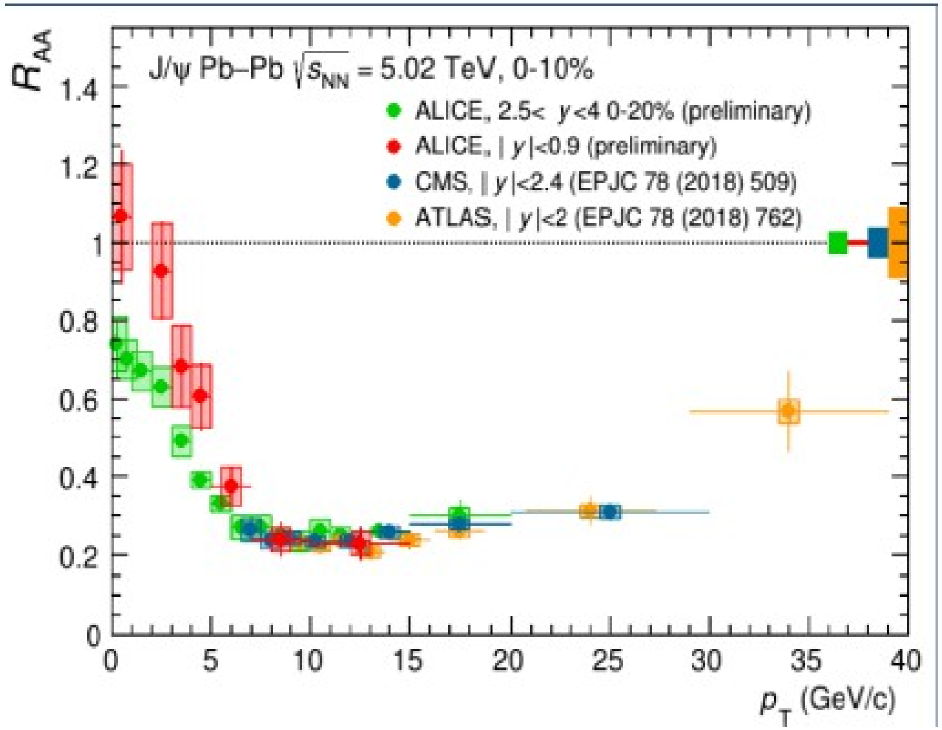
binding energies:
J/psi 600 MeV
hypertriton 2.2 MeV
Lambda S.E. 0.2 MeV

from review: hypernuclei and other loosely bound objects produced in nuclear collisions at the LHC,
pbm and Benjamin Doenigus,
Nucl. Phys. A987 (2019) 144, arXiv:1809.04681

charmonium at LHC: peaks at mid-y and strong enhancement at low transverse momentum

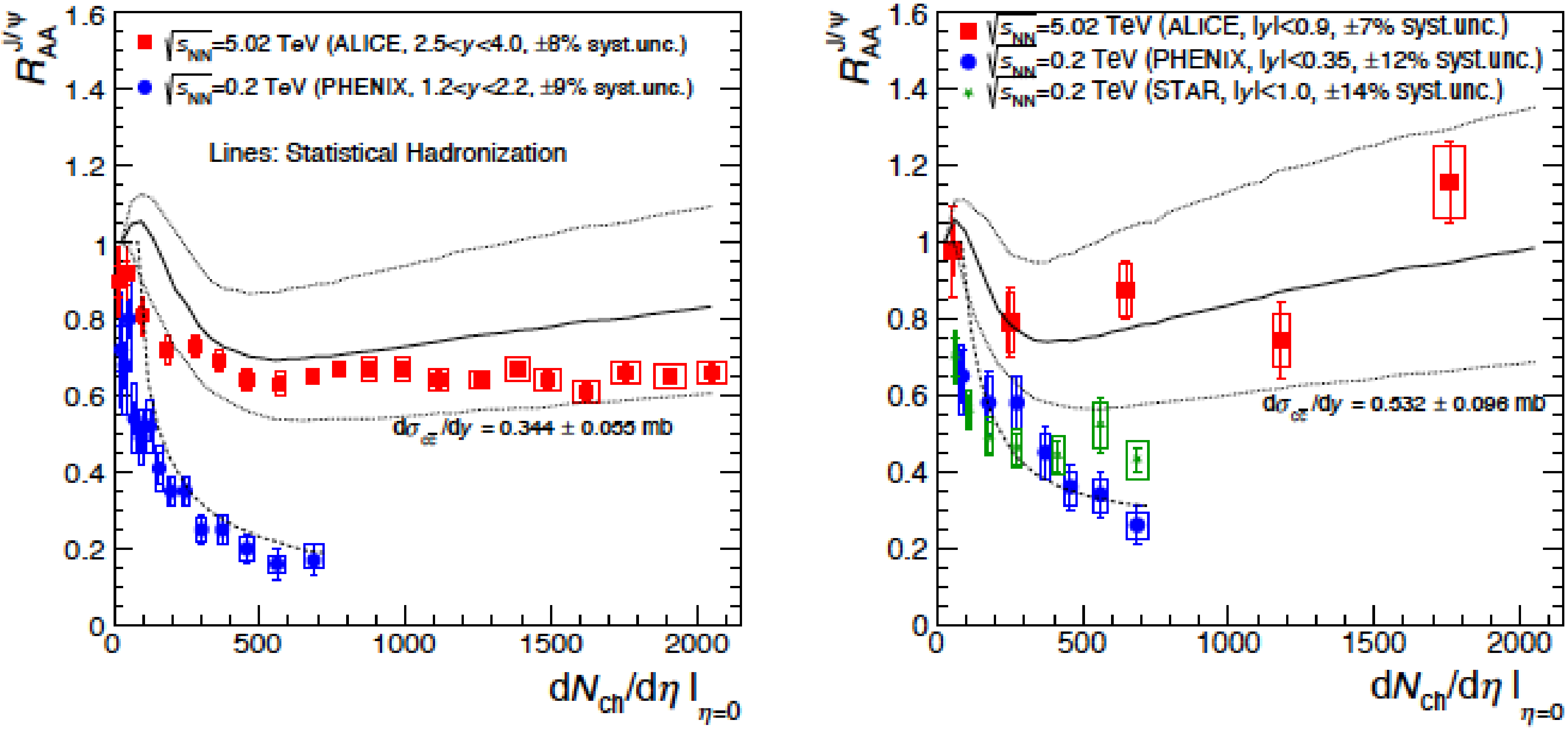
nuclear modification factor: $R_{\mathrm{AA}}(p_{\mathrm{T}}) = \dfrac{\mathrm{d}N^{\mathrm{AA}}/\mathrm{d}p_{\mathrm{T}}}{\langle N_{\mathrm{coll}}\rangle \mathrm{d}N^{\mathrm{pp}}/\mathrm{d}p_{\mathrm{T}}}$





RHIC and LHC data compared to SHMc predictions

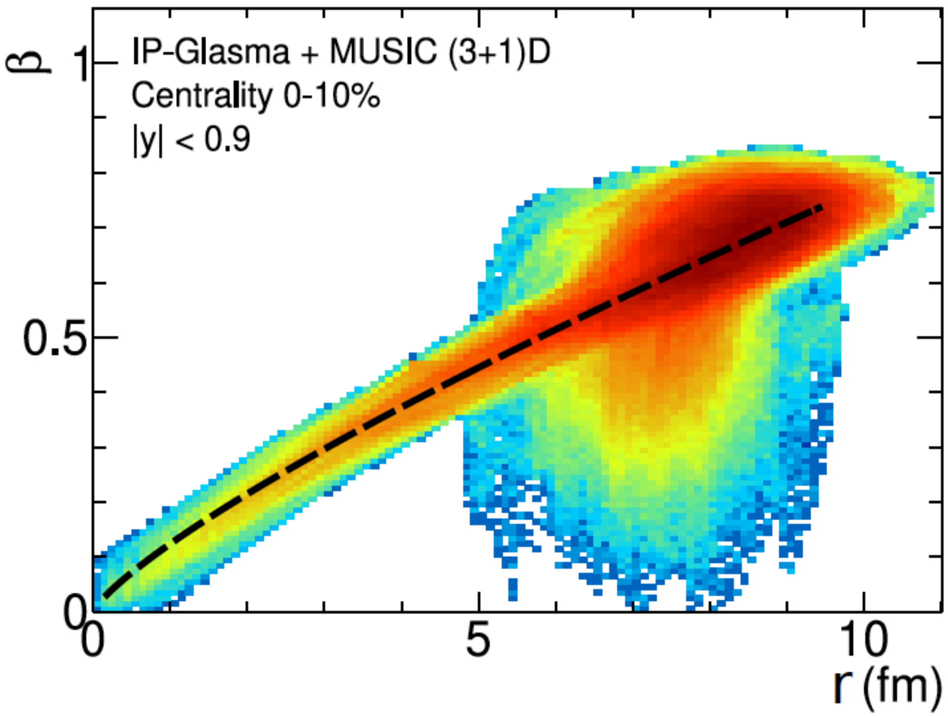
note the energy dependence of the nuclear modification factor R_{AA}



the band with the model predictions at LHC energy is due to the uncertainties in the pp open charm cross section and the necessary shadowing corrections

beyond yields: transverse momentum distributions

assume thermalization of charm quarks in QGP, charm quarks follow collective flow
use hydro velocity profile at pseudocritical temperature from MUSIC (3+1) D
tuned to light flavor observables



$$\beta(r) = \beta_{\max} \frac{r^n}{r_{\max}^n}$$

$$\beta_{\max} = 0.62$$

$$n = 0.85$$

$$V = 2\pi \int_0^{r_{\max}} \mathrm{d}r \; r\tau(r) u^\tau \left[1 - \beta(r) \frac{\partial \tau}{\partial r}\right]$$

$$V = 4997 \, \mathrm{fm}^3$$

and blast wave parametrization of spectral shape with T = 156.5 MeV and
a fireball volume per unit rapidity for central PbPb collisions V = 4997 fm^3
sensitivity to shape of freeze-out surface: backup