Detectors in Nuclear and Particle Physics

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6. Momentum Measurements

Deflection of track of charged particle in magnetic spectrometer
Lorentz force $\rightarrow$ circular orbit of curvature radius ρ in homogeneous magnetic field

$$\frac{mv^2}{\rho} = q\vec{v} \times \vec{B} = qv_\perp \cdot |\vec{B}| \qquad v_\perp : \text{ component of } \vec{v} \perp \text{ to } \vec{B}$$

$$\rho = \frac{p^2}{qp_\perp B} \qquad p_\perp : \text{ analogue}$$

and for $\vec{p} \perp B$

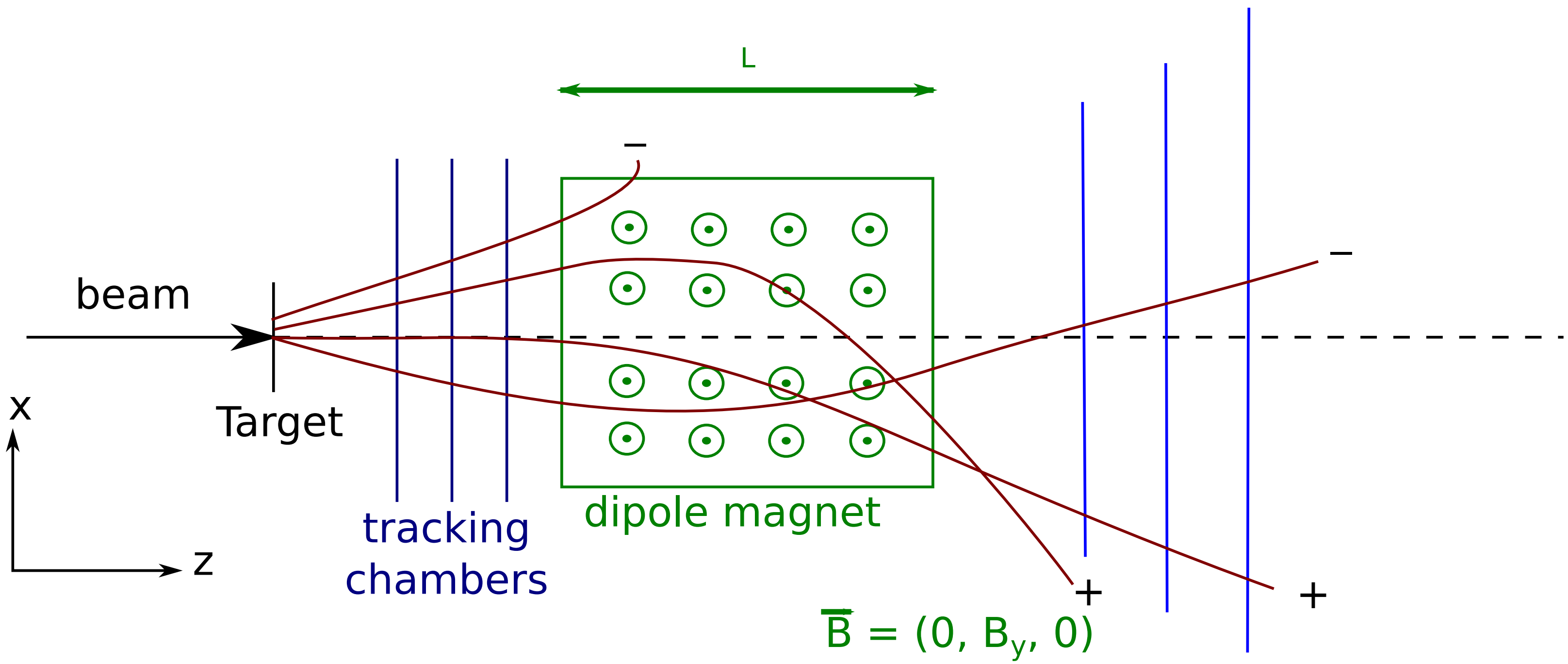
$$\rho = \frac{p}{qB}$$

units: for ρ in m
p in GeV/c
B in T
q in units of e

$$\boxed{\rho = \frac{p}{0.3\, qB} \qquad \text{or} \qquad p = 0.3\, q\rho B}$$

6.1 Forward Spectrometer

Mainly in fixed target experiments (but also LHCb) or ALICE forward muon spectrometer



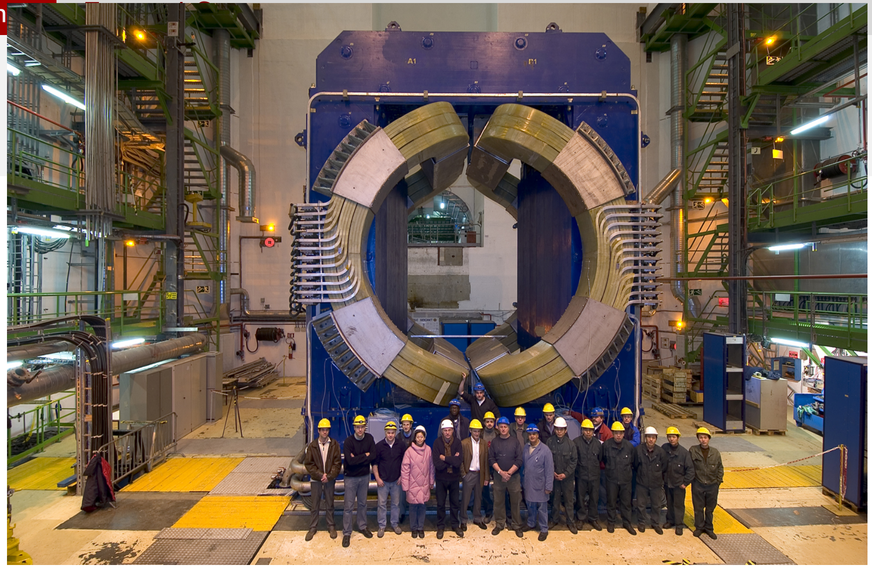
magnetic field gives (additional) $p_\perp$-kick $\Delta p_\perp$
typically $p \gg p_\perp, \Delta p_\perp \rightarrow$ Lorentz force always approximately in x-direction and

$$\Delta p_x = 0.3\, L\, q\, B$$

or for magnetic field not constant over entire path

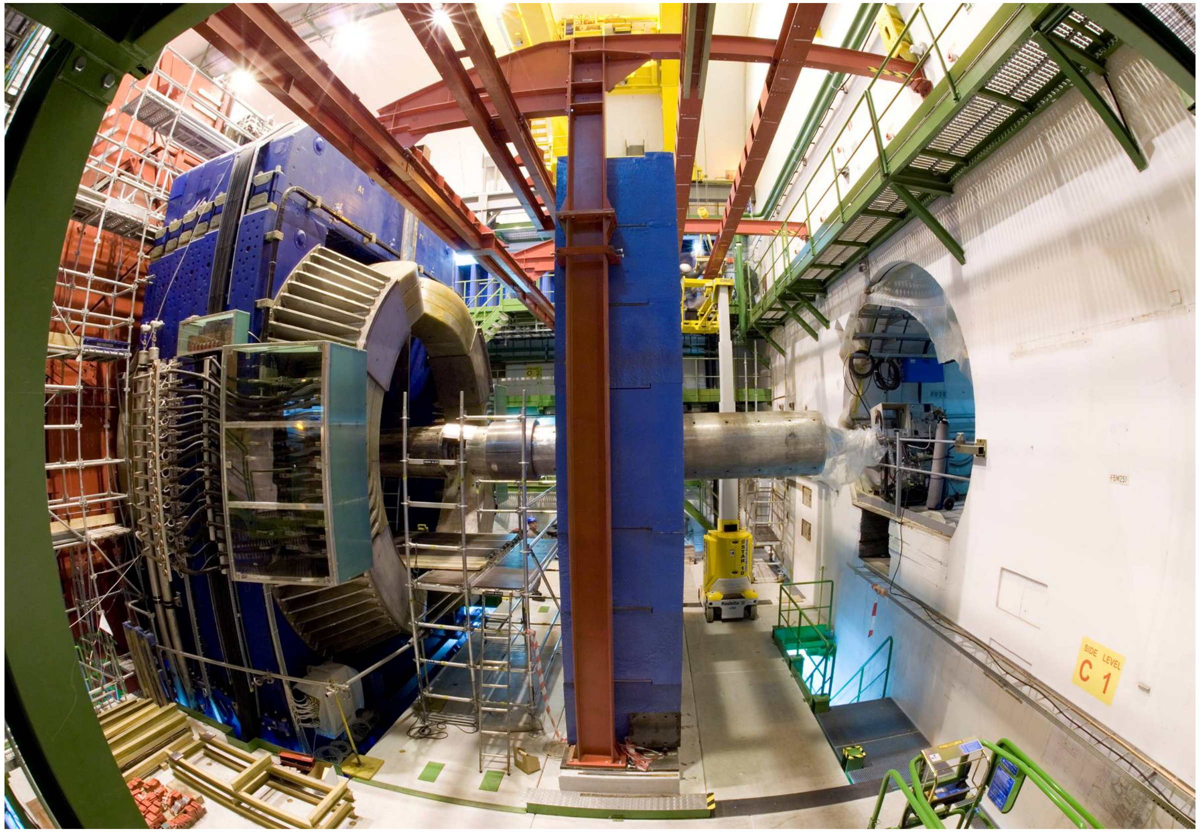
$$\Delta p_x = 0.3\, q \int_L B \mathrm{d}L$$

ALICE (Di)-Muon Spectrometer



Dipole magnet

Muon chambers

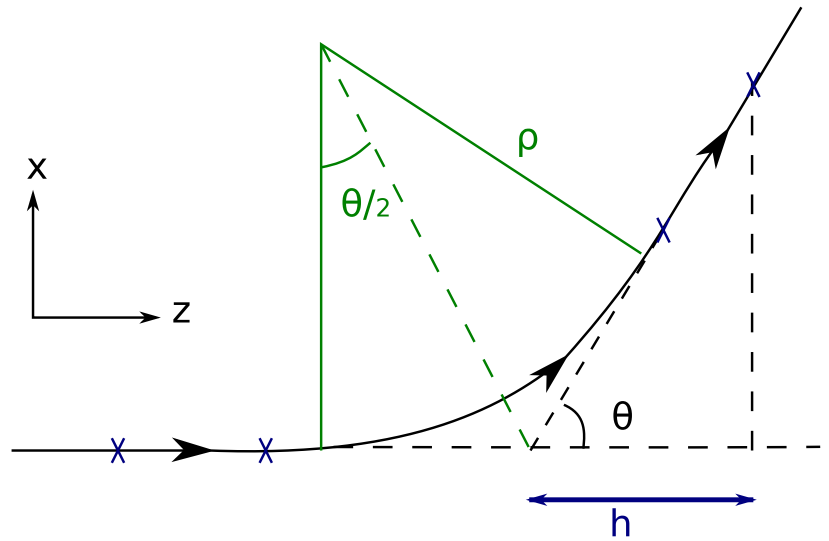


Muon absorber and filter

Example: proton of

$$\begin{aligned} p &= 10 \text{ GeV/c} \simeq p_z \\ \int BdL &= 6 \text{ Tm} \\ \Delta p_x &= 1.8 \text{ GeV/c} \\ \Delta\theta_x &= 10^\circ \end{aligned}$$

about the limit for this approximation



for $\rho \gg L$

$$\theta \approx \frac{L}{\rho} = \frac{LqB_y}{p}$$

$$\Delta p_x = p \sin\theta \approx p\theta = LqB_y$$

$$\text{or} \qquad \approx q \int_0^L B_y \mathrm{d}L$$

Momentum resolution

$$p = q\rho B_y = q\frac{L}{\theta}B_y \qquad \frac{dp}{d\theta} = qLB_y\frac{1}{\theta^2} = \frac{p}{\theta} \quad \Rightarrow \quad \frac{dp}{p} = \frac{d\theta}{\theta} \qquad \frac{\sigma_p}{p} = \frac{\sigma_\theta}{\theta}$$

accuracy of angular measurement $\equiv$ accuracy of momentum measurement

minimum: two measured points before and two after deflection

in practice always 3 or more measurements, since detectors need to be aligned relative to each other (best done with straight tracks)

in case all measured points have identical resolution σ_x:

$n/2$ points before
$n/2$ points after deflection
lever arm h (see Fig. previous page)

$$\Rightarrow \quad \sigma_\theta = \sqrt{\frac{8}{n}}\frac{\sigma_x}{h}$$

$$\boxed{\frac{\sigma_p}{p} = \frac{\sqrt{8/n}\,\sigma_x}{h}\frac{p}{qLB_y} = \frac{\sqrt{8/n}\,\sigma_x}{h}\frac{p}{\Delta p_x}}$$

contribution of space point resolution to momentum resolution

typical form $\quad \frac{\sigma_p}{p} = \text{const}\cdot p \quad$ with const $= 10^{-2}\dots 10^{-4}$ i.e. 1% - 0.01%

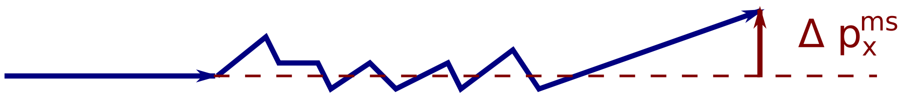
Example: 6 measurements each with $\sigma_x = 200\ \mu$m, $h = 5$ m, deflection 1°

$$\theta_x = \frac{\Delta p_x}{p} = 0.017 \quad \text{deflection)} \quad \text{for } p = 10\ GeV/c$$

$$\Rightarrow \frac{\sigma_p}{p} = 3\cdot 10^{-3} = 3\cdot 10^{-4}p$$

Effect of multiple scattering

multiple Coulomb scattering along particle trajectory of length L contributes to $p_\perp$-broadening perpendicular to direction of propagation



$$\Delta p_\perp^{ms} = p \sin\theta_{rms} \simeq p\theta_{rms} = \frac{q \cdot 19.2\ \mathrm{MeV/c}}{\beta}\sqrt{\frac{L}{X_0}}$$

where X_0 is the radiation length. In the direction of deflection (x) this means:

$$\Delta p_x^{ms} = \frac{q \cdot 19.2\ \mathrm{MeV/c}}{\beta\sqrt{2}}\sqrt{\frac{L}{X_0}} = \frac{q \cdot 13.6\ \mathrm{MeV/c}}{\beta}\sqrt{\frac{L}{X_0}}$$

for sufficiently large momenta independent of p

contribution to momentum resolution:

$$\boxed{\left(\frac{\sigma_p}{p}\right)_{ms} = \frac{\Delta p_x^{ms}}{\Delta p_x} = \frac{13.6\ \mathrm{MeV/c}\sqrt{L/X_0}}{e\int B_y \mathrm{d}L}}$$

where Δp_x is the deflection due to magnetic field (see above).

total momentum resolution

Example: as above

$$e\int B dL = \Delta p_x = 0.17\ \mathrm{GeV/c}$$

$$L = 15\ \mathrm{m}$$

$$\text{material: air,}\quad X_0 = 304\ \mathrm{m}$$

$$\left(\frac{\sigma_p}{p}\right)_{ms} = 1.8\%$$

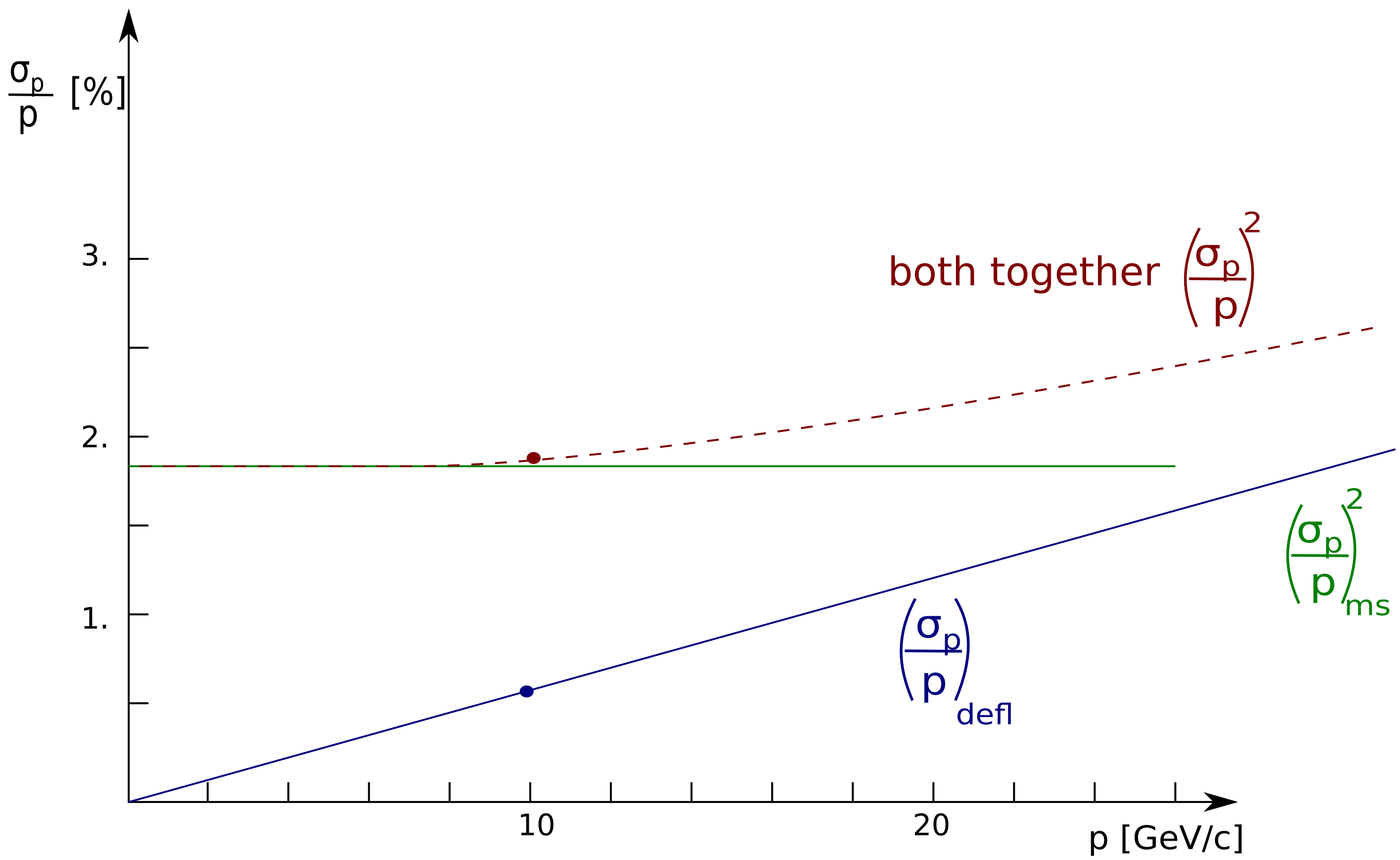
$$\text{vs.}\ \left(\frac{\sigma_p}{p}\right)_{defl} = 0.03\%\, p \quad \Rightarrow$$ multiple scattering dominates at small momenta

$$\boxed{\left(\frac{\sigma_p}{p}\right)^2 = \left(\frac{\sigma_p}{p}\right)^2_{ms} + \left(\frac{\sigma_p}{p}\right)^2_{defl}}$$

momentum resolution in magnetic spectrometer

Example:

$$\left(\frac{\sigma_p}{p}\right)^2 = (1.8\%)^2 + (0.06\% \cdot p)^2$$



Multiple scattering particularly relevant if **magnetized iron** is used, as frequently done for measurements of muon momentum

- advantage: high B-field, stops π, K before they decay into μ
- disadvantage: worsens momentum resolution by multiple scattering

$$X_0^{Fe} = 1.76\ \text{cm}, \qquad B = 1.8\ \text{T}, \qquad L = 3\ \text{m}, \qquad \Delta p_x = 1.6\ \text{GeV/c}$$

$$\Rightarrow \qquad \left(\frac{\sigma_p}{p}\right)_{ms} = 11\%$$

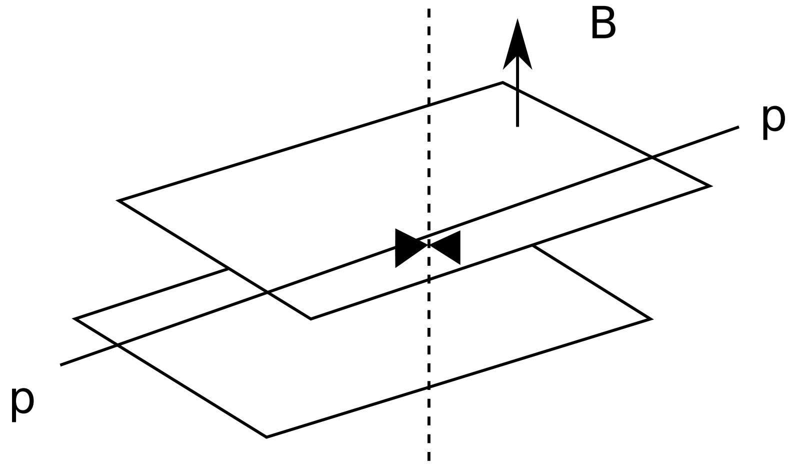
depending on desired momentum range, accuracy of deflection measurement can be chosen accordingly

6.2 Solenoidal and Toroidal Fields - mostly at Colliders

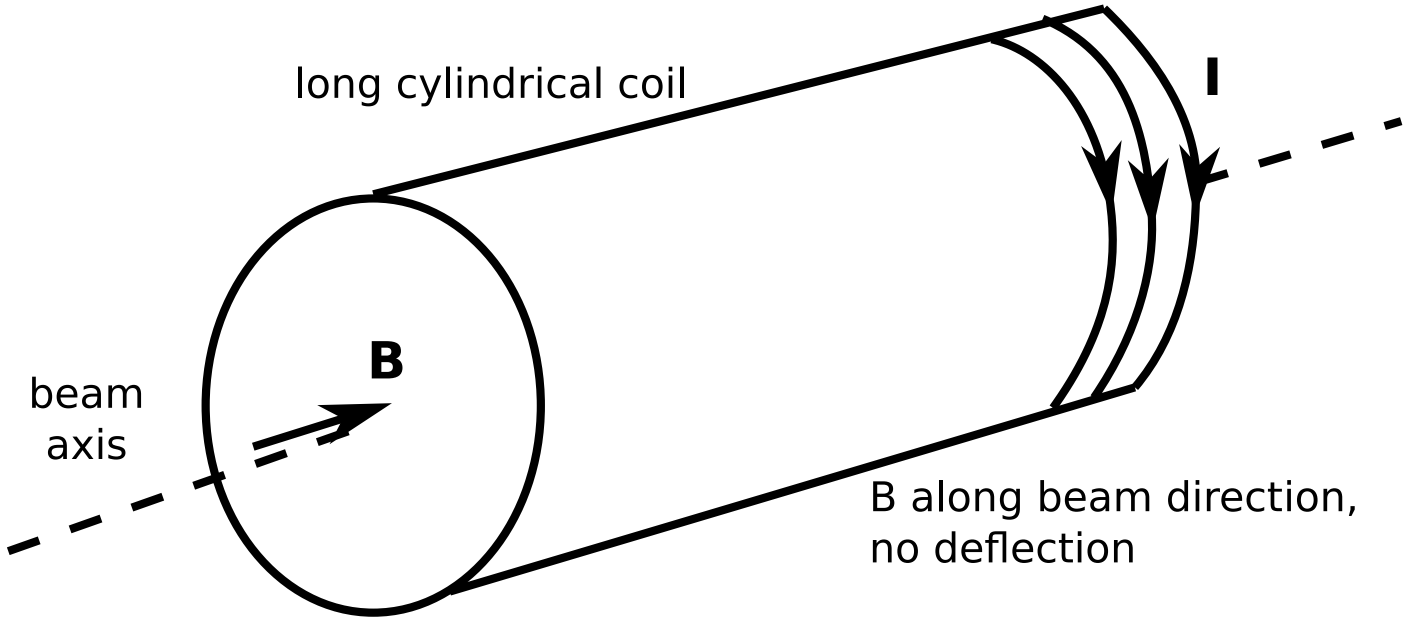
Normally 4π coverage desired, leading to special spectrometer configuration

dipole disfavored

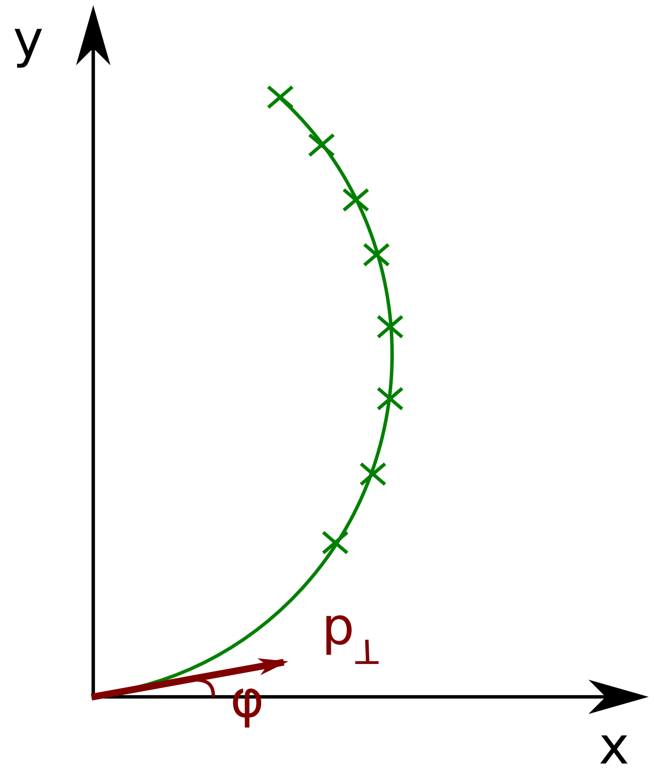
- deflects beam which must be compensated
- not nice symmetry for 4π experiment



Solenoid



measure momentum component $p_\perp$ perpendicular to beam



beam and B-field along z-axis

particle produced with momentum $\vec{p}$

completely characterized by p_x, p_y, p_z, where p_x and p_y can be written as:

$$|p_\perp|, \varphi : \qquad p_x = |p_\perp| \cos\varphi$$
$$p_y = |p_\perp| \sin\varphi$$

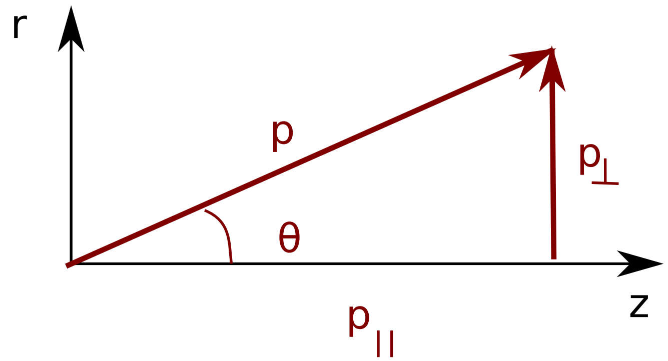
need to measure at least 3 points of track

circular in xy-plane

$\Rightarrow$ ρ (radius of curvature) or $p_\perp$ and φ

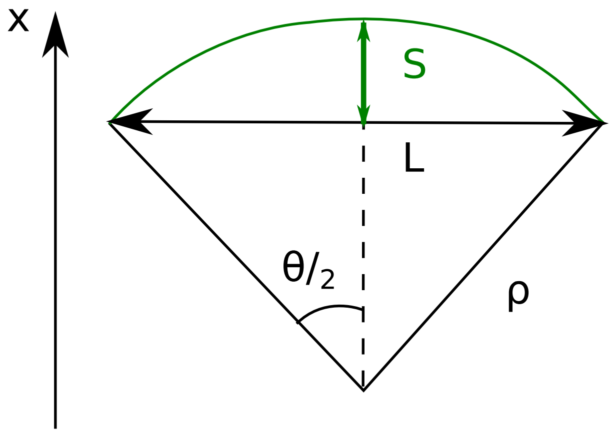
measurement of θ:

$$p_{||} = \frac{p_\perp}{\tan\theta}$$
$$p = \frac{p_\perp}{\sin\theta}$$



complete measurement of particle momentum

Sagitta Method



$$\text{Sagitta} \quad S = \rho - \rho\cos\frac{\theta}{2} = \rho\left(1-\cos\frac{\theta}{2}\right)$$
$$= 2\rho\sin^2\frac{\theta}{4}$$
$$\text{for small } \theta \quad S \simeq \frac{\rho\theta^2}{8}$$

$$\text{with } \theta = \frac{qBL}{p_\perp} \quad \text{and} \quad \sin\theta/2 \simeq \theta/2 = \frac{L/2}{\rho}$$

$$\boxed{S = \frac{qL^2B}{8p_\perp}}$$

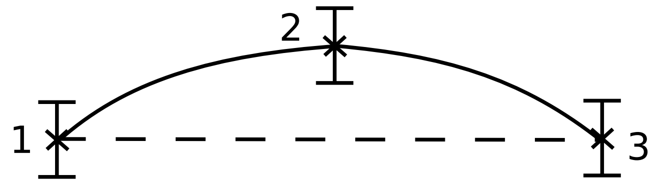
B in T, L in m, $p_\perp$ in GeV/c, q in e

$$S(\mathrm{m}) = \frac{0.3\,qL^2B}{8p_\perp}$$

Measurement of at least 3 points with coordinates $x_1,\ x_2,\ x_3$

$$S = x_2 - \frac{x_1 + x_3}{2}$$

$$\sigma_S = \sqrt{\frac{3}{2}}\sigma_x$$



$$\boxed{\Rightarrow \frac{\sigma_p}{p} = \frac{\sigma_S}{S} = \frac{\sqrt{3/2}\,\sigma_x 8p}{0.3qBL^2}}$$

Measurement of N equally spaced points:

$$\boxed{\frac{\sigma_p}{p} = \frac{\sigma_x}{0.3qBL^2}\sqrt{\frac{720}{(N+4)}}p}$$

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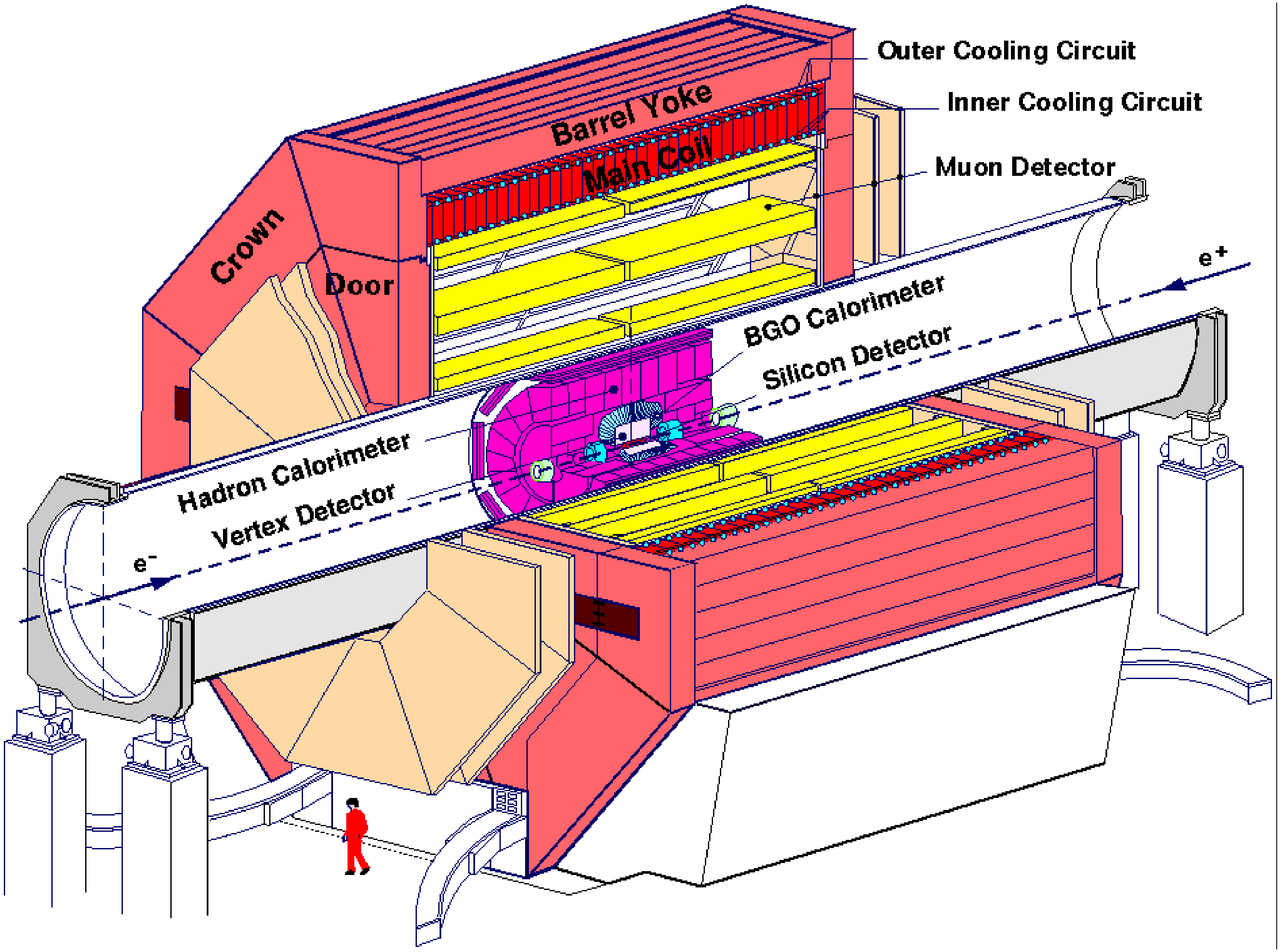
- example

$$\left.\begin{array}{lcl} B & = & 0.5\ \mathrm{T} \\ L & = & 2\ \mathrm{m} \\ \sigma_x & = & 400\ \mu\mathrm{m} \\ N & = & 150 \end{array}\right\} \frac{\sigma_p}{p} \simeq 1.4 \cdot 10^{-3} p$$

similar to ALICE TPC, usage of former L3 Magnet

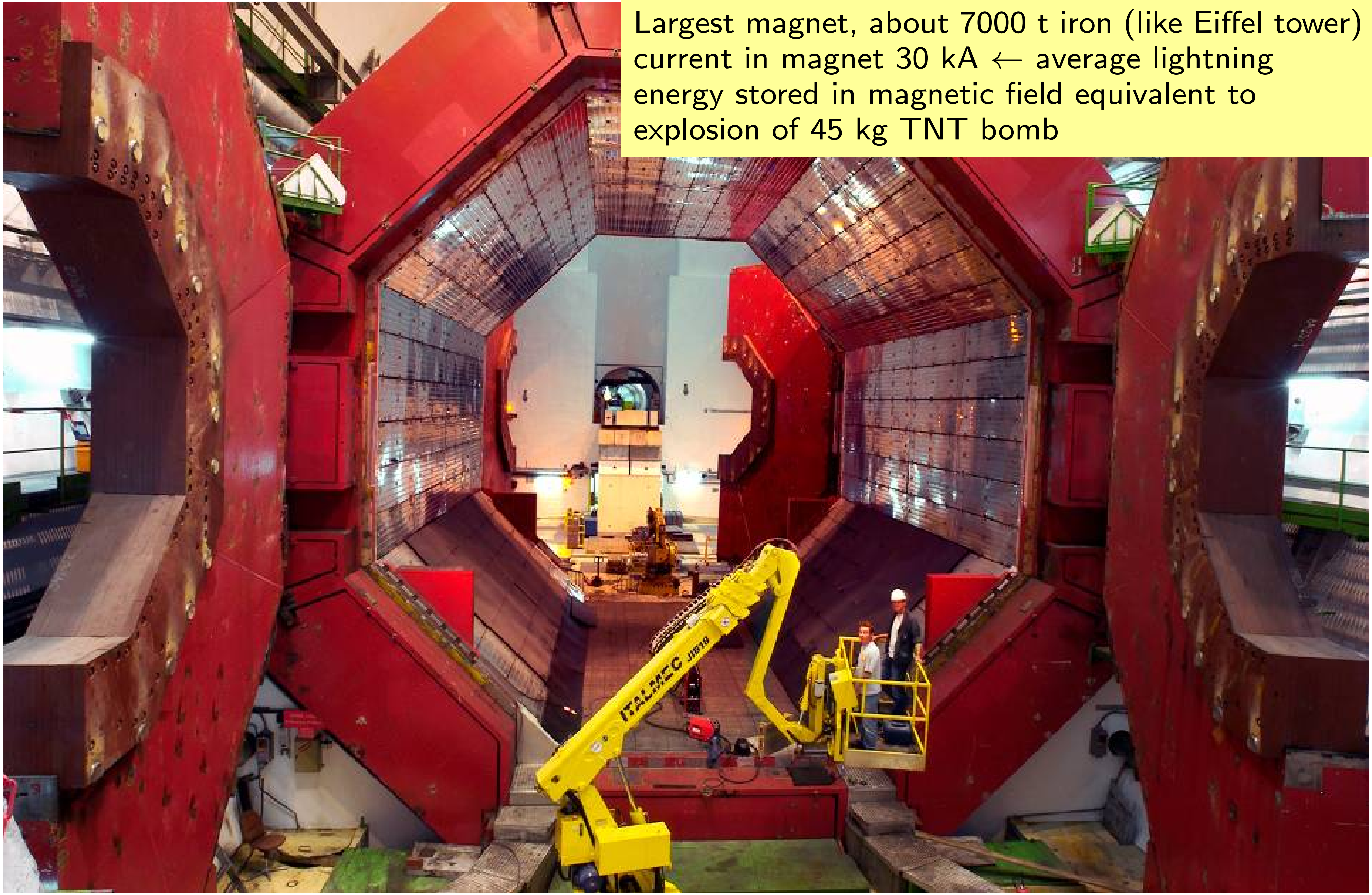
L3



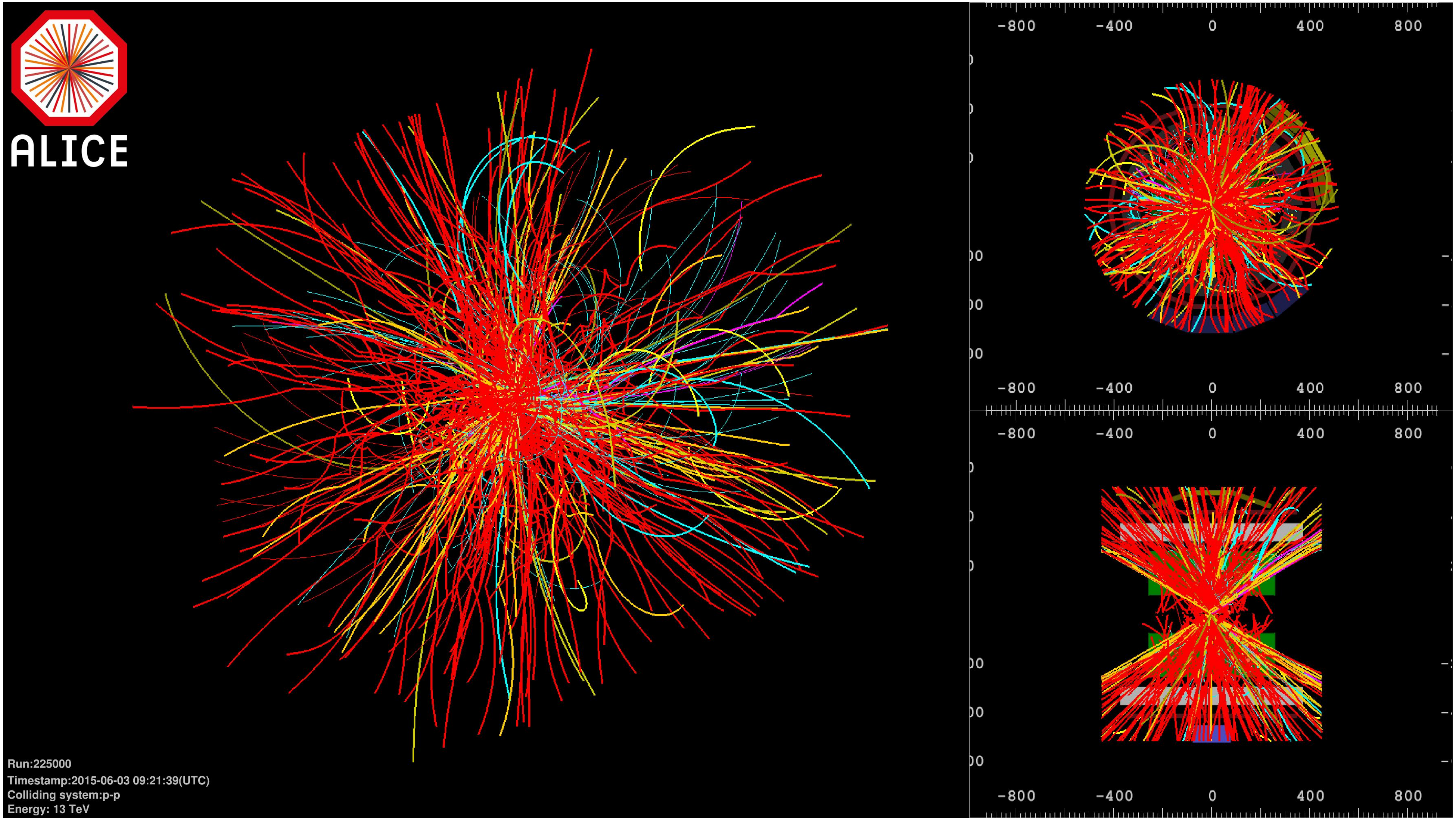
$\sigma_S = 90\ \mu\text{m} \qquad \frac{\sigma_{p_\perp}}{p_\perp} = 2.5\%$ at 45 GeV (typical Z^0 decay product)

Construction site ALICE 2004 - the solenoid and the iron return yoke

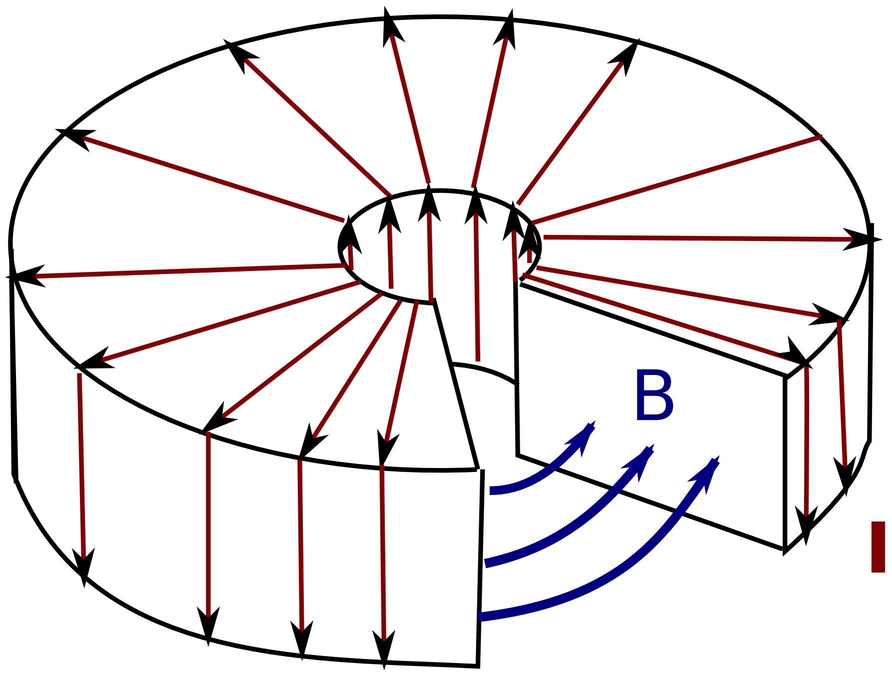
Largest magnet, about 7000 t iron (like Eiffel tower)
current in magnet 30 kA $\leftarrow$ average lightning
energy stored in magnetic field equivalent to
explosion of 45 kg TNT bomb



ALICE first 13 TeV pp collisions



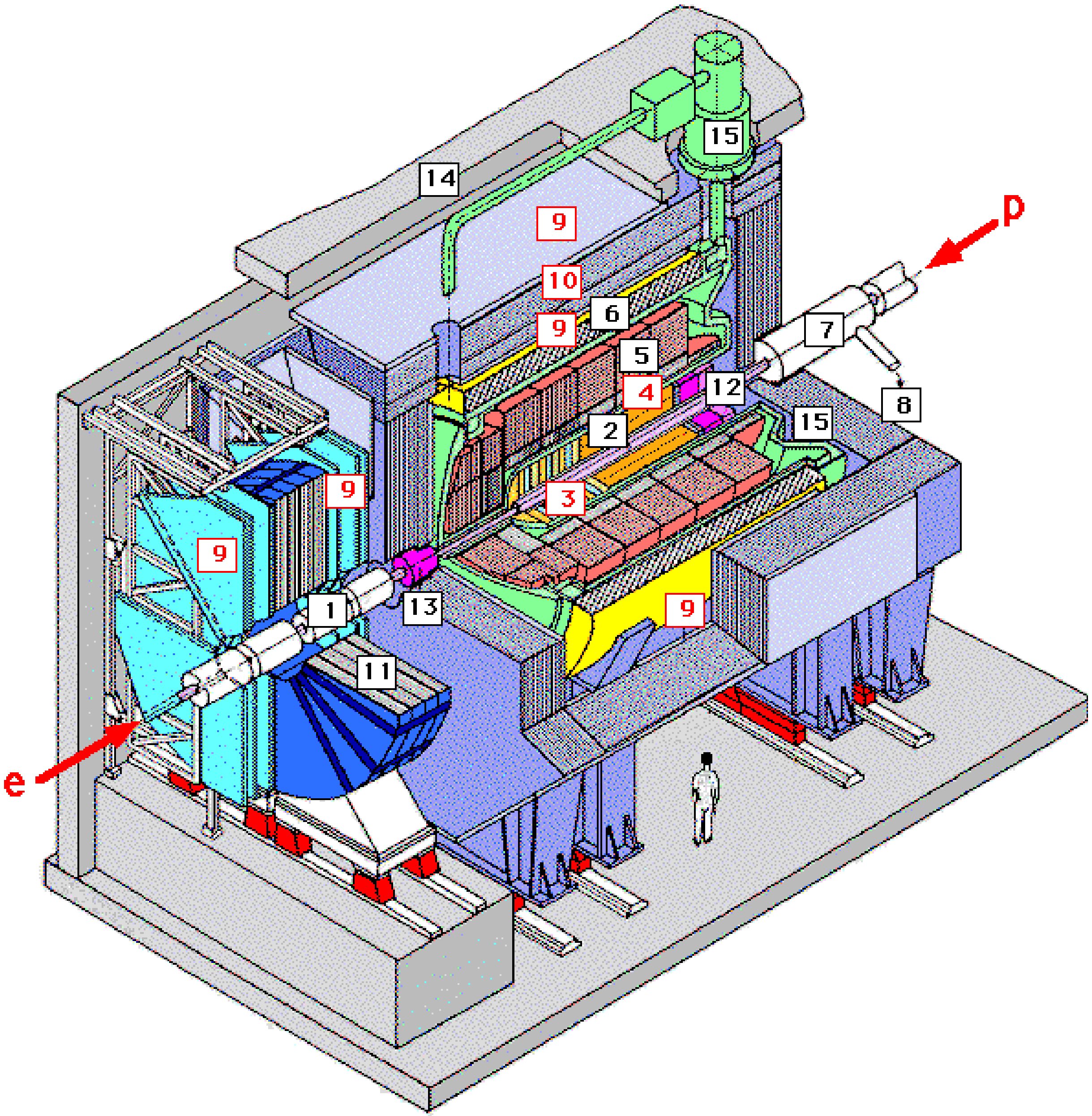
Toroid



- on axis vanishing B-field
- no deflection of beam
- fill with iron-core e.g. for muon measurement in end caps

Example: H1 forward muon spectrometer

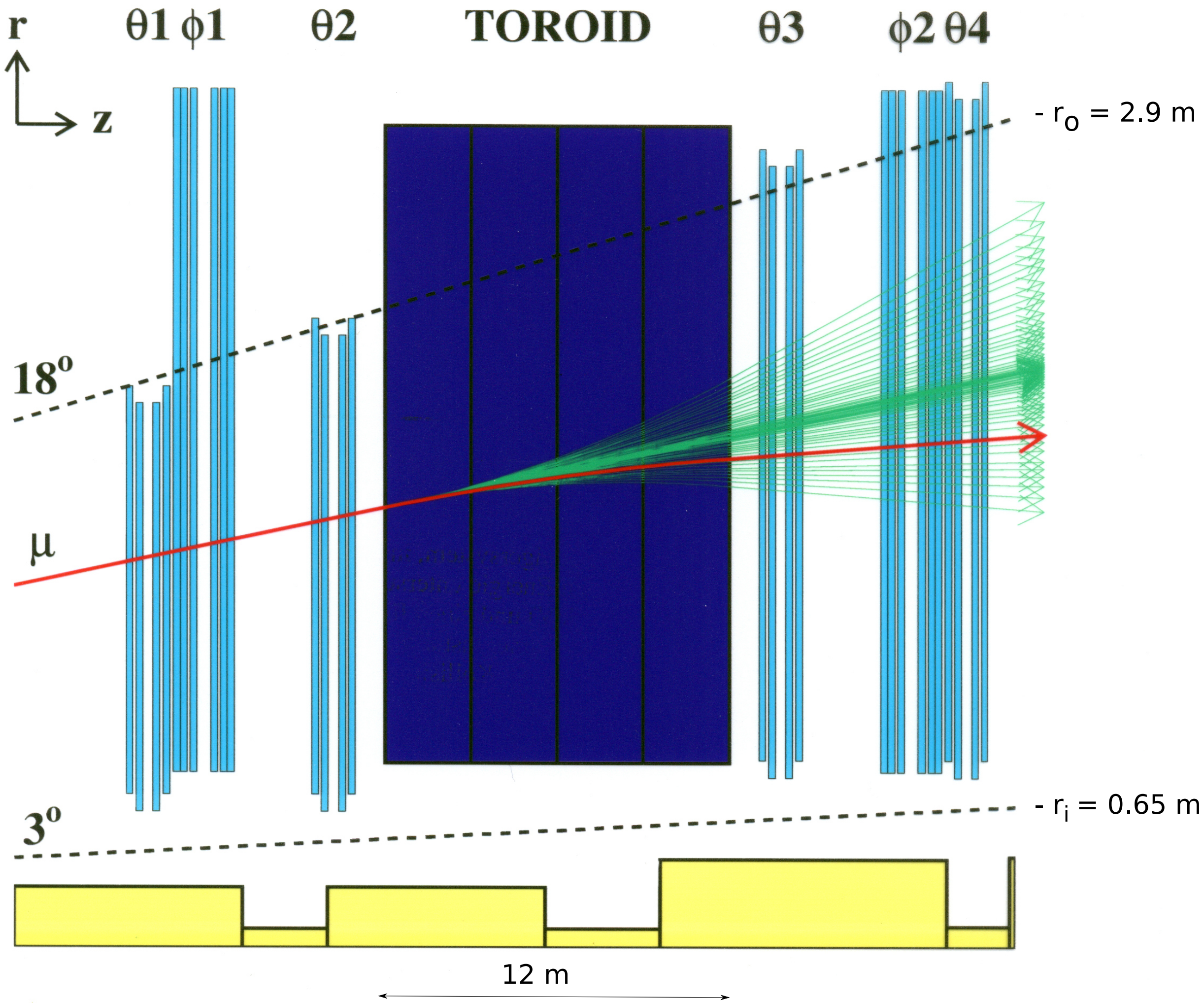
H1 experiment at HERA



Central solenoid plus forward muon toroid to measure high energy muons between 3° and 17°

drift chamber planes before and after toroid

Toroidal magnet: 12 segments with 15 turns each (Cu), 150 A
$\rightarrow B = 1.6$ T filled with Fe core



deflection in polar angle → momentum

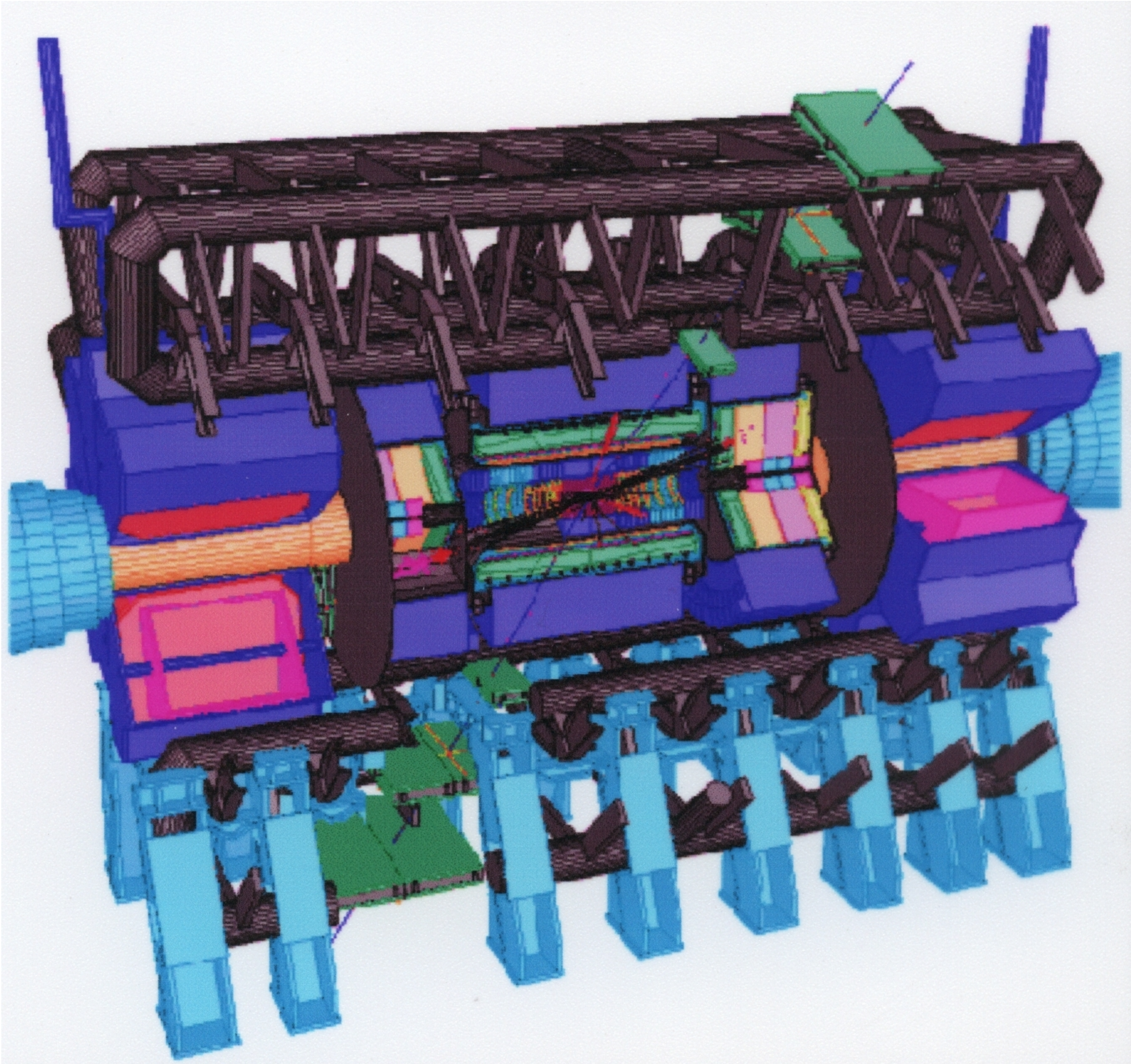
$$\frac{\sigma_p}{p} = 24 - 36\%$$

$$\text{for } p = 5 - 200 \text{ GeV/c}$$

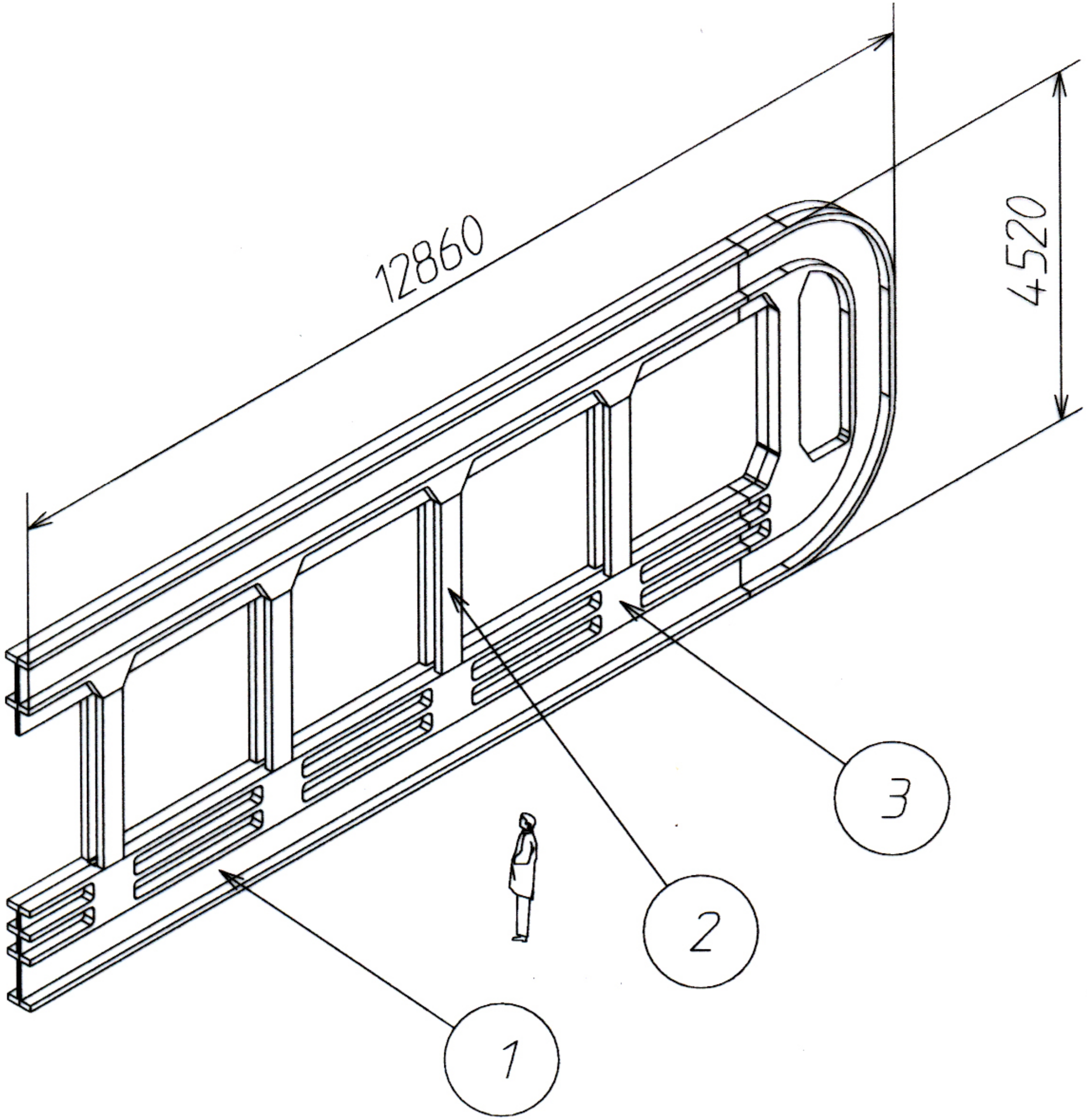
dominated by multiple scattering

$$\frac{\sigma_p}{p} = 0.24 \oplus 1.3 \cdot 10^{-3} p$$

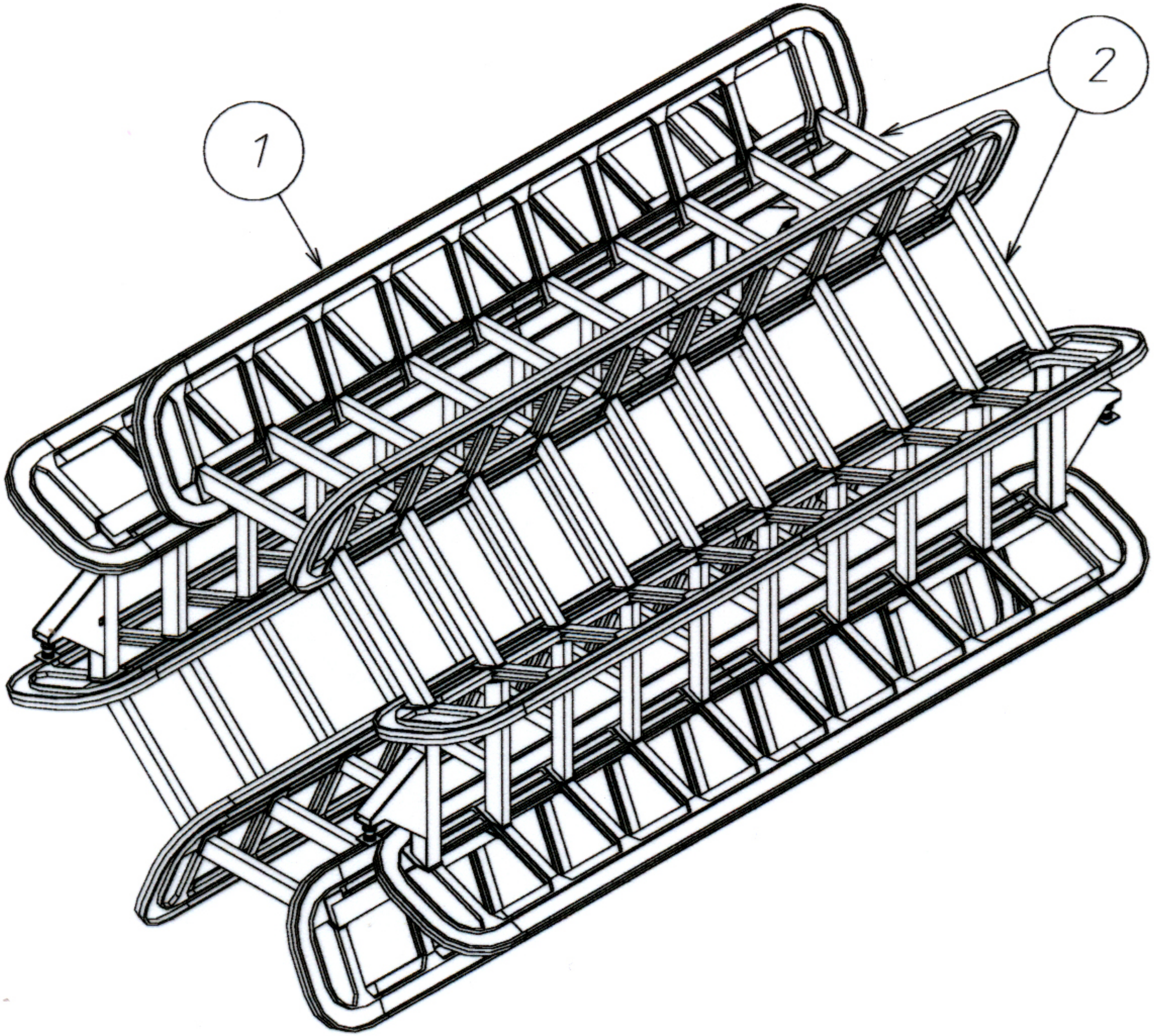
ATLAS - A Toroidal LHC ApparatuS



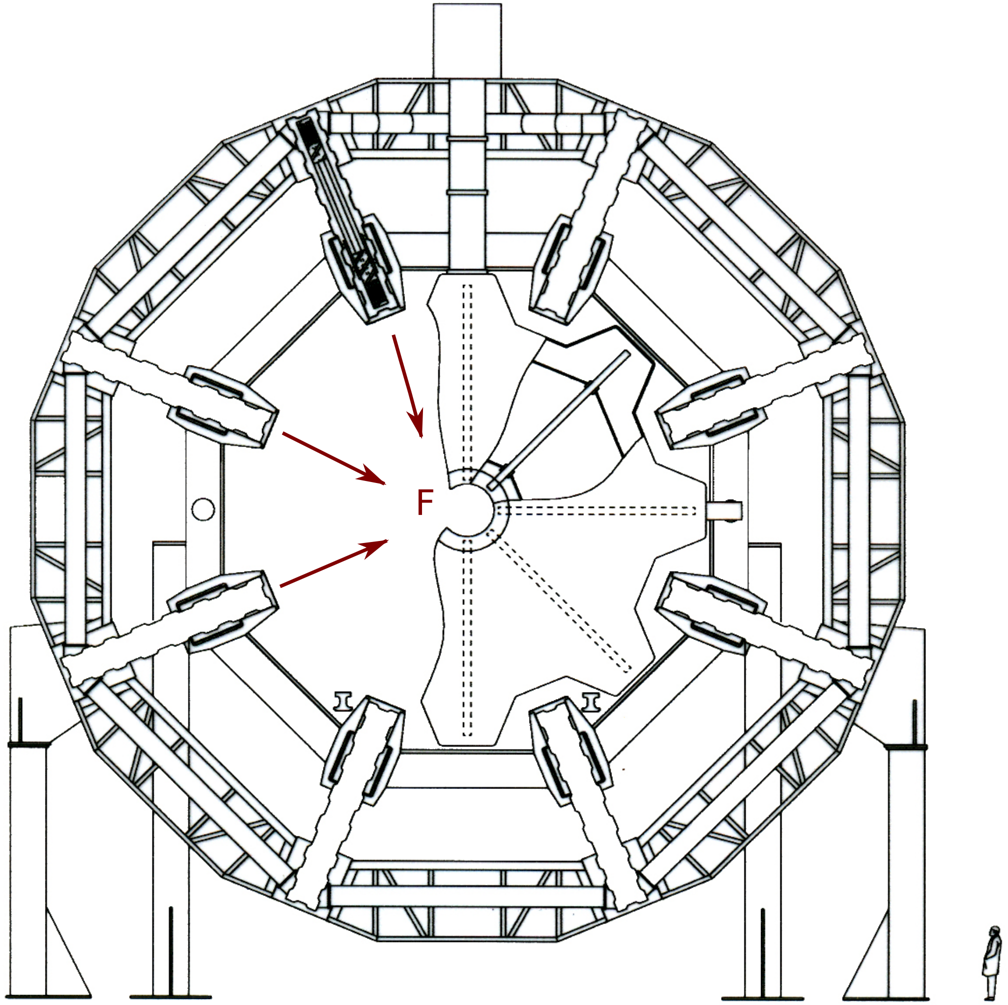
'air core toroid' central barrel
$L = 26$ m, $D_i = 9.4$ m, $D_0 = 19.5$ m



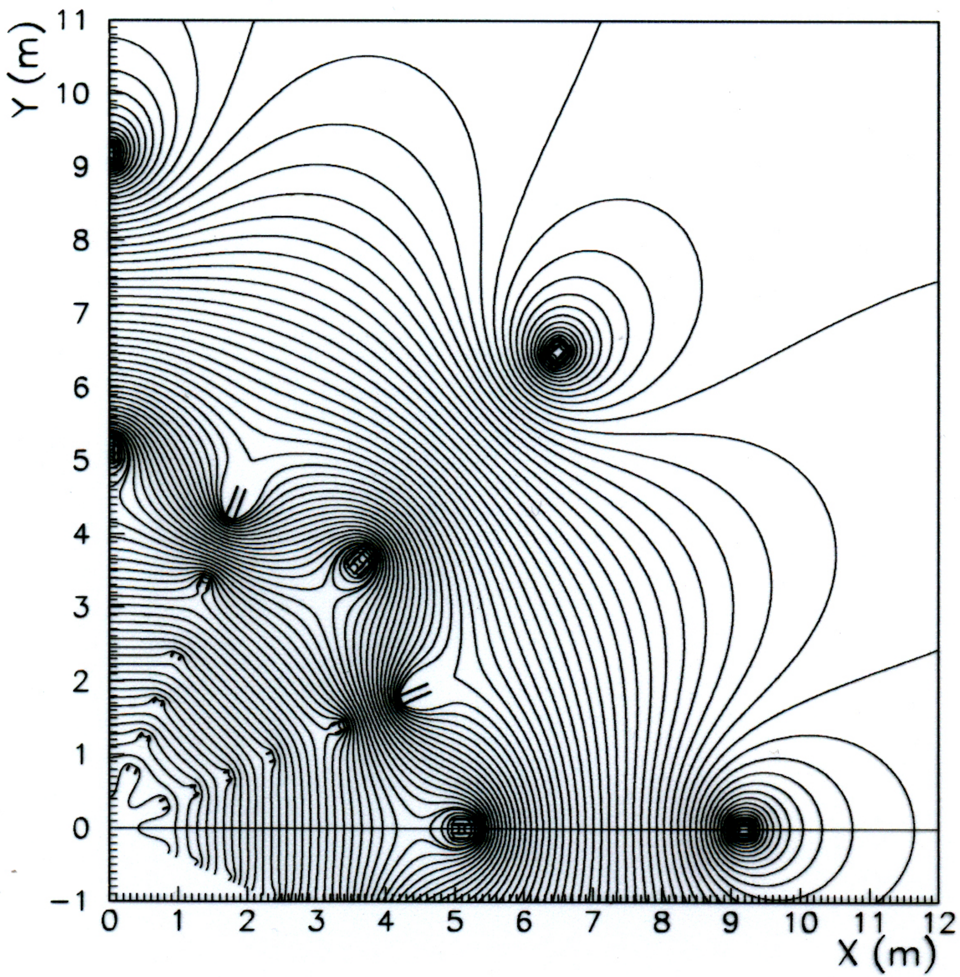
8 flat coils, super-conducting, 70 km super-conducting cable
20 kA, $\int BdL = 3 - 9$ Tm
energy stored in magnetic field 1490 MJ



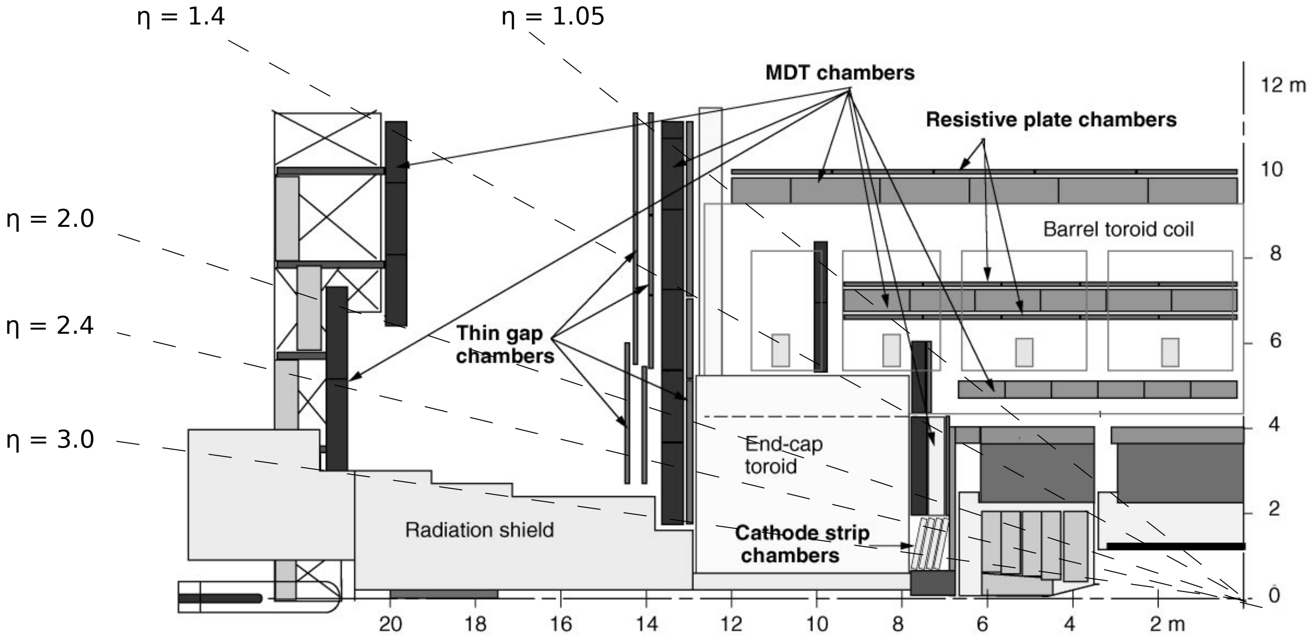
ATLAS cavern

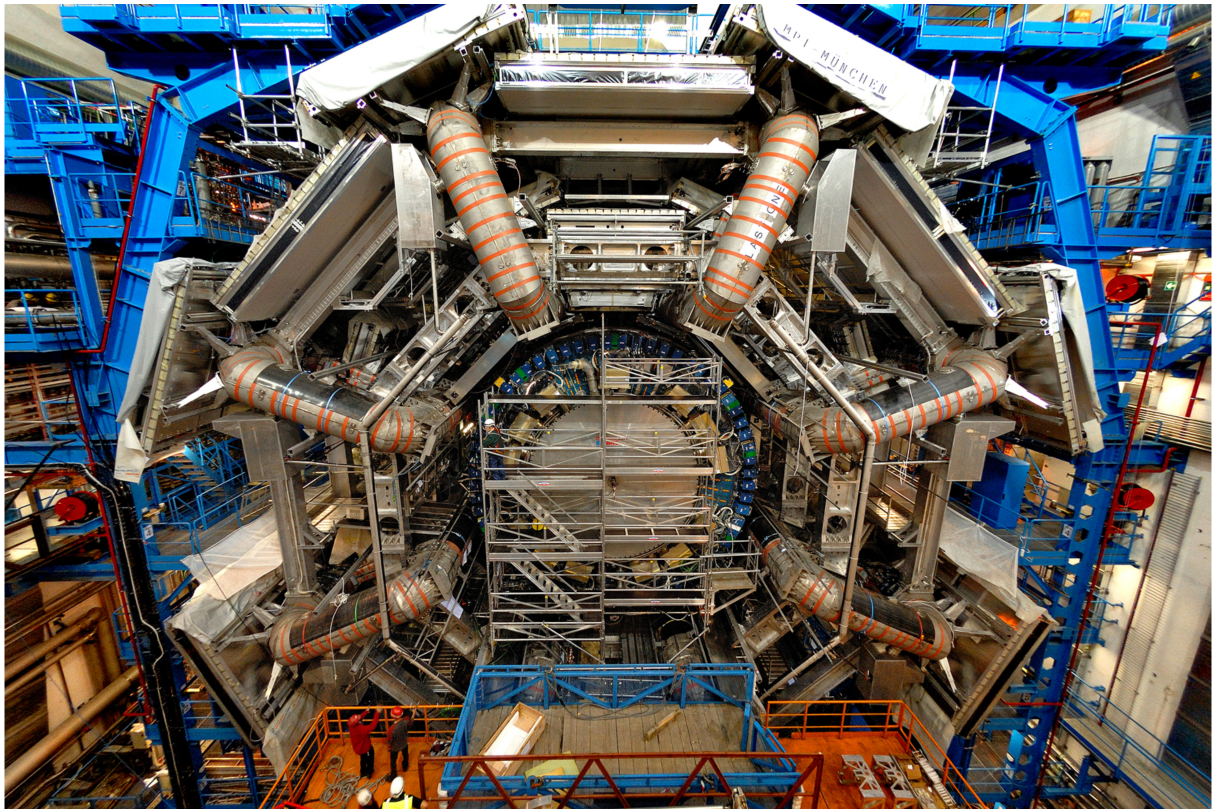


with 'current on' forces on coils radially inward



B-field monitored by 5000 Hall probes attached to muon chambers





Principle of momentum measurement of muon tracks with monitored drift-tube array:

3 layers, each consisting of
2 multilayers

total 1200 muon chambers of $2 \times 3.5\ \mathrm{m}^2$

total 300000 channels

ATLAS monitored drift tube arrays

drift tubes Al-Mn ⌀ 3 cm

400 μm wall thickness

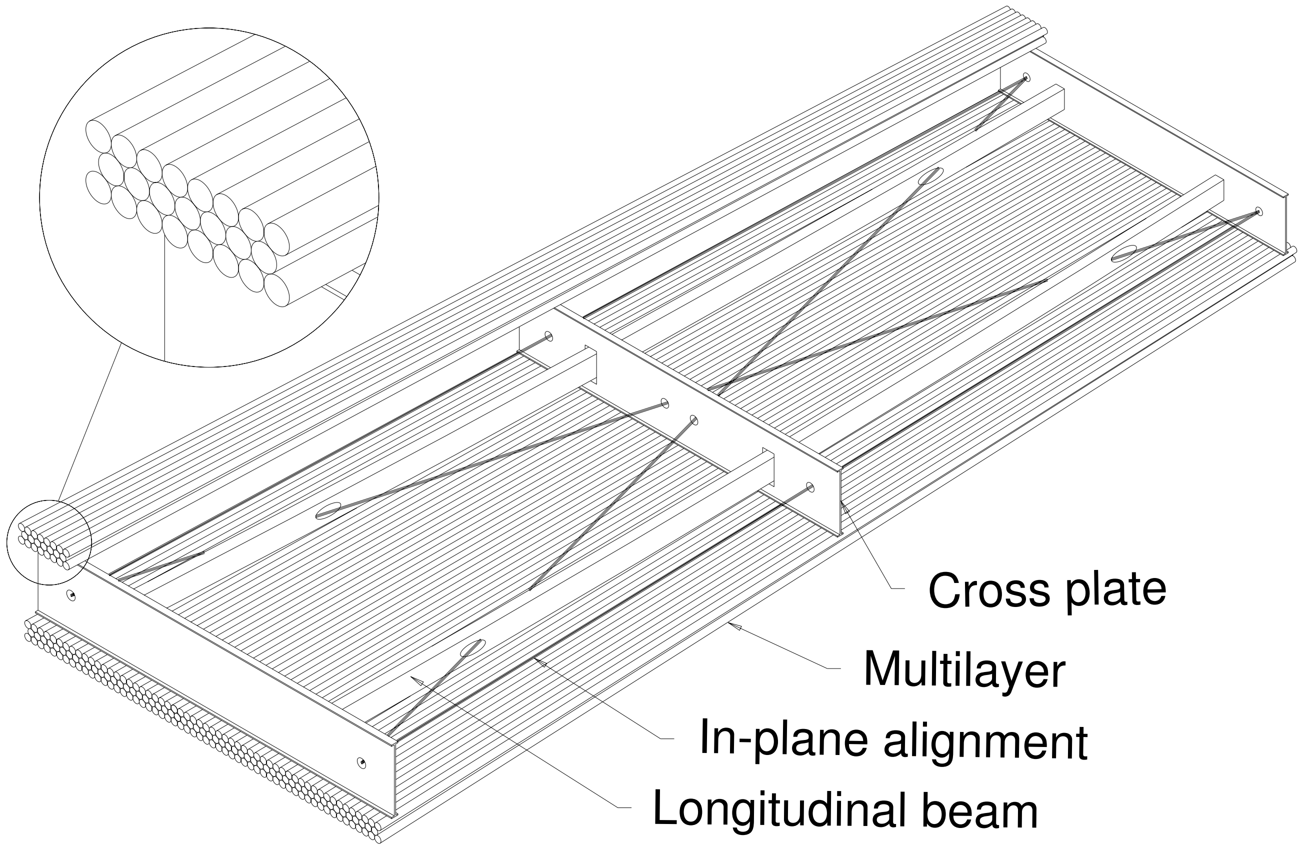
1.4 – 6.3 m long 50 μm wire
centered in tube to 20 μm

operation at 3 bar

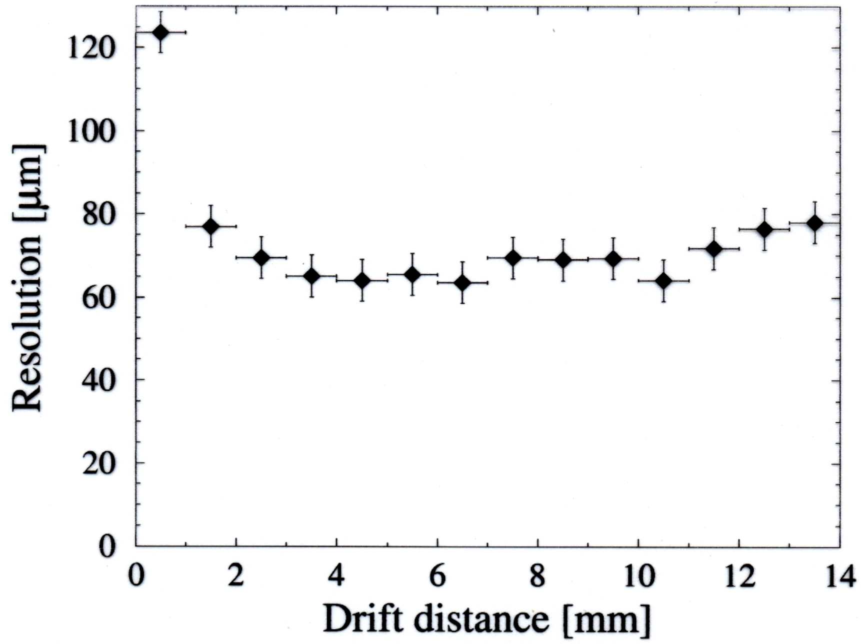
drift time $\leq$ 600 ns

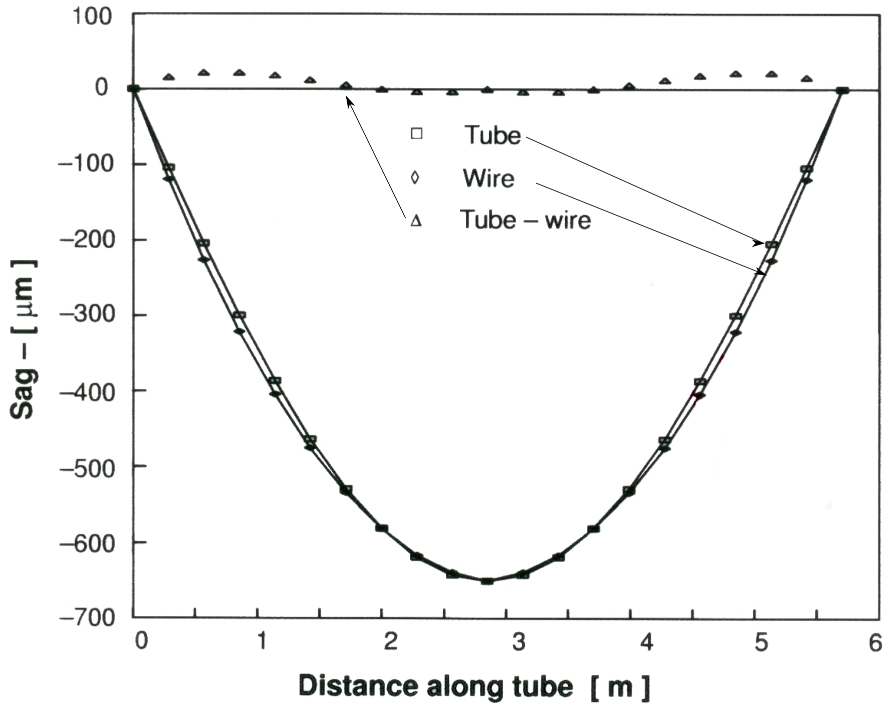
gas gain $<$ 50000
(only 'streamers'
to avoid $\leftrightarrow$ 'aging')

gas: Ar/C_2H_6/CO_2/N_2
86 : 5 : 4 : 5

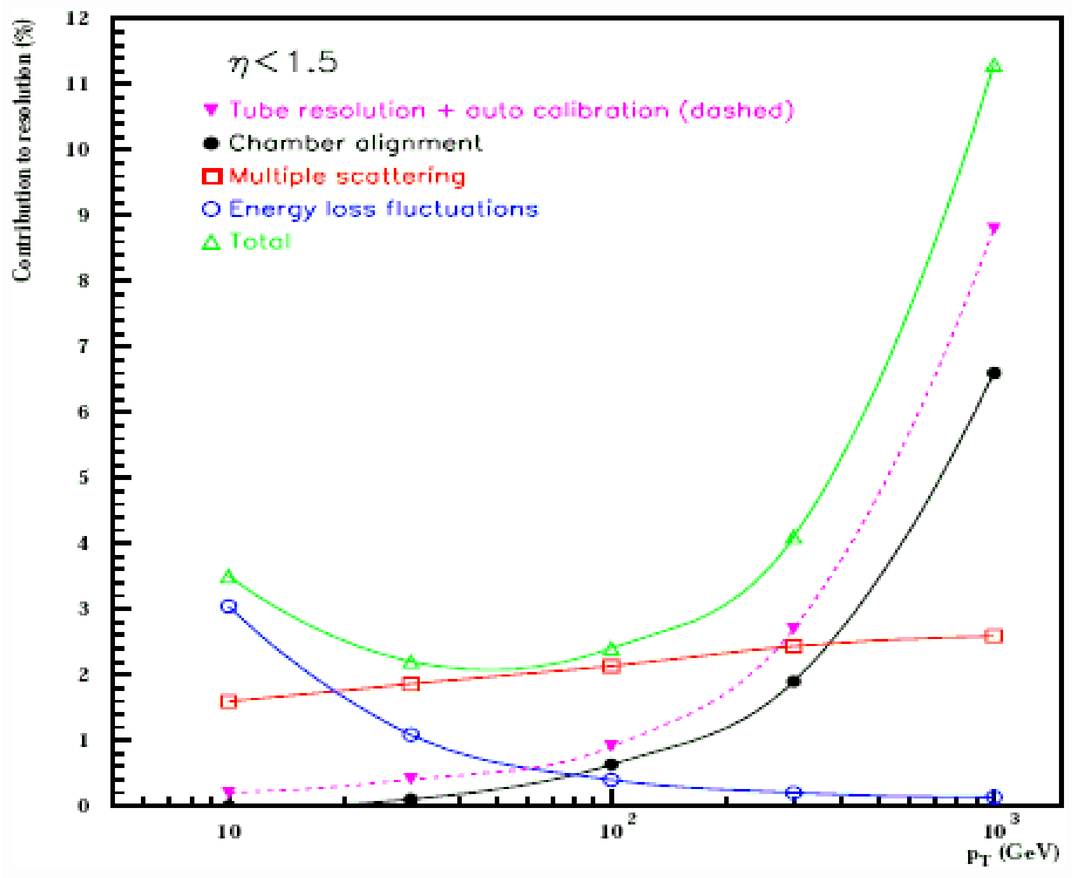


position resolution ATLAS monitored drift tube array



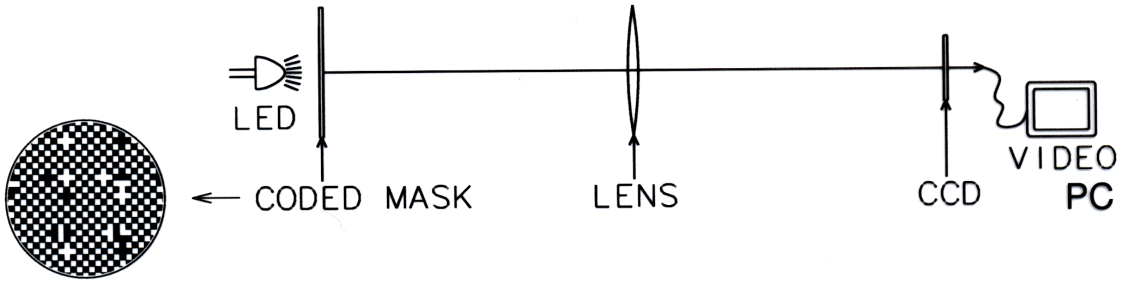


difficulty: gravitational sag of long tubes and wires

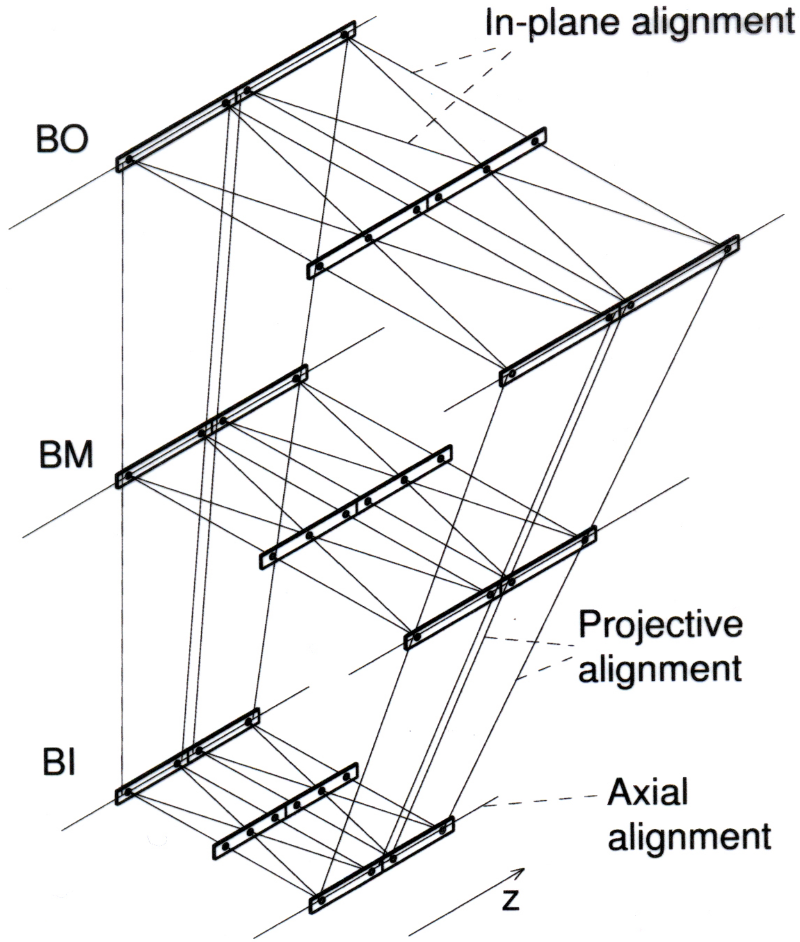


single tube position resolution 80 μm
$\rightarrow \sigma_{p_t}/p_t = 10\%$ at 1000 GeV

Need to know exactly where tubes and wires are!



Rasnick System



Alignment system for 3 layers of MDT's

13000 CCD cameras
→ align each muon chamber to 0.05 mm precision