Detectors in Nuclear and Particle Physics

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2. Interactions of particles and matter

- very compact presentation, since material should be largely known (see also chapter 3 of my lecture 'Experimentalphysik 5' WS 2008/2009 and Skript - to be found on my webpage)
- some additional material, useful relations, tables, figures[1]
- more emphasis on some aspects that are new beyond PEP4 and important for detectors

[1] Good, but very compact presentation of material, including many references in *Review of Particle Physics, Chin. Phys. C40 (2016) 100001 and 2017 update, ch. 34 "Passage of particles through matter" by P. Bichsel, D.E. Groom & S.R. Klein*

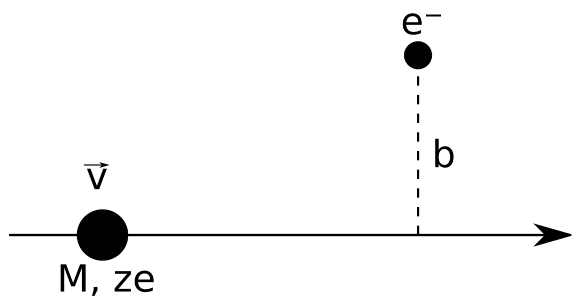
2.1 Electronic energy loss dE/dx

consider particle X with $Mc^2 \gg m_e c^2$
Coulomb interaction between particle X and atom
cross section dominated by inelastic collisions with electrons

$$
\begin{array}{llll}
\text{atom} + X & \rightarrow & \text{atom}^+ + e^- + X & \text{ionization} \\
 & \rightarrow & \text{atom}^* + X & \text{excitation} \\
 & & \quad \hookrightarrow \text{atom} + \gamma & \text{de-excitation}
\end{array}
$$

(for electrons also bremsstrahlung, see below)
classical derivation: *N. Bohr 1913*
quantum mechanical derivation: *H. Bethe, Ann. d. Physik 5 (1930) 325* and *F. Bloch, Ann. d. Physik 16 (1933) 285*

Bohr: particle with charge ze moves with velocity v through medium with electron density n, electrons considered free and, during collision, at rest



$$\Delta p_\perp = \Delta p = \frac{2ze^2}{bv} \qquad \Delta p_{||} \text{ averages to zero}$$

$$\Delta E(b) = \frac{\Delta p^2}{2m_e} \qquad \text{energy transfer onto one electron at distance } b$$

per pathlength $\mathrm{d}x$ in the distance between b and $b + \mathrm{d}b$, $n 2\pi b \, \mathrm{d}b \, \mathrm{d}x$ electrons are found [2]

[2] here and in the following $e^2 = 1.44$ MeV fm (contains $4\pi\epsilon_0$)

$$-\mathrm{d}E(b) = \frac{n4\pi z^2 e^4}{m_e v^2}\frac{\mathrm{d}b}{b}\mathrm{d}x$$

diverges for $b \to 0$ *Bohr*: choose relevant range $b_{min} - b_{max}$

b_{min} relative to heavy particle electron is located only within the Broglie wavelength

$$\Rightarrow b_{min} = \frac{\hbar}{p} = \frac{\hbar}{\gamma m_e v}$$

b_{max} duration of perturbation should be shorter than period of electron: $b/\gamma v \leq 1/\langle \nu \rangle$

$$\Rightarrow b_{max} = \frac{\gamma v}{\langle \nu \rangle}$$

integrate over b with these limits:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi z^2 e^4}{m_e c^2 \beta^2} n \ln \frac{m_e c^2 \beta^2 \gamma^2}{\hbar \langle \nu \rangle}$$

electron density $n = \frac{N_A \rho Z}{A}$
average revolution frequency of electron $\langle \nu \rangle \leftrightarrow$ mean excitation energy $I = \hbar \langle \nu \rangle$

Bethe-Bloch equation

considering quantum mechanical effects and some other corrections

Bethe-Bloch equation

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = Kz^2 \frac{Z}{A}\rho\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2}\right]$$

describes mean rate of energy loss in the range $0.1 \leq \beta\gamma \leq 1000$

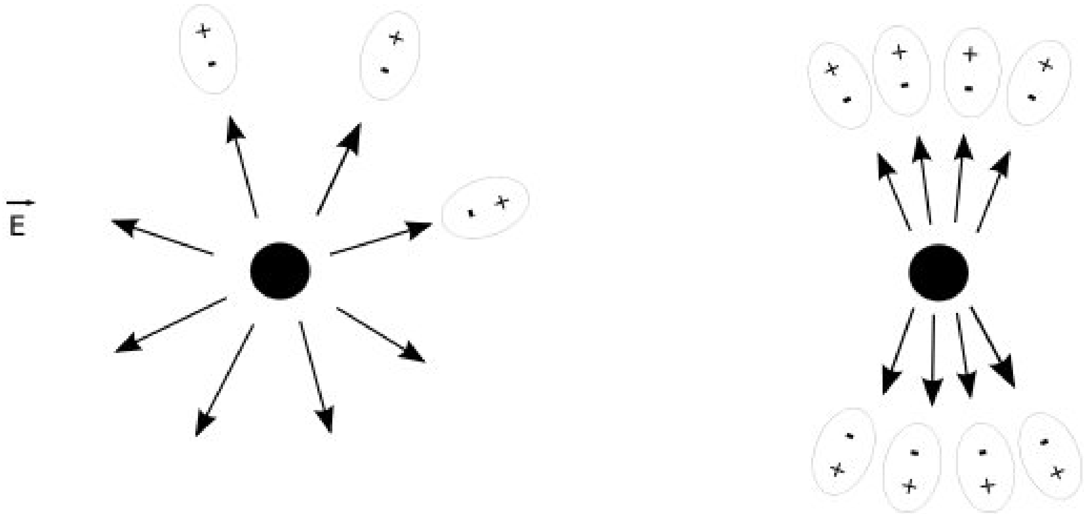
$\frac{K}{A} = \frac{4\pi N_A r_e^2 m_e c^2}{A}$ with classical electron radius $r_e = \frac{e^2}{m_e c^2}$

$T_{max} \approx 2m_e c^2\beta^2\gamma^2$ max. energy transfer in a single collision, for $M \gg m_e$

$I = (10 \pm 1) \cdot Z$ eV mean excitation energy (for elements beyond aluminum)

$\delta/2$ 'density correction' (see next page)

with increasing particle energy → Lorentz contraction of electric field, corresponding to increase of contribution from large *b* with ln $\beta\gamma$



left: for small γ, right: for large γ

but: real media are polarized, effectively cuts off long-range contributions to logarithmic rise, term $-\delta/2$ leads to Fermi plateau high energy limit

$$\frac{\delta}{2} \to \ln \frac{\hbar\omega_p}{I} + \ln \beta\gamma - \frac{1}{2}$$

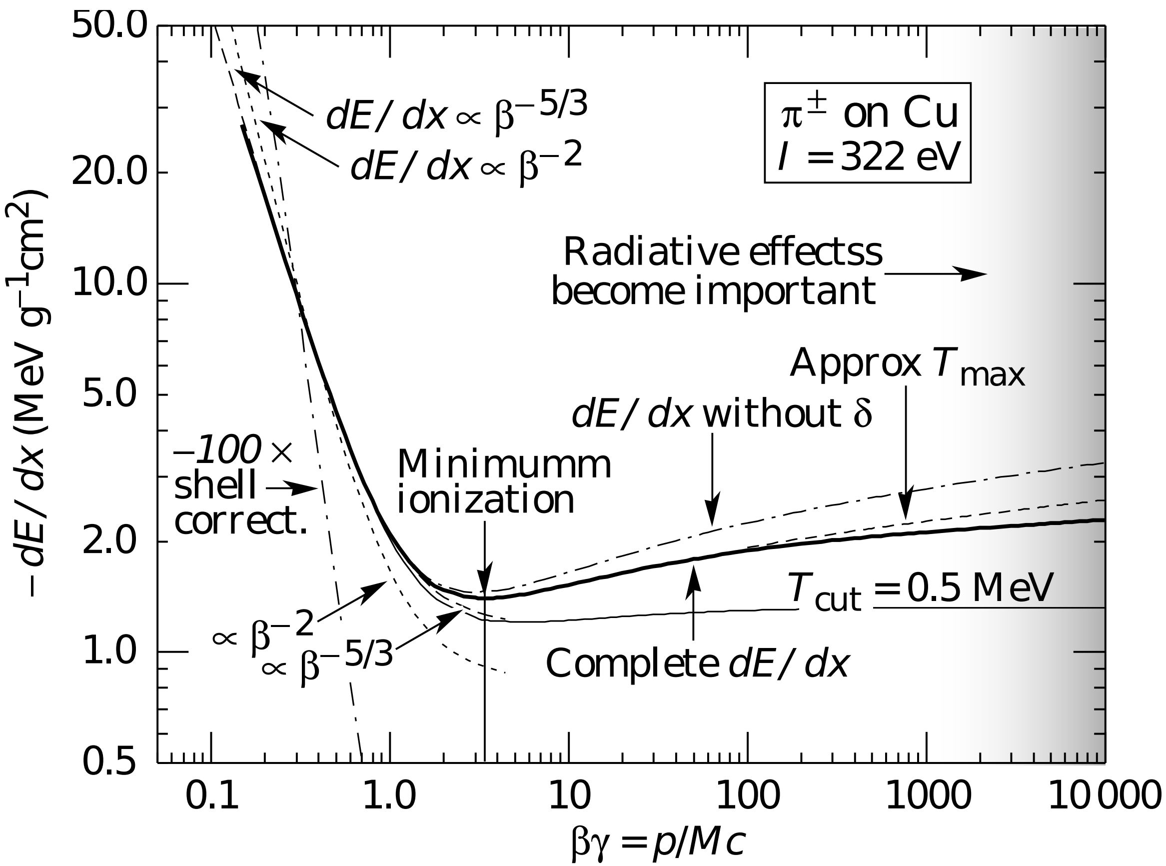
with plasma energy

$$\hbar\omega_p = \sqrt{4\pi n r_e^3 m_e c^2}/\alpha$$

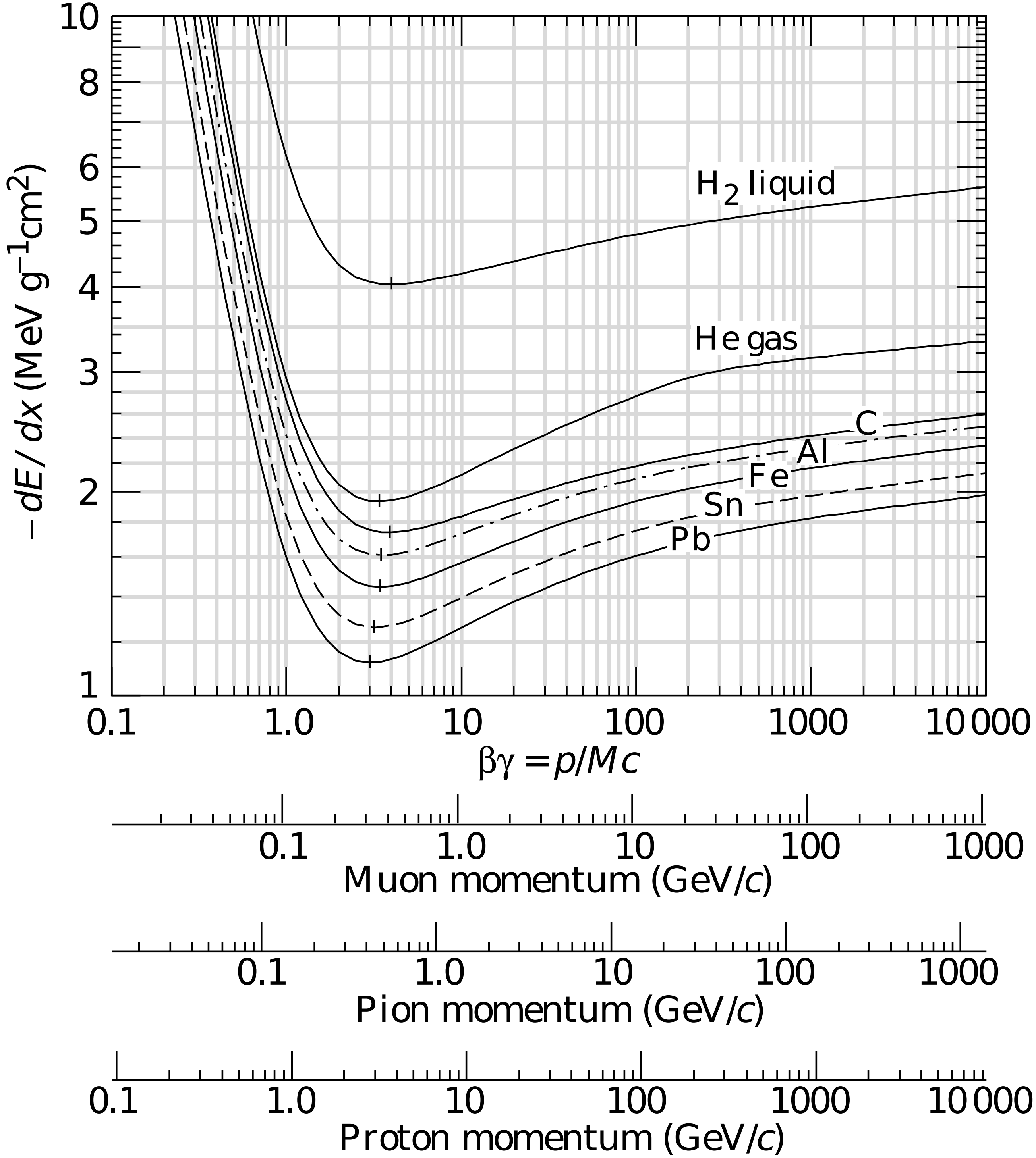
$\Rightarrow -\frac{\mathrm{d}E}{\mathrm{d}x}$ increases more like $\ln \beta\gamma$ than $\ln \beta^2\gamma^2$ and I should be replaced by plasma energy

remark: plasma energy $\propto \sqrt{n}$
i.e. correction much larger for liquids and solids, leading to smaller relativistic rise

one more (small) correction:
'shell correction' $\Rightarrow$ for $\beta c \cong v_e$ capture processes possible



Energy loss rate in copper. The function without the density effect correction is also shown, as is the shell correction and two low-energy approximations.



10
8
6
5
4
3
2
1
$-dE/dx$ (MeV g^{-1}cm^2)
H_2 liquid
He gas
C
Al
Fe
Sn
Pb
0.1
1.0
10
100
1000
10 000
$\beta\gamma = p/Mc$
0.1
1.0
10
100
1000
Muon momentum (GeV/c)
0.1
1.0
10
100
1000
Pion momentum (GeV/c)
0.1
1.0
10
100
1000
10 000
Proton momentum (GeV/c)

General behavior of $\mathrm{d}E/\mathrm{d}x$

- at low energies / velocities decrease as approx. $\beta^{-5/3}$ up to $\beta\gamma > 1$
- broad minimum at

$$\beta\gamma \cong \left.\begin{matrix} 3.5\ (Z=7) \\ 3.0\ (Z=100) \end{matrix}\right\} 1-2\ \frac{\mathrm{MeV\ cm^2}}{g}$$

  'minimally ionizing particle'

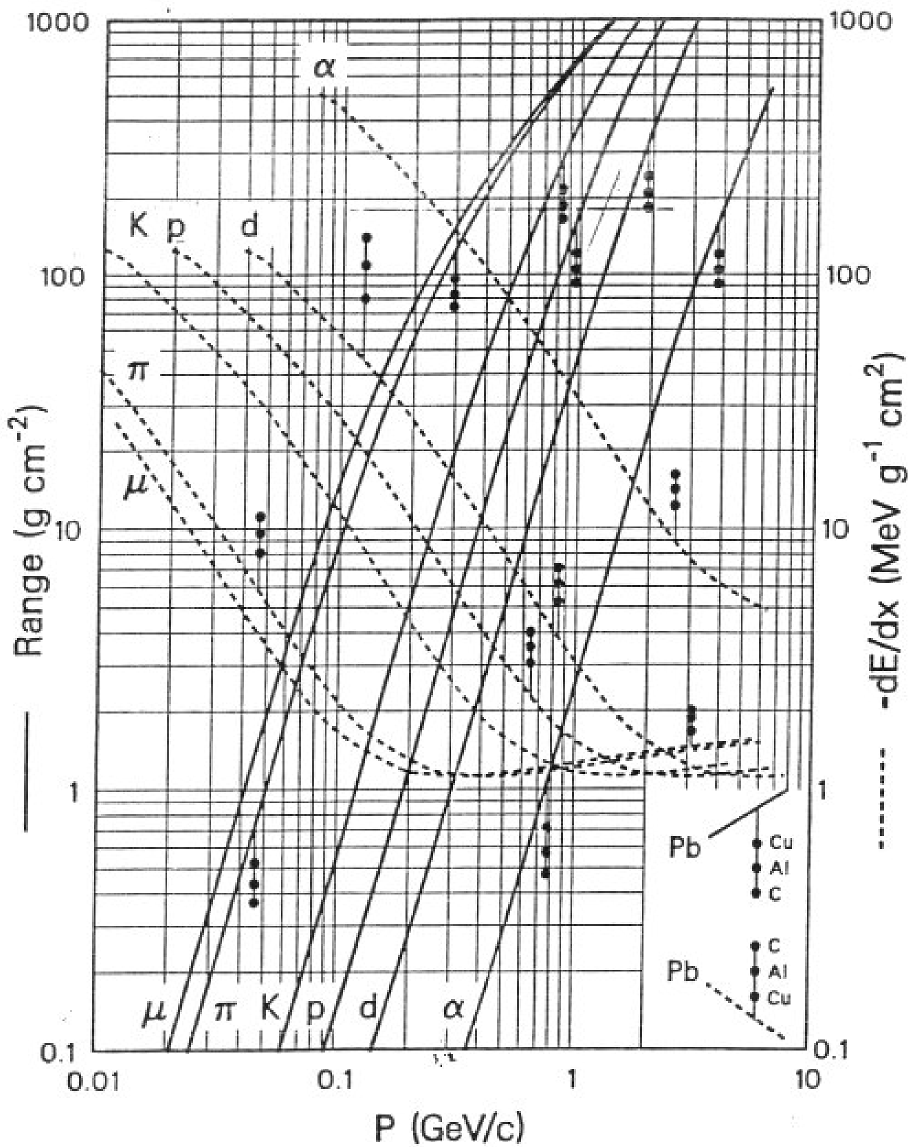
- logarithmic rise and 'Fermi plateau'
  density correction would lead to plateau at high energy, except for energy transfer to few very energetic electrons ($T_{max} \propto \beta^2\gamma^2$). Treated explicitly beyond a certain T_{cut}
  logarithmic rise about 20% in liquids/solids and about 50% in gases
- very low velocities ($v < v_{\mathrm{electron}}$) cannot be treated this way
  for $10^{-3} \leq \beta \leq \alpha \cdot z$: $\quad -\frac{\mathrm{d}E}{\mathrm{d}x} \propto \beta$ non-ionizing, recoil of atomic nuclei
  for $\beta \cdot c \cong v_e$ also capture processes important (shell correction)

Range

Integration over changing energy loss from initial kinetic energy E down to zero

$$R = \int_E^0 \frac{\mathrm{d}E}{\mathrm{d}E/\mathrm{d}x}$$

concept only useful for low energy hadrons (such that $\mathrm{R} \leq \lambda_i$) and for muons



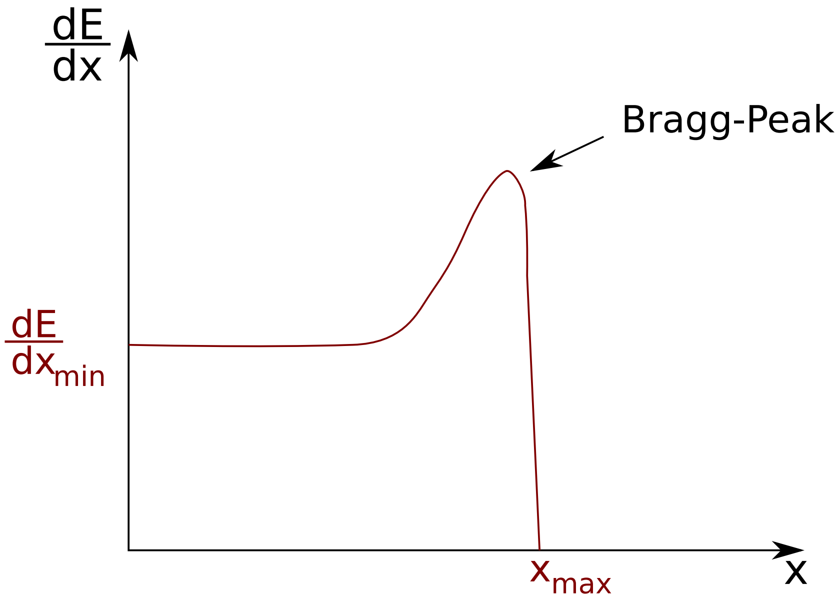
Mean range and energy loss due to ionization in lead, copper, aluminum and carbon

Energy deposition of particles stopped in medium

for $\beta\gamma \simeq 3.5$ $\quad \langle \frac{dE}{dx} \rangle \simeq \frac{dE}{dx}_{\text{min}}$

for $\beta\gamma \leq 3.5$ steep rise $\langle \frac{dE}{dx} \rangle \gg \frac{dE}{dx}_{\text{min}}$ down to very small energies,

then decrease again

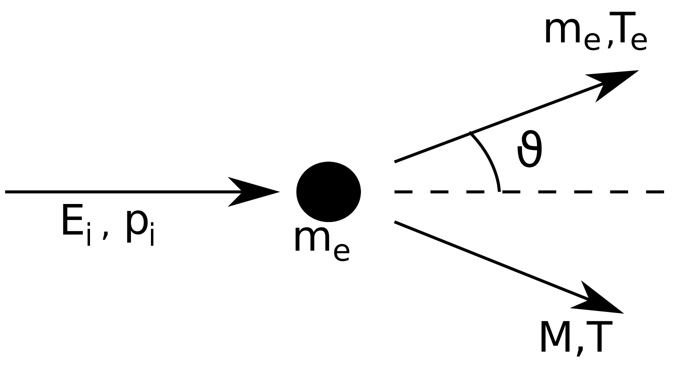


Application: tumor therapy - one can deposit precise dose in well defined depth of material (body), determined by initial beam energy, proton therapy
lately also with heavy ions as ^{12}C; HIT tumor center has started operation in Heidelberg (collaboration DKFZ & GSI)
precise 3D irradiation profile by suitably shaped absorber (custom made for each patient)

Energy loss curve vs depth showing Bragg peak

Delta-Electrons

Electrons liberated by ionization having an energy in excess of some value (e.g. T_{cut}) are called δ-electrons (initial observation in emulsions, hard scattering $\rightarrow$ energetic electrons)



$$T_e = 2m_e \frac{\vec{p}_i^2 \cos^2\theta}{(E_i + m_e)^2 - \vec{p}_i^2 \cos^2\theta}$$

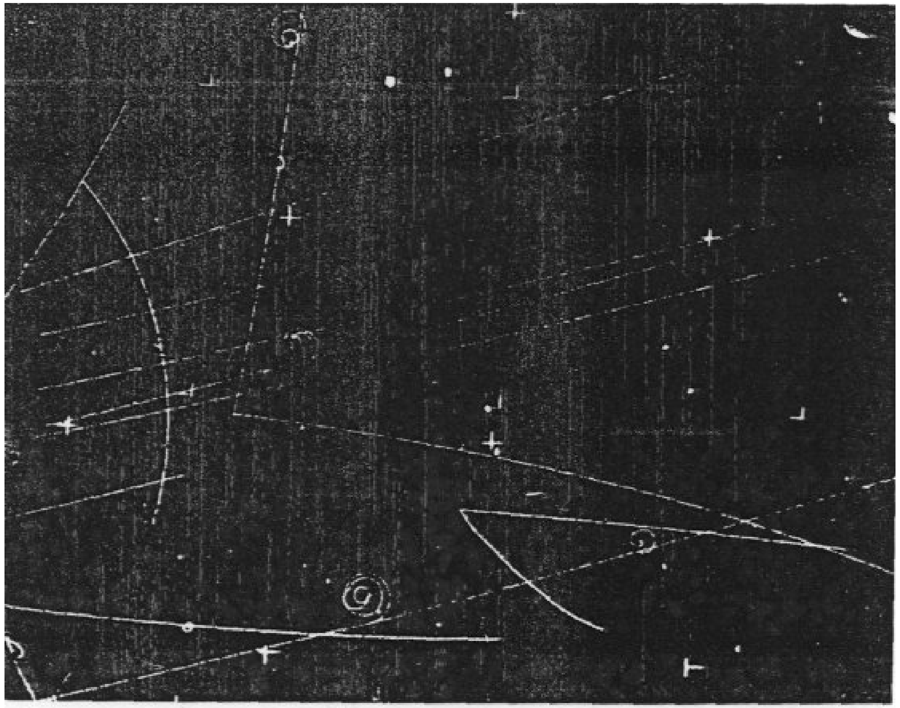
$$\Rightarrow \quad T_e^{\text{max}} = \frac{2m_e \vec{p}_i^2}{(E_i + m_e)^2 - \vec{p}_i^2}$$

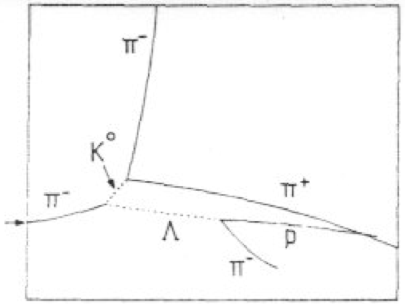
$$\approx \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\frac{m_e \gamma}{M} + \left(\frac{m_e}{M}\right)^2} \quad \text{for} |\vec{p}_i| \gg M, m_e$$

Massive highly relativistic particle can transfer practically all its energy to a single electron!
Probability distribution for energy transfer to a single electron:

$$\frac{d^2 W}{\text{dx dE}} = 2m_e c^2 \pi r_e^2 \frac{z^2}{\beta^2} \cdot \frac{Z}{A} N_A \cdot \rho \cdot \frac{1}{E^2}$$

unpleasant: often this electron is not detected as part of the ionization trail
$\rightarrow$ broadening of track and of energy loss distribution





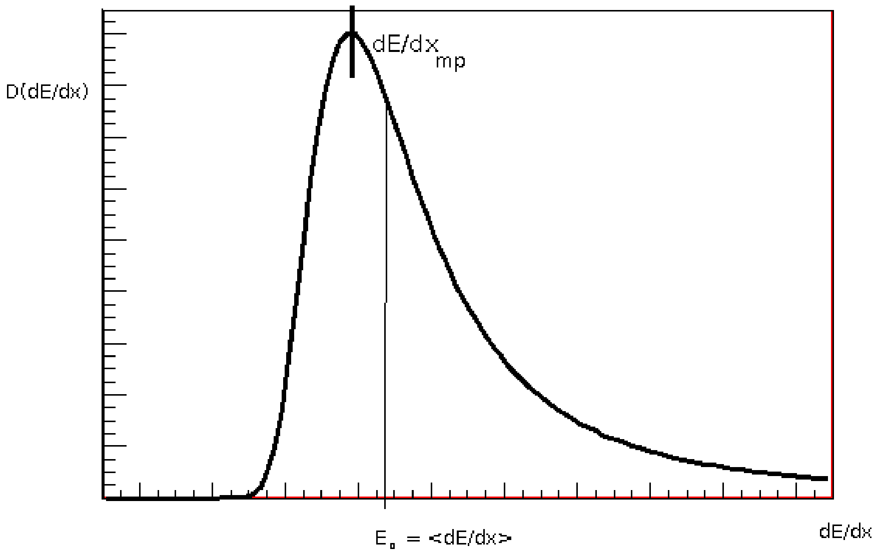
A bubble chamber picture of the associated production reaction $\pi^- + p \to K^0 + \Lambda$. The incoming pion is indicated by the arrow, and the unseen neutrals are detected by their decays $K^0 \to \pi^+ + \pi^-$ and $\Lambda \to \pi^- + p$. This picture was taken in the 10-inch (25 cm) bubble chamber at the Lawrence Berkeley Radiation Laboratory. The spirals are δ electrons.

Energy loss distribution for finite absorber thickness

Energy loss by ionization is distributed statistically: 'energy loss straggling'
Bethe-Bloch formula describes the *mean energy loss*
strong fluctuations about mean: first considered by *Bohr 1915*

$$\sigma^2 = \langle E^2 \rangle - E_0^2 \cong 4\pi n z^2 e^4 \Delta x$$

σ: standard deviation of Gaussian distribution with mean deposited energy E_0 and tail towards high energies due to δ-electrons (actual solution complicated problem)



'Landau distribution' for thin absorber
Vavilov (1957): correction for thicker absorber
approximation:

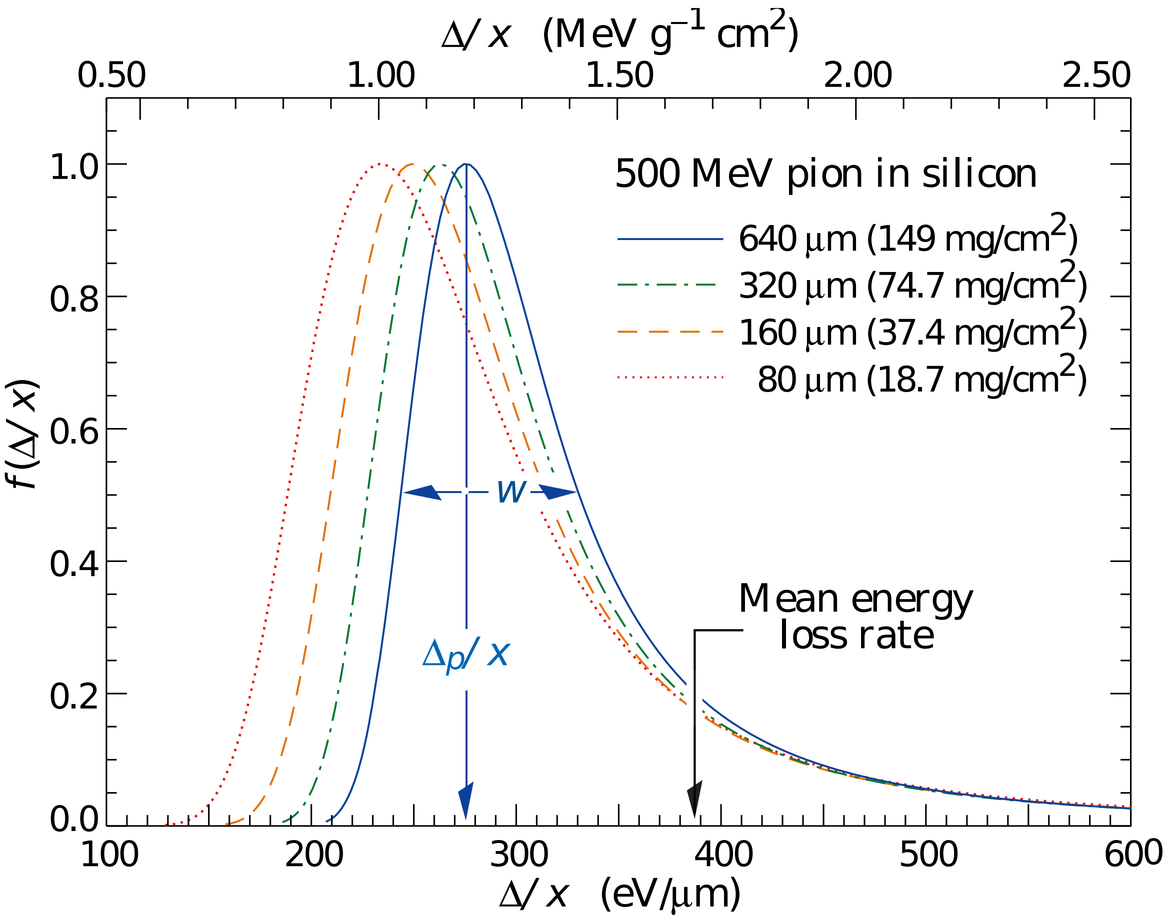
$$D\left(\frac{dE}{dx}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\underbrace{\frac{\frac{dE}{dx} - \frac{dE}{dx}_{\mathrm{mp}}}{\xi}}_{\lambda} + e^{-\lambda}\right)\right)$$

ξ is a material constant

more precise: Allison & Cobb (using measurements and numerical solution)
Ann. Rev. Nuclear Sci. 30 (1980) 253

Energy loss distribution normalized to thickness x
with increasing thickness:

- most probable dE/dx shifts to large values
- relative width shrinks
- asymmetry of distribution decreases



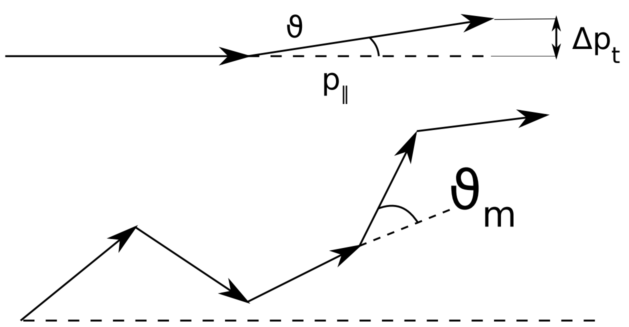
Straggling functions in silicon for 500 MeV pions, normalized to unity at the most probable value Δ_p/x. The width w is the full width at half maximum.

Multiple (Coulomb) scattering

In deriving energy loss by ionization we had considered the

transverse momentum transfer to electron $\Delta p_\perp \simeq \frac{2ze^2}{bv}$

there is a corresponding momentum transfer to primary particle that is losing energy. But here, most visible: deflection by target nuclei due to factor Z



$$\theta \simeq \frac{\Delta p_\perp}{p_{||}} \simeq \frac{\Delta p_\perp}{p}$$
$$= \frac{2Zze^2}{b}\frac{1}{pv}$$

after k collisions

$$\langle \theta_k^2 \rangle = \sum_{m=1}^{k} \theta_m^2 = k\langle \theta^2 \rangle$$

for very thin absorber: single collision dominates, Rutherford scattering $d\sigma/d\Omega \propto \sin^{-4}\theta/2$
for a few collisions: difficult
for many collisions (> 20): statistical treatment ‘Molière theory’ *(G.Z. Molière 1947, 1948)*

Molière theory: averaging over many collisions and integration over b, angular distribution roughly Guassian
the rms deflection angle projected to a plane is

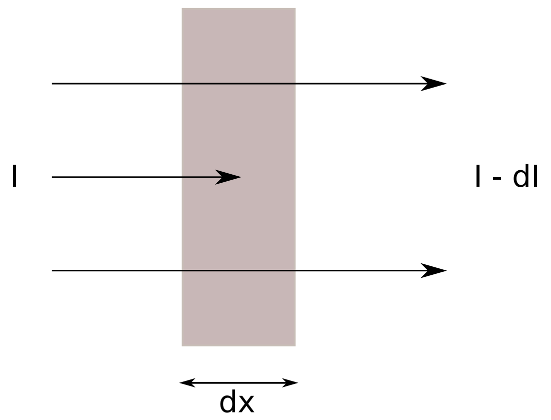
$$\sqrt{\langle\theta^2(x)\rangle} = \theta_{\rm rms}^{\rm plane} = \frac{13.6\ \rm MeV}{\beta pc} z \sqrt{\frac{x}{X_0}} (1 + 0.038 \ln \frac{x}{X_0})$$

X_0: ‘radiation length’, material constant

in 3D: $\theta_{\rm rms}^{\rm space} = \sqrt{2}\ \theta_{\rm rms}^{\rm plane}$ $\quad 13.6 \rightarrow 19.2$

at small momenta this multiple scattering effect limits the momentum and vertex resolution

2.2 Interaction of photons with matter



characteristic for photons: in a single interaction a photon can be removed out of beam with intensity I

$$\begin{aligned} \mathrm{d}I &= -I\mu \,\mathrm{d}x \\ \mu(E,Z,\rho) &\rightarrow \text{absorption coefficient} \end{aligned}$$

Lambert-Beer law of attenuation:

$$I = I_0 \exp -\mu x$$

- mean free path of photon in matter: $\lambda = 1/n\sigma = 1/\mu$

to become independent of state (gaseous, liquid) and reduce variations $\rightarrow$ introduce

$$\text{mass absorption coefficient} \quad \tau = \frac{\mu}{\rho} = N_A \frac{\sigma}{A}$$

example: $E_\gamma = 100$ keV, in iron $Z = 26$, $\lambda = 3$ g/cm^2 or 0.4 cm

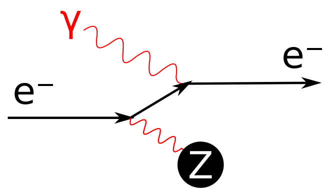
3 processes, in order of growing importance with increasing photon energy E

- photo effect
- Compton scattering (incoherent off an electron)
- pair production (in nuclear field)

also present, but for energy loss not as important

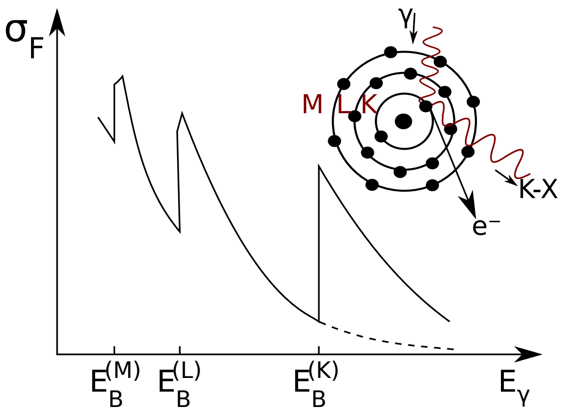
- Rayleigh scattering (coherent, atom neither ionized nor excited) $\gamma + e_b \rightarrow \gamma + e_b$
- photo nuclear absorption $\gamma +$ nucleus $\rightarrow (p \; or \; n) +$ nucleus
- pair production (in electron field)

Photo effect I



$$\gamma + \text{atom} \quad \rightarrow \quad \text{atom}^{+} + e^{-}$$

$E_e = h\nu - I_b$
$h\nu$: γ energy
I_b: binding energy of electron; K, L, M absorption edges
since binding energy strongly Z-dependent, strong Z-dependence of cross sections



$$I \ll E_\gamma \ll mc^2 \qquad \sigma_{Ph} = \alpha \pi a_b Z^5 \left(\frac{I_0}{E_\gamma} \right)^{\frac{7}{2}}$$

$$a_b = 0.53 \cdot 10^{-10}\text{m} \qquad I_0 = 13.6 \text{ eV}$$

for $E_\gamma = 0.1$ MeV

$\sigma_{\text{Ph}}(Fe) = 29$ b

$\sigma_{\text{Ph}}(Pb) = 5$ kb

for $E_\gamma \gg mc^2 \quad \sigma_{\text{Ph}} \propto \dfrac{Z^5}{E_\gamma}$

Photo effect II

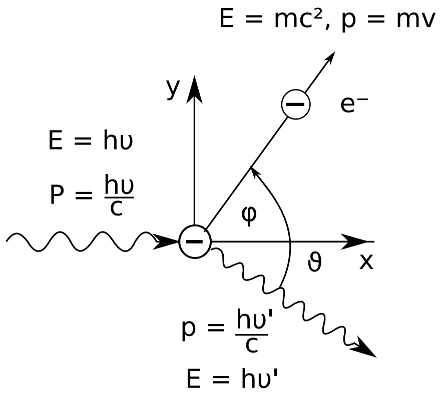
The excited atom emits either

$$\text{char. X-rays} \quad \text{atom}_{\text{K}}^{+*} \rightarrow \text{atom}_{\text{LM}}^{+*} + \gamma$$

$$\text{or Auger electrons} \quad \text{atom}_{\text{K}}^{+*} \rightarrow \text{atom}_{\text{LM}}^{++*} + e^{-}$$

Auger electrons have small energy that is deposited locally
X-ray $\rightarrow$ photo effect again, range may be significant
this ‘fluorescence yield’ increases with Z.

Compton scattering



$$\frac{1}{E_{\gamma'}} - \frac{1}{E_\gamma} = \frac{1}{m_e c^2}(1-\cos\theta) \leq \frac{2}{m_e c^2}$$

recoil of electrons

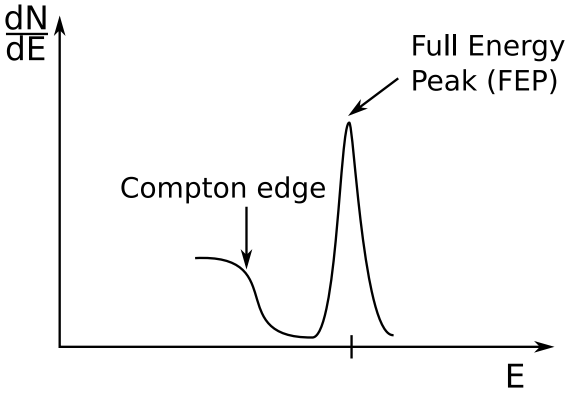
$$T_e = \frac{\frac{E_\gamma}{m_e c^2}(1-\cos\theta)}{\frac{E_\gamma}{m_e c^2}(1-\cos\theta)+1} E_\gamma$$

$$\left(\frac{T_e}{E_\gamma}\right)_{\text{max}} = \frac{E_\gamma}{m_e c^2}\frac{2}{1+\frac{2E_\gamma}{m_e c^2}}$$

and $\Delta E = E_\gamma - T_{e,max} = \frac{E_\gamma}{1+\frac{2E_\gamma}{m_e c^2}} \rightarrow \frac{m_e c^2}{2}$ for $E_\gamma \gg m_e c^2$

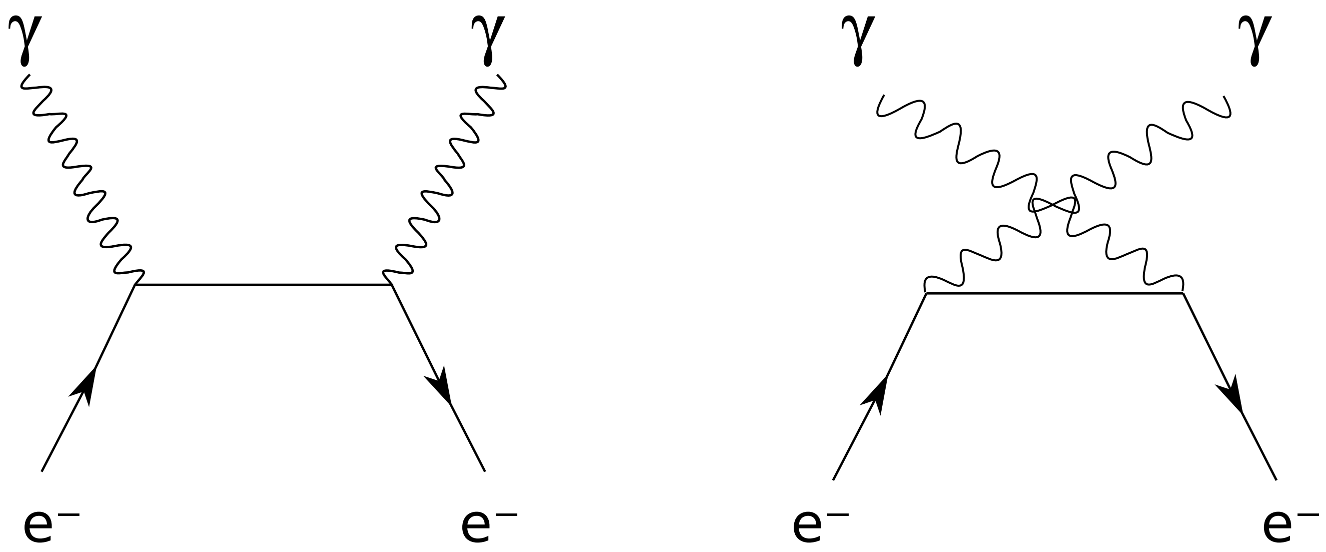
gives rise to 'Compton edge' in measured γ spectrum

Compton edge: in case scattered photon is not absorbed in detector, a minimal amount of energy is missing from the 'full energy peak' (asymptotically half electron rest mass)



FEP = 'full energy peak':
photo effect and Compton effect with scattered photon absorbed
intensity depends on detector volume

Cross section: calculation in QED - *1929 O. Klein and Y. Nishina*



Klein-Nishina diagrams

- order of magnitude given by Thomson cross section

$$\sigma_{\mathrm{Th}} = \frac{8\pi}{3} r_e^2 = 0.66 \text{ b}$$

$$\gamma + e^- \rightarrow \gamma + e^- \qquad E_\gamma \rightarrow 0$$

Compton cross section

$$E_\gamma \ll m_e c^2 \qquad \sigma_c = \sigma_{\mathrm{Th}} \left(1 - 2\mathcal{E}\right)$$

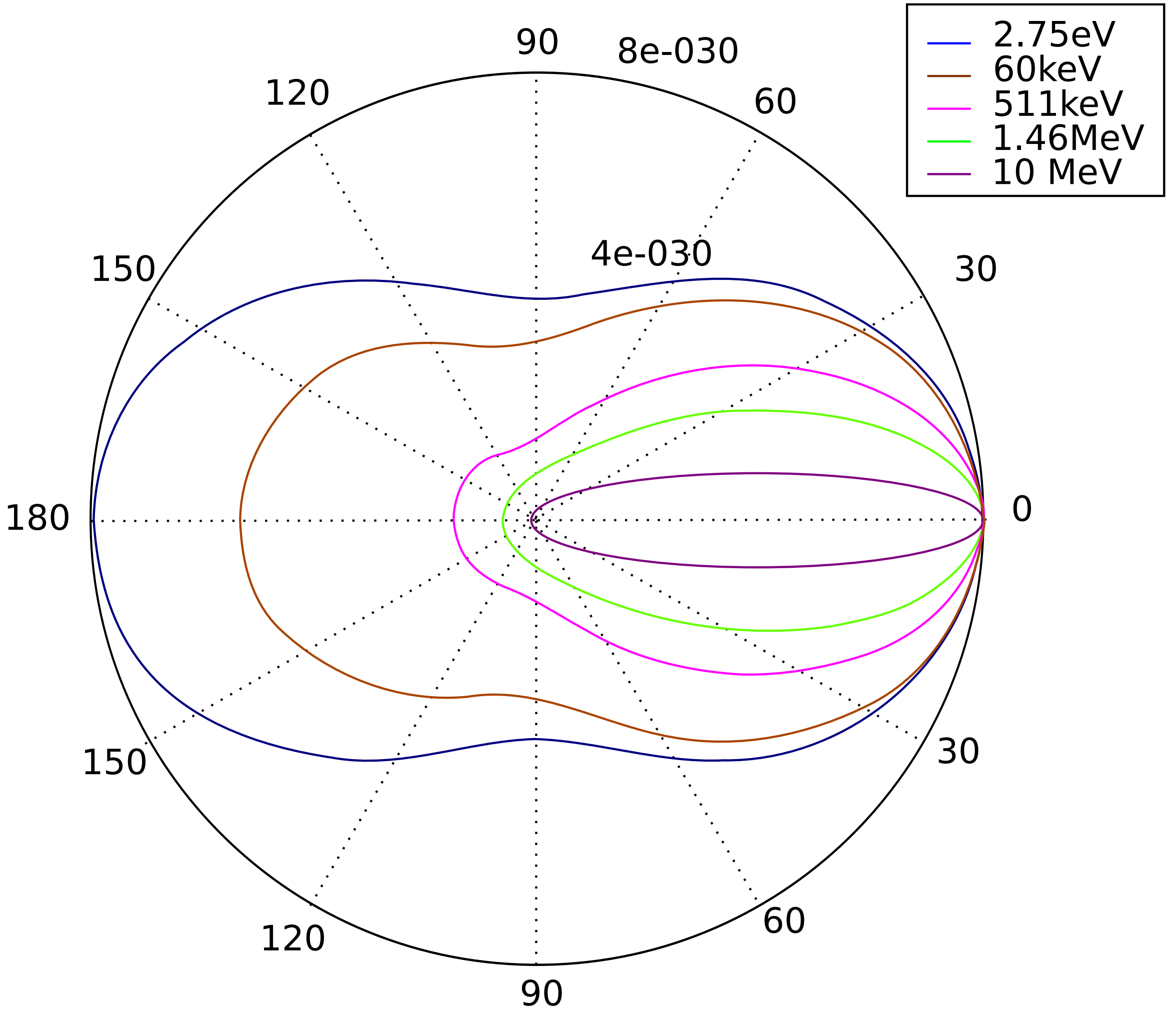
$$E_\gamma \gg m_e c^2 \qquad \sigma_c = \frac{3}{8} \sigma_{\mathrm{Th}} \frac{1}{\mathcal{E}} \left(\ln 2\mathcal{E} + \frac{1}{2} \right)$$

with $\mathcal{E} = \frac{E_\gamma}{m_e c^2}$

angular distribution from QED: Klein-Nishina formula

$$\frac{d\sigma_c}{d\Omega} = \frac{r_e^2}{2}\frac{1}{(1+\mathcal{E}(1-\cos\theta))^2}\left[1+\cos\theta+\frac{\mathcal{E}^2(1-\cos\theta)^2}{1+\mathcal{E}(1-\cos\theta)}\right] \qquad \mathcal{E}=\frac{E_\gamma}{m_e c^2}$$

angular distribution of scattered photon for high γ-energies forward peaked



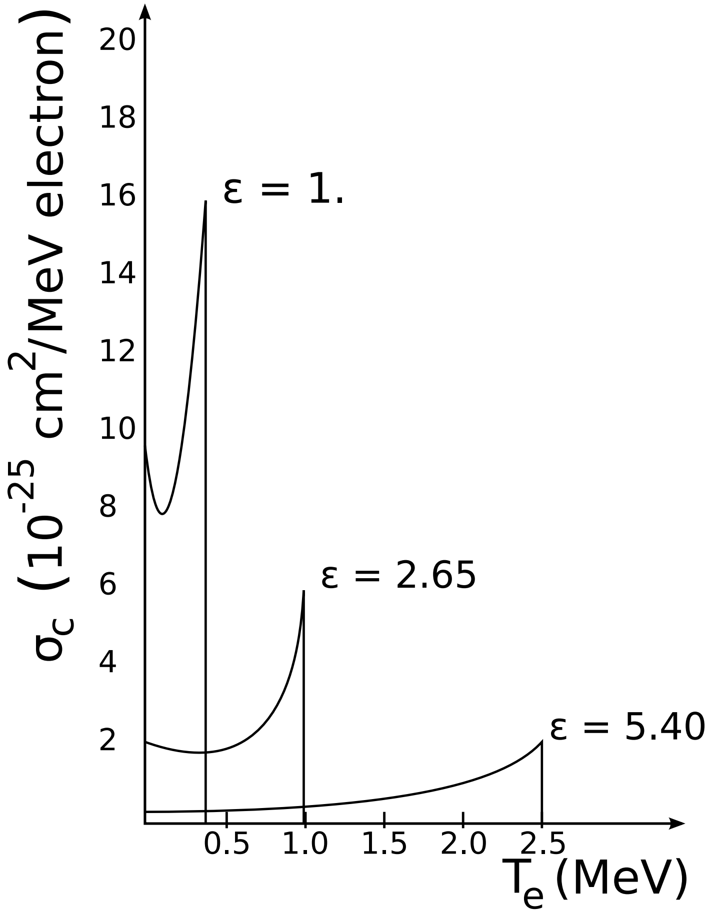
Spectrum of recoil electrons from Klein-Nishina formula after angular integration

$$\frac{d\sigma_c}{dT_e} = \frac{\pi r_e^2}{m_e c^2 \mathcal{E}^2}\left[2 + \frac{s^2}{\mathcal{E}(1-s)^2} + \frac{s}{1-s}\left(s - \frac{2}{\mathcal{E}}\right)\right]$$

$$\mathcal{E} = \frac{E_\gamma}{m_e c^2}$$

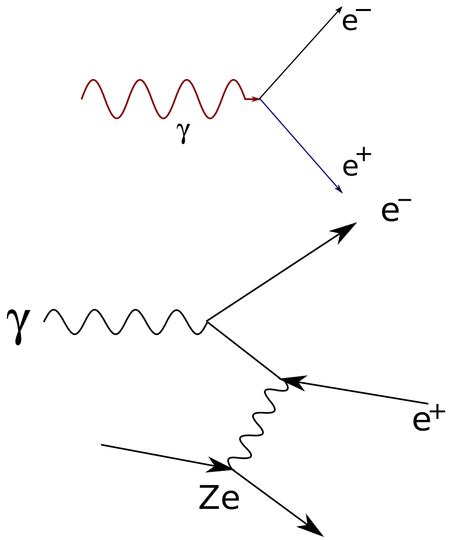
$$s = \frac{T_e}{E_\gamma}$$

$$T_e^{max} = E_\gamma\left(1 - \frac{m_e c^2}{2E_\gamma}\right) \qquad \text{for large } E_\gamma$$



mass absorption coefficient $\frac{\mu_c}{\rho} = \frac{N_A}{A} Z\sigma_c \propto \frac{Z \ln E_\gamma}{E_\gamma}$

Pair production (Bethe-Heitler process) I



not possible in free space but in Coulomb field of atomic nucleus to absorb recoil

energy threshold

$$E_\gamma \geq 2 m_e c^2 + 2 \frac{m_e^2 c^4}{m_K c^2}$$

Cross section: for low energies, impact parameters small, photon sees 'naked' nucleus
with increasing E_γ, range of impact parameter b is growing up to $b \geq a_{\text{atom}}$, complete screening $\rightarrow$
saturation of cross section for $E_\gamma \gg m_e c^2$

$$\sigma_{\text{pair}} = 4 Z^2 \alpha r_e^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) \simeq \frac{7}{9} \underbrace{4 \alpha r_e^2 Z^2 \ln \frac{183}{Z^{1/3}}}_{(A/N_A) X_0}$$

X_0: 'radiation length' (g/cm^2), to obtain length (cm): $\rho \, X_0$

mass absorption coeff. $\quad \frac{\mu_{\text{p}}}{\rho} = \frac{N_A}{A} \sigma_p \simeq \frac{7}{9} \frac{1}{X_0}$

Pair production (Bethe-Heitler process) II

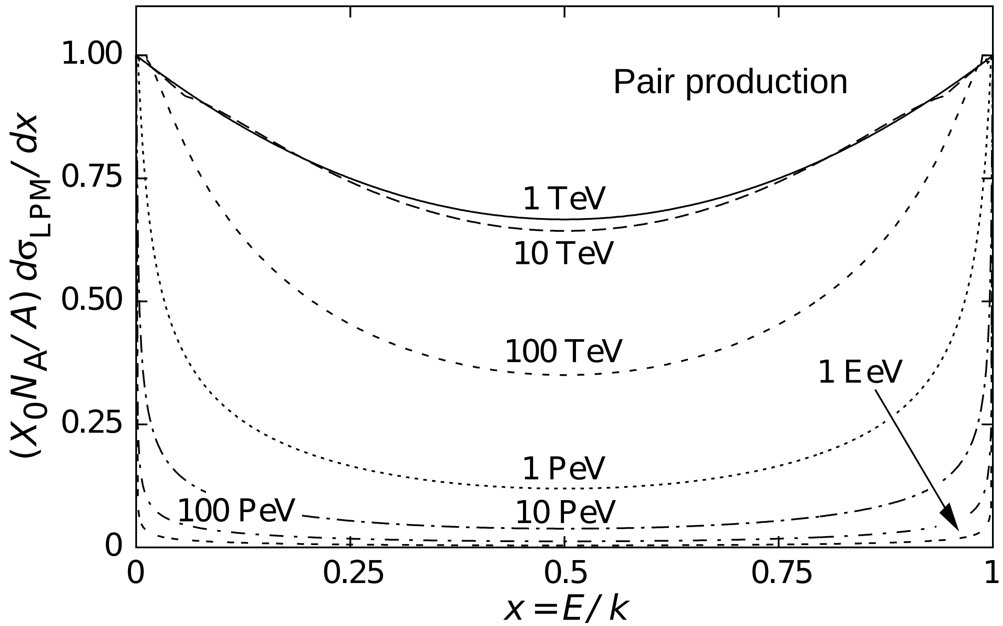
	ρ (g/cm^3)	X_0 (cm)
liq H_2	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
air	0.0012	30 420

the angular distribution of produced electrons is narrow in forward cone with opening angle of $\theta = m_e/E_\gamma$

definition of radiation length X_0 in terms of energy loss of electron by bremsstrahlung see below

Fractional electron (or positron) energy x

$x = E/k$ = electron energy/photon energy
cross section necessarily symmetric between x and $(1 - x)$



at ultrahigh energies new effect - **Landau Pomeranchuk Migdal effect**: quantum mechanical interference between amplitudes from different scattering centers; relevant scale **formation length** - length over which highly relativistic electron and photon split apart.

interference (generally) destructive $\rightarrow$ reduced cross section for a given, very high photon energy k: if electron (or positron) energy are above some value given by

$E(k - E) > kE_{LPM} \Rightarrow$ effect visible, cross section reduced

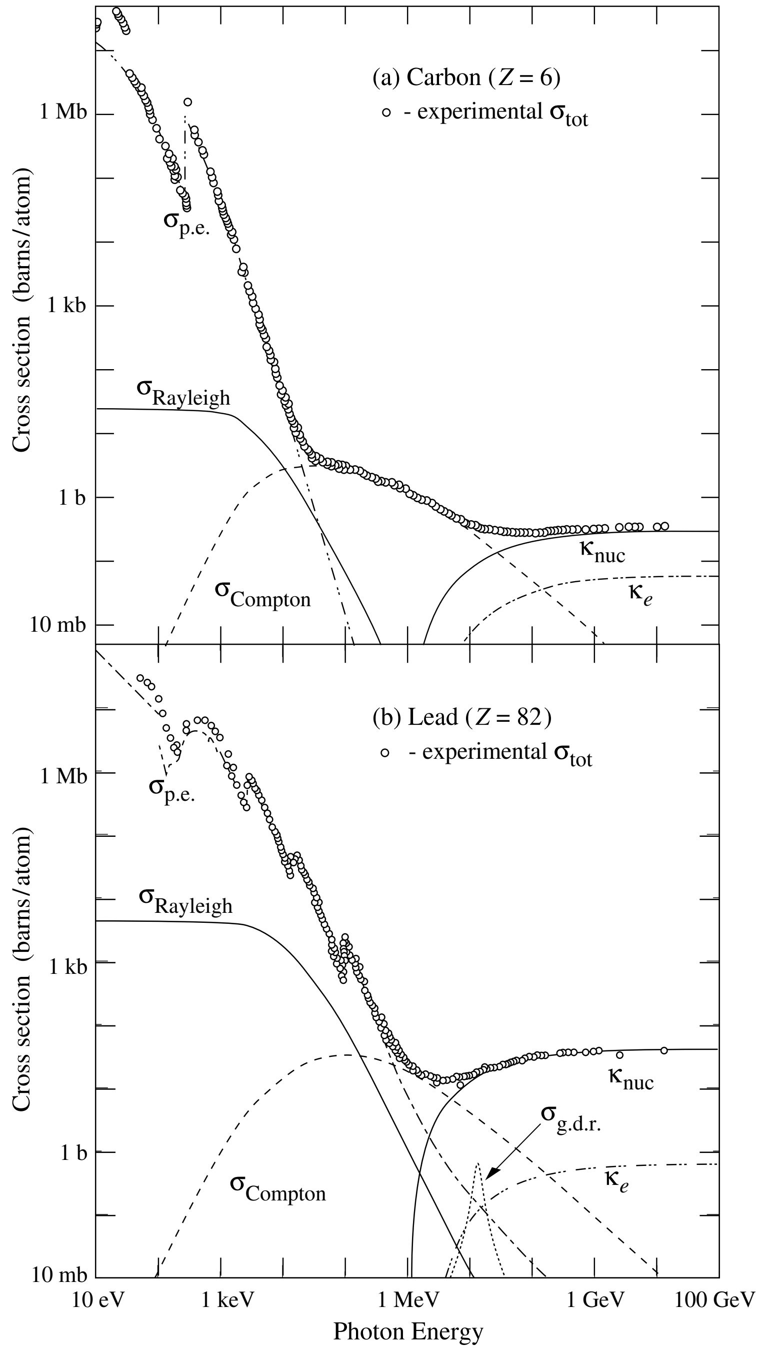
$E_{LPM} = 7.7\, X_0$ TeV/cm

e.g. for Pb $E_{LPM} = 4.3$ TeV
take $k = 100$ TeV, suppression for $E > 4.5$ TeV or $x = 0.045$
(see also bremsstrahlung below)

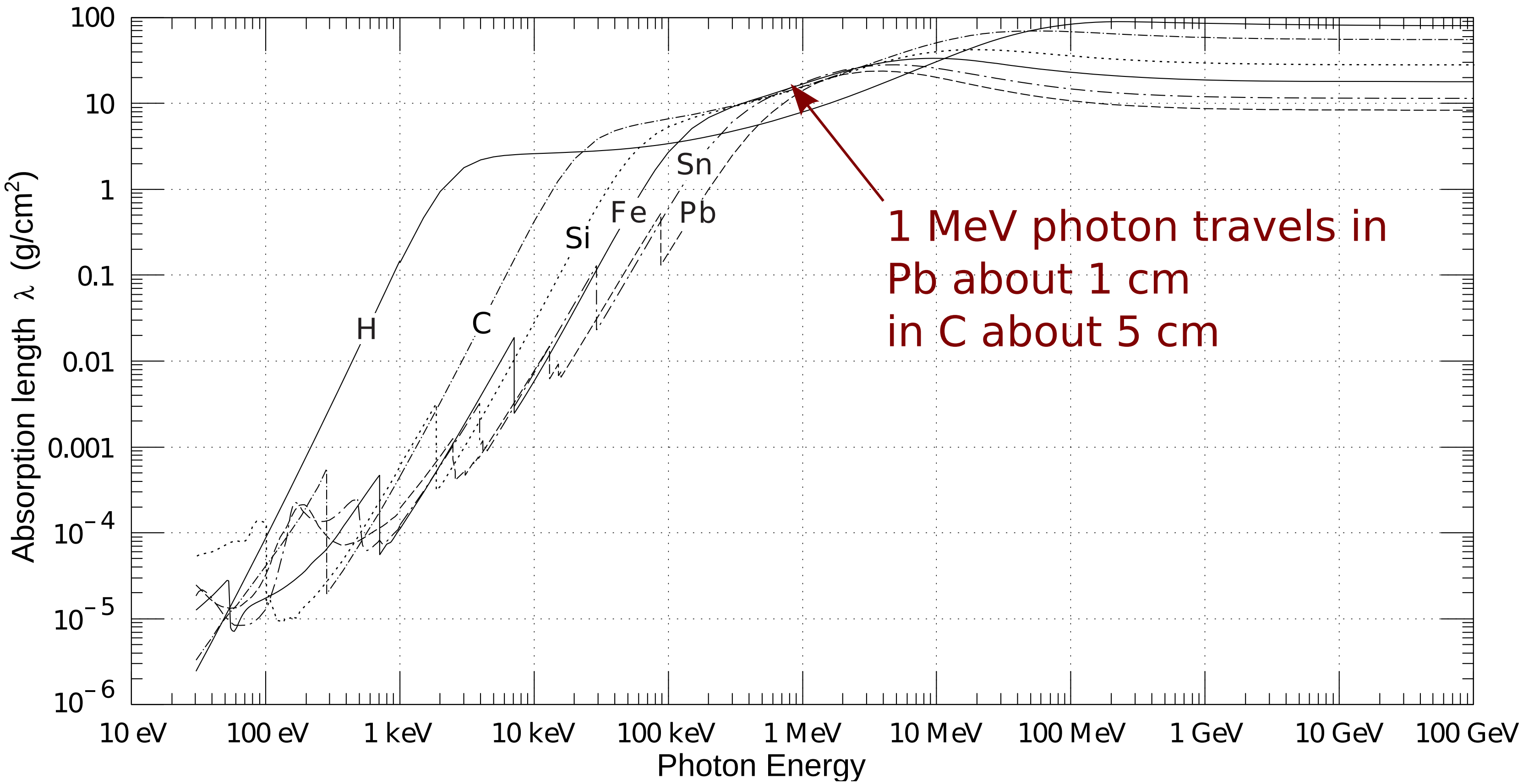
Total absorption coefficient

$$\sigma_{tot} = \sigma_{Ph} + \sigma_c + \sigma_p$$
$$\mu = \mu_{Ph} + \mu_c + \mu_p$$
$$\mu_i = n\sigma_i = \frac{N_A \rho}{A}\sigma_i$$

photon total cross sections as a function of energy in carbon and lead



The photon mass attenuation length λ

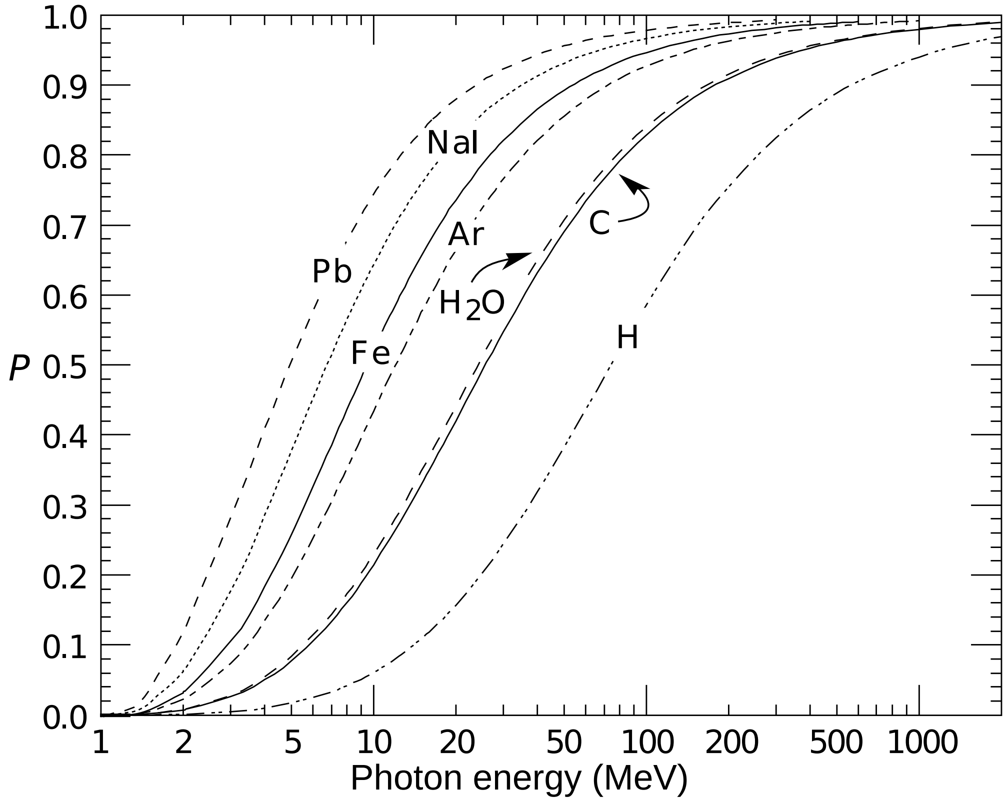


The photon mass attenuation length (or mean free path) $\lambda = \rho/\mu$ for various elemental absorbers as a function of photon energy. The mass attenuation coefficient is μ/ρ, where ρ is the density. The intensity I remaining after traversal of thickness t (in mass/unit area) is given by $I = I_0 \exp -t/\lambda$. The accuracy is a few percent. For a chemical compound or mixture, $1/\lambda_{\mathrm{eff}} \approx \sum_{\mathrm{elements}} w_Z/\lambda_Z$, where w_Z is the proportion by weight of the element with atomic number Z. Since coherent processes are included, not all processes result in energy deposition.

with increasing photon energy pair creation becomes dominant

for Pb beyond 4 MeV
for H beyond 70 MeV

Probability P that a photon interaction will result in conversion to an e^+e^- pair. Except for a few-percent contribution from photonuclear absorption around 10 or 20 MeV, essentially all other interactions in this energy range result from Compton scattering off an atomic electron. For a photon attenuation length λ, the probability that a given photon will produce an electron pair (without first Compton scattering) in thickness t of absorber is $P[1 - \exp(-t/\lambda)]$



2.3 Interaction of electrons
Energy loss by ionization

Modification of **Bethe-Bloch equation**

m_e small $\rightarrow$ deflection important

identical particles $\rightarrow$ $W_{max} = T/2$

quantum mechanics: after scattering, no way to distinguish between incident electron and electron from ionization.

$\rightarrow$ for relativistic electrons

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 4\pi N_A r_e^2 m_e c^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{\gamma m_e c^2 \beta \sqrt{\gamma - 1}}{\sqrt{2} I} + F(\gamma) \right]$$

considers kinematics of $e^- + e^-$ collision and screening

positrons: for small energies energy loss a bit larger (annihilation)
also: they are not identical particles

remark: for same β the energy loss by ionization for e^- and p equal within 10%

Ionization yield (also valid for heavy particles)

Mean energy loss by ionization and excitation can be transformed into mean number of electron-ion pairs produced along track of ionizing particle
total ionization = primary ionization + secondary ionization due to energetic primary electron

$$n_t = n_p + n_s$$

mean energy W to produce an electron-ion pair

$$n_t = \frac{\Delta E}{W}$$

$W >$ ionization potential I_0 since

- also ionization of inner shells
- excitation that may not lead to ionization

$$n_t \approx (2-6) n_p$$

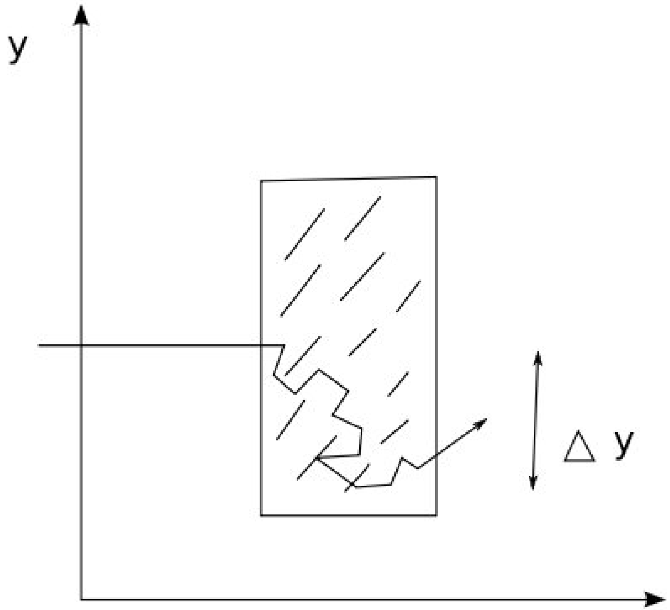
typical values

	I_0 (eV)	W (eV)	n_p (cm^{-1})	n_t (cm^{-1})
H_2	15.4	37	5.2	9.2
N_2	15.5	35	10	56
0_2	12.2	31	22	73
Ne	21.6	36	12	39
Ar	15.8	26	29	94
Kr	14.0	24	22	192
Xe	12.1	22	44	307
CO_2	13.7	33	34	91
CH_4	13.1	28	16	53
		in gases $\approx$ 30 eV	diff. due to ρ and Z	diff. due to electronic struct.

Solid state detectors:

	W (eV)	
Si	3.6	additional factor 10^3 due to density
Ge	2.85	$\rightarrow$ many more electron ion pairs!

Lateral straggeling



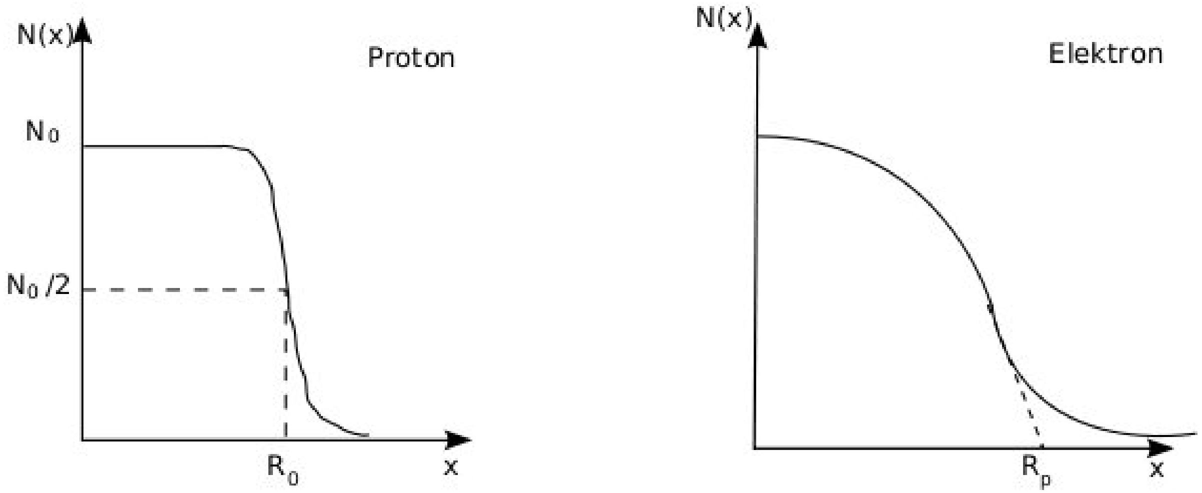
Important difference: electron - heavy particle
heavy particle: track more or less straight
electron: can be scattered into large angles
pathlength $\gg$ range

transverse deflection of an electron of energy $E = E_c$ (see below) after traversing distance X_0 (one radiation length)

$$\Delta y = R_M = \frac{21\ \text{MeV}}{E_c} X_0 \qquad \text{'Molière radius'}$$

	E_c (MeV)	R_M (cm)	X_0 (cm)
Pb	7.2	1.6	0.56
scint.	80	9.1	42
NaI	12.5	4.4	2.6

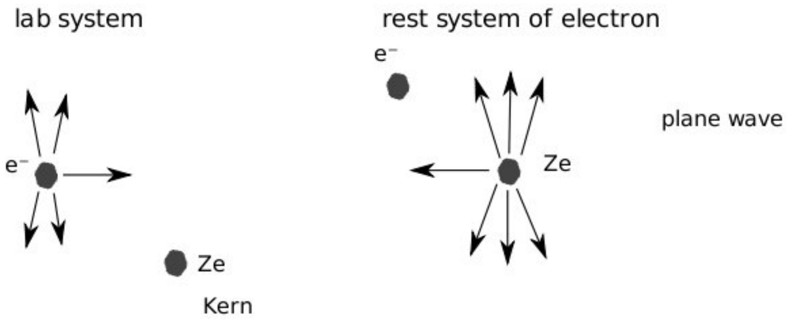
Consequence of lateral straggeling:
range of electrons much more diffuse in comparison to protons



R_p: extrapolated range
rule of thumb: $R_p \left(\frac{\text{g}}{\text{cm}^2}\right) = 0.52\ T - 0.09 \quad \text{for} \quad T = 0.5 - 3\ \text{MeV}$

2.4 Bremsstrahlung

QED process (Fermi 1924, Weizsäcker-Williams 1938)



electron is hit by plane electromagnetic wave (for large v)
$E \perp B$ and both $\perp v$; quanta are scattered by electron and appear as real photons

note: graph closely related to pair creation

in Coulomb field of nucleus electron is accelerated
amplitude of electromagnetic radiation $\propto$ acceleration $\propto 1/m_e c^2$

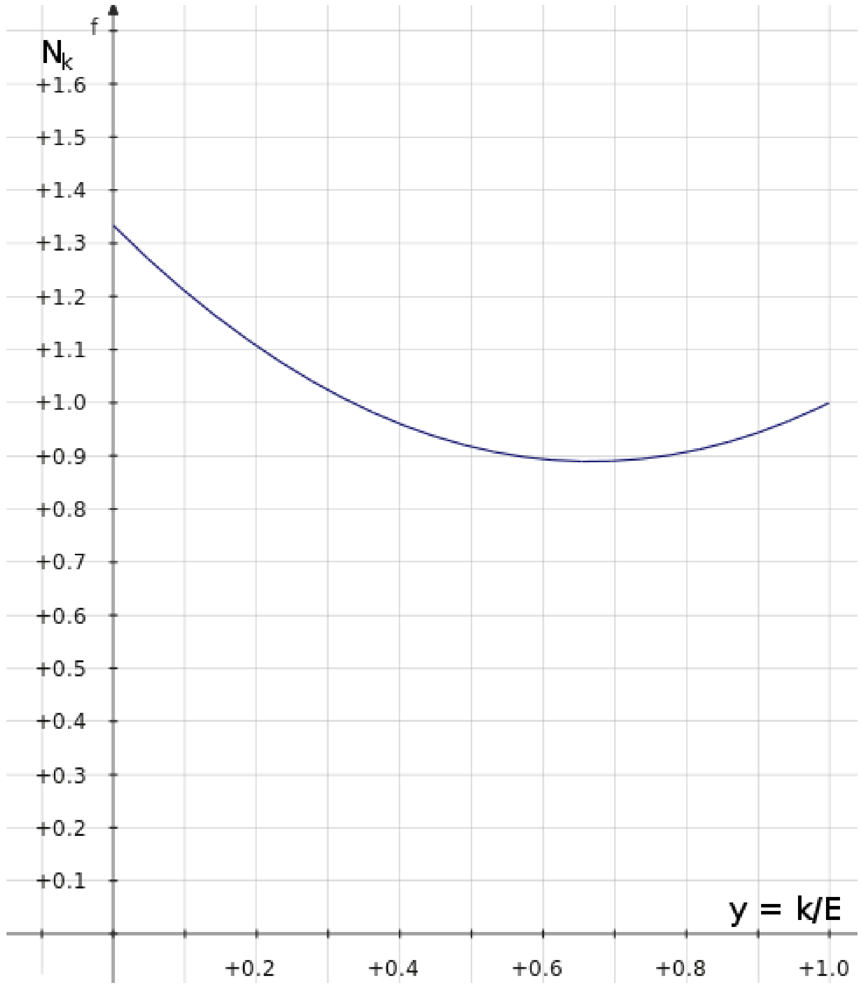
$$\sigma_{\text{brems}} \propto \frac{Z^2\alpha^3}{(m_e c^2)^2}$$

spectrum of photons $\propto \frac{1}{k}$, approximately

$$\frac{d\sigma}{dk} \simeq \frac{A}{X_0 N_A}\frac{1}{k}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$$

with $y = k/E$ (corrections later)
$\rightarrow$ normalized bremsstrahlung cross section (in number of photons per radiation length)

$$N_k = \frac{X_0 N_A}{A} k \frac{d\sigma}{dk} = \left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$$



from this compute N_γ in interval dk and from this energy loss

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{\frac{1}{3}}}$$

remark:

$$r_e^2 = \frac{e^4}{(m_e c^2)^2} = \alpha^2 \left(\frac{\hbar c}{m_e c^2}\right)^2 \leftrightarrow -\frac{\mathrm{d}E}{\mathrm{d}x} \propto \frac{\alpha^3}{(m_e c^2)^2}$$

considering also interaction with electrons in atom

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 4\alpha N_A \frac{Z(Z+1)}{A} r_e^2 E \ln \frac{287}{Z^{\frac{1}{2}}} = \frac{E}{X_0}$$

so $E(x) = E_0 \exp(-x/X_0)$

$\Rightarrow$ X_0 **is distance over which energy decreases to** $1/e$ **of initial value**

for mixtures:

$$\frac{1}{X_0} = \sum_i \frac{w_i}{X_{0i}}$$ w_i weight fraction of substance i

Critical energy

$$-\frac{\mathrm{d}E}{\mathrm{d}x} \quad \text{by ionization} \quad \propto \ln E$$
$$-\frac{\mathrm{d}E}{\mathrm{d}x} \quad \text{by bremsstrahlung} \quad \propto E$$

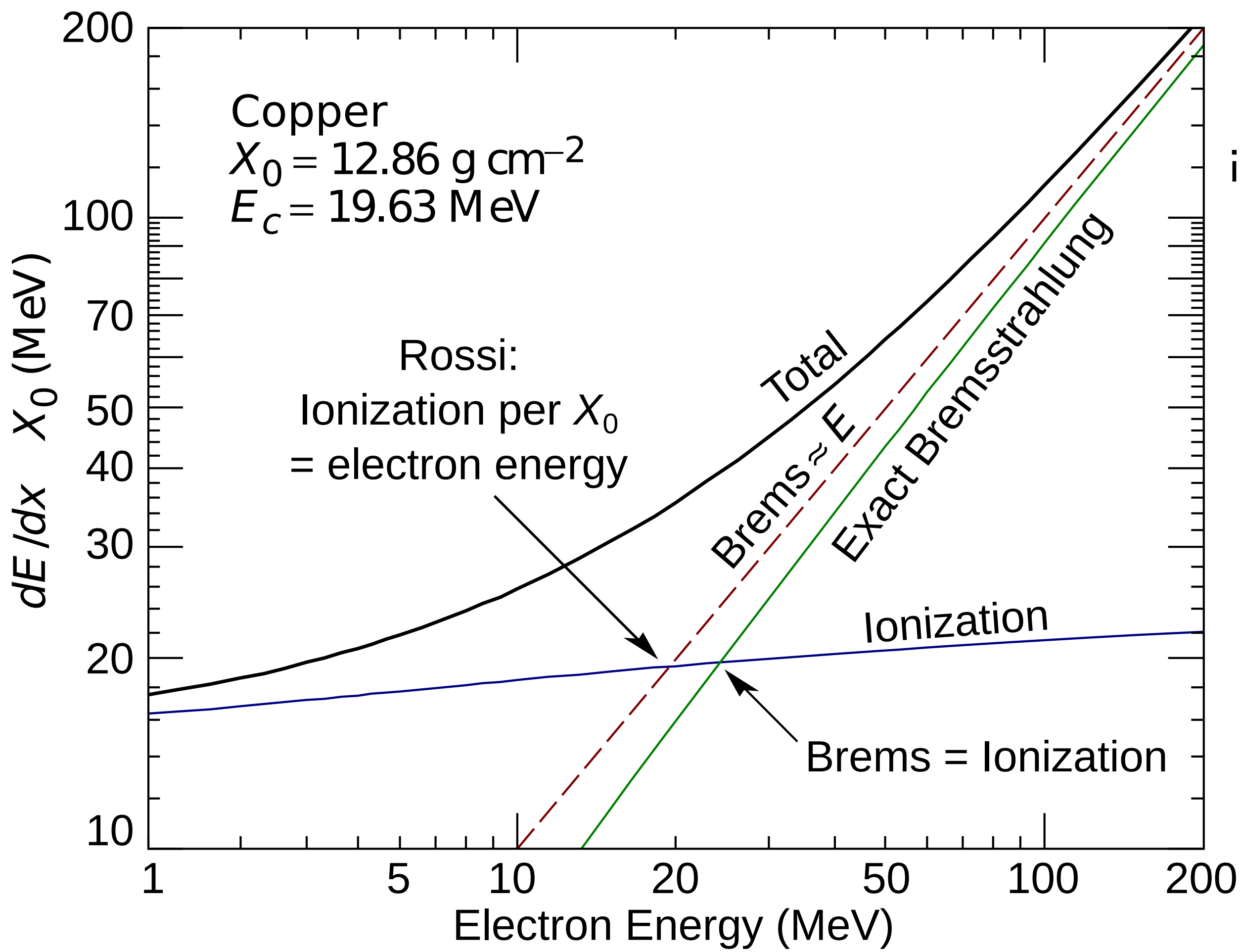
$\rightarrow$ existence of crossing point beyond which bremsstrahlung dominates

at critical energy E_c $\quad \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{ion}} = \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{brems}}$

for electrons and $Z > 13$ $\quad E_c = \frac{580}{Z}$ MeV

for muons negligible $\quad E_c = \frac{24}{Z}$ TeV $\quad$ due to $\left(\frac{m_\mu}{m_e}\right)^2 = 4.3 \cdot 10^4$

Critical energy for electrons in Cu

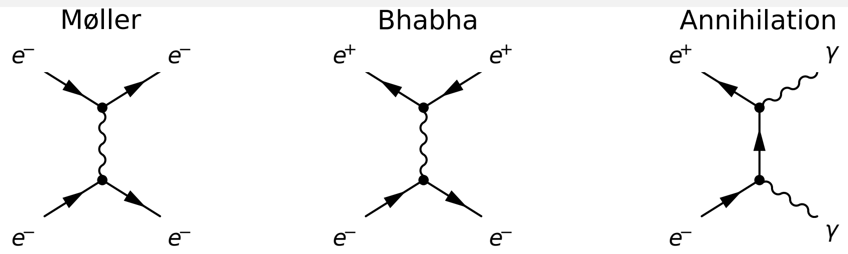


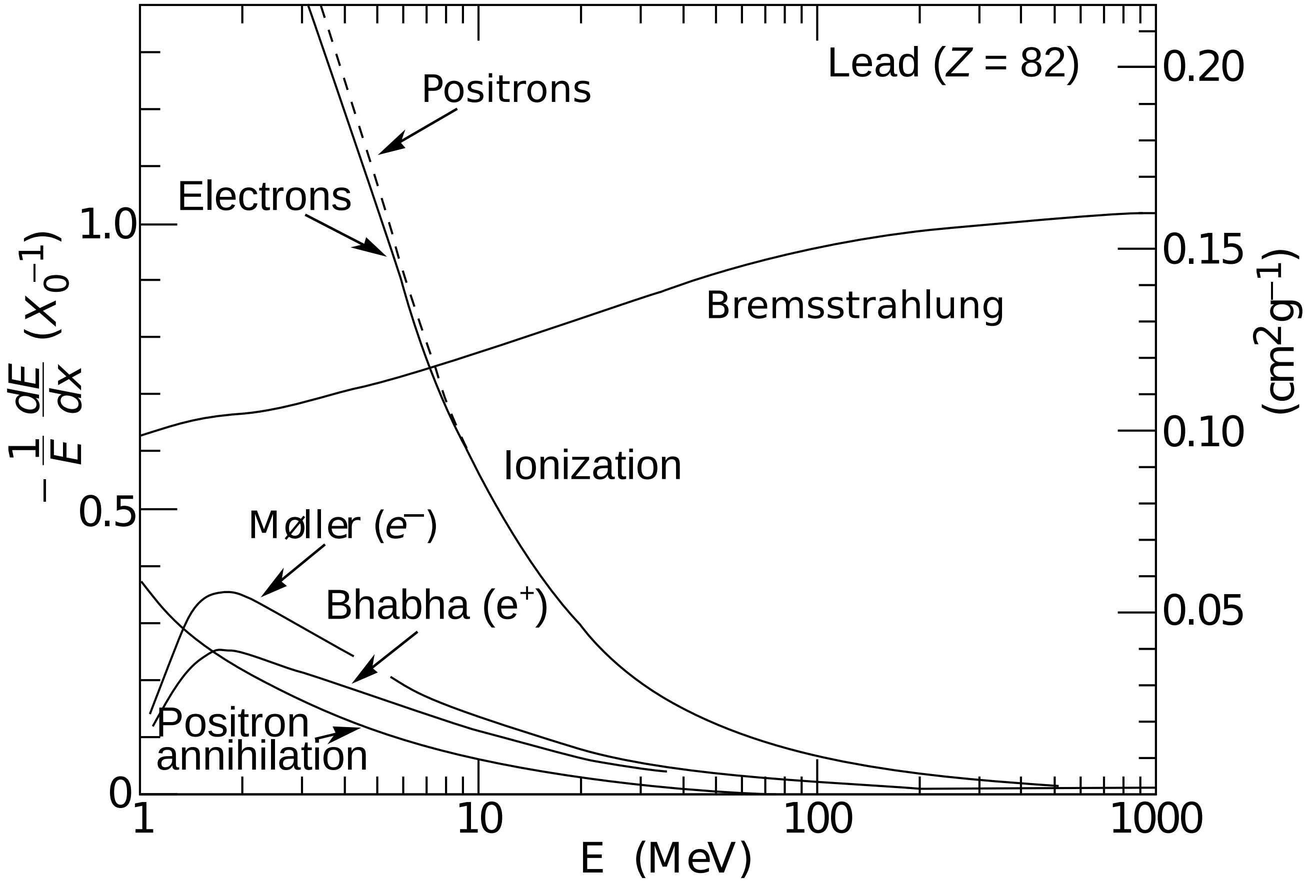
in the literature alternative definitions

- energy at which loss rates of ionization and bremsstrahlung equal
- energy at which ionization energy loss per rad. length is equal to electron energy (Rossi) (equivalent for approximation $\left.\frac{dE}{dx}\right|_{brems} = \frac{E}{X_0}$) good for transverse em shower description

Total energy loss of electrons and positrons

at small energies also

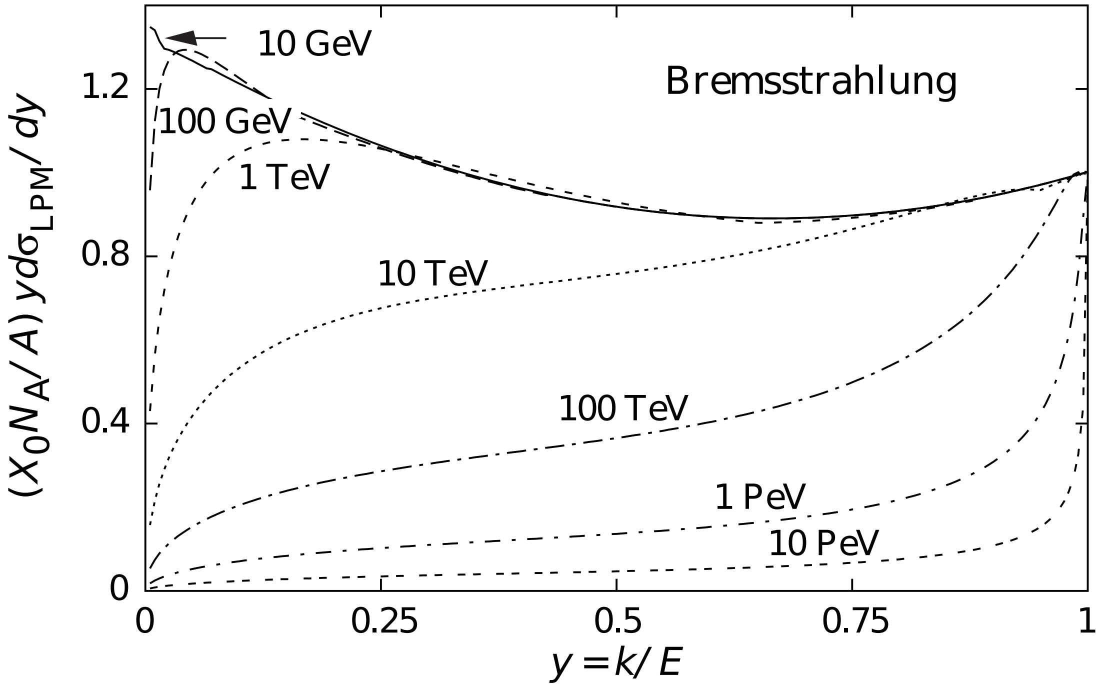




fractional energy loss per radiation length in lead as a function of electron or positron energy; electron (positron) scattering is considered as ionization, when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering, when it is above.

Quantum-mechanical suppression of bremsstrahlung I

normalized bremsstrahlung cross section:



normalized bremsstrahlung cross section $k\,\mathrm{d}\sigma_{\mathrm{LPM}}/\mathrm{d}k$ in lead versus the fractional photon energy $y = k/E$. The vertical axis has units of photons per radiation length.

for small photon energies: again LPM effect important, successive radiations interfere.
radiation spread over formation length and if distance between successive radiations comparable to formation length $\rightarrow$ destructive interference

for Pb and electron of 10 GeV suppression for $k < 23$ MeV
for Pb and electron of 100 GeV suppression for $k < 2.3$ GeV Important for very high energies,

Quantum-mechanical suppression of bremsstrahlung II

e.g. air showers of cosmic ray interactions

- in bremsstrahlung process nucleus absorbs longitudinal momentum

$$c|\vec{q}_{||}| \simeq c|\vec{p}_e| - c|\vec{p}_e{}'| - c|\vec{p}_\gamma| \simeq \frac{E_\gamma}{2\gamma^2}$$

- corresponding to uncertainty principle momentum transferred over finite length scale (formation length)

$$L_F = \frac{\hbar c}{q_{||}c} = \frac{2\gamma^2 \hbar c}{E_\gamma}$$

$$\text{e.g. } E = 25 \text{ GeV} \quad E_\gamma = 100 \text{ MeV} \quad q_{||} = 20 \ \frac{\text{meV}}{\text{c}} \rightarrow L_F = 10 \ \mu\text{m}$$

Semi-classically: photon emission and exchange of photon with nucleus take place over length L_F
Alternative: quantum transport approach

Quantum-mechanical suppression of bremsstrahlung III

Semi-classically: photon emission and exchange of photon with nucleus take place over length L_F *but only* if electron and photon remain coherent over this length.
Destruction of coherence via

a) Landau-Pomeranchuk-Migdal effect: decoherence by multiple scattering when

$$\sqrt{\theta^2_{ms}} = \frac{21\text{MeV}}{E}\sqrt{\frac{L_F}{X_0}} \geq \theta_\gamma = \frac{m}{E} = \frac{1}{\gamma}$$

for $E = 25$ GeV and Au target, suppression for $E_\gamma \leq 10$ MeV

b) dielectric effect
phase shift of photons by coherent forward scattering off the electrons in material; strong suppression for

$$E_\gamma \leq \gamma\hbar\omega_p \quad \text{or} \quad \frac{E_\gamma}{E} \leq 10^{-4}$$

c) at large y screening may be incomplete

consequence of a-b: at very high photon and electron energies: strong suppression of bremsstrahlung and pair production
dominance of photonuclear and electronuclear interactions of em interactions

2.5 Cherenkov effect

Particle of mass M and velocity $\beta = v/c$ propagates through medium with real part of dielectric constant

$$\epsilon_1 = n^2 = \frac{c^2}{c_m^2}$$

in case

$$\beta > \beta_{\text{thr}} = \frac{1}{n} \text{ or } v > c_m$$

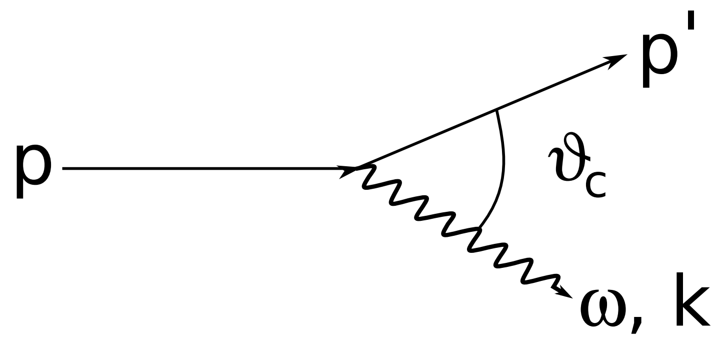
real photons can be emitted with

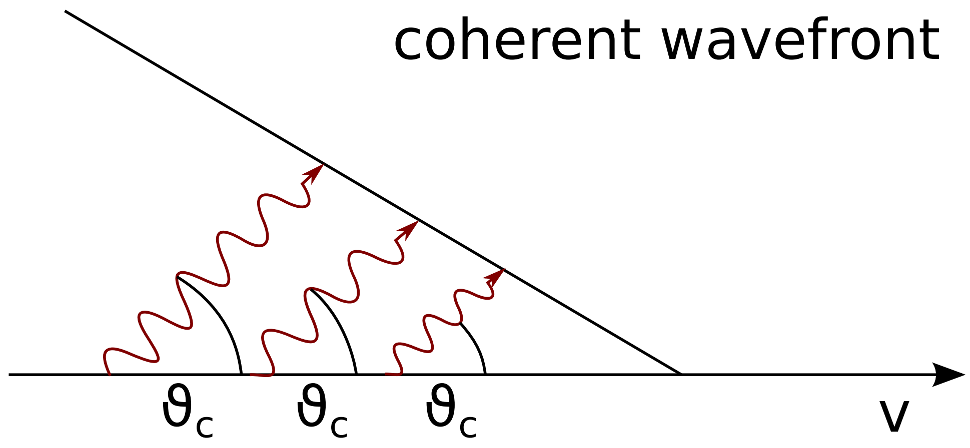
$$\begin{aligned} |p| &\simeq |p'| \\ \omega &\ll \gamma M c^2 \end{aligned}$$

emission under angle

$$\cos\theta_c = \frac{\omega}{k \cdot v} = \frac{1}{n\beta}$$

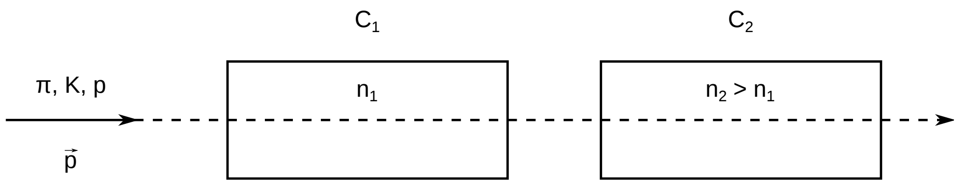
Cherenkov 1934





Applications

a) threshold detector: principle - if Cherenkov radiation observed $\Rightarrow \beta > \beta_{\text{thr}}$
e.g. separation of π/K/p of given momentum p



$$\text{choose } \begin{matrix} n_2 \\ n_1 \end{matrix} \text{ such that } \begin{matrix} \beta_\pi, \beta_K > \frac{1}{n_2} & \beta_p < \frac{1}{n_2} \\ \beta_\pi > \frac{1}{n_1} & \beta_k, \beta_p < \frac{1}{n_1} \end{matrix}$$

light in C_1 and C_2: π
light in C_1 and not in C_2: K
no light in C_1 and C_2: p

b) measurement of θ_c in medium with known $n \Rightarrow \beta$
(RICH, DIRC, DISC detectors)

Spectrum and number of radiated photons

over range in ω where $\epsilon_1 > \frac{1}{\beta^2}$

$$dN_\gamma \propto d\nu = \frac{d\lambda}{\lambda^2}$$ blue dominated

for distance x and frequency interval $d\nu$:

$$N_\gamma = x \underbrace{\frac{\alpha}{\hbar c}}_{370/\text{eV}\cdot\text{cm}} \int_{\omega_1}^{\omega_2} \underbrace{(1 - \frac{1}{\beta^2 n^2(\omega)})}_{\sin^2\theta_c} \hbar d\omega$$

for interval $d\omega$, where $n(\omega)$ varies not much, e.g. gases around visible wavelengths:

$$300\text{ nm} < \lambda < 600\text{ nm:}\quad N_\gamma = 750 \sin^2\theta_c\ /\text{cm}$$

	$(n-1)$	$(\beta\gamma)_{thr}$	θ_c^∞(deg)	$N_\gamma^\infty(cm^{-1})$
H_2	$0.14 \cdot 10^{-3}$	59.8	0.96	0.21
N_2	$0.3 \cdot 10^{-3}$	40.8	1.4	0.45
Freon 13	$0.72 \cdot 10^{-3}$	26.3	2.2	1.1
H_2O	0.33	1.13	41.2	165
lucite	0.49	0.91	47.8	412

typical photon energy: $\simeq 3$ eV

in water $\left.\frac{dE}{dx}\right|_{cher} = 0.5$ keV/cm $= 0.5$ keV/g/cm^2

cf. ionization $\left.\frac{dE}{dx}\right|_{ion} \geq 2$ MeV/g/cm^2

$\rightarrow$ energy loss by Cherenkov radiation negligible

danger: emission of scintillation light by excited atoms can fake Cherenkov radiation!

measurement of β via ring radius requires minimum number of detected photo electrons

$$n_e = N_\gamma \cdot \epsilon_{\text{lightcoll}} \cdot \eta \simeq N_\gamma \cdot 0.8 \cdot \text{ quantum efficiency } \simeq 20\%\, N_\gamma$$

example: require for reconstruction of ring in RICH $n_e \geq 4$ and efficiency should be 90%
n_e follows Poisson distribution
for a given $\langle n_e \rangle$ P(4) + P(5) + P(6) + $\ldots \geq 0.9$

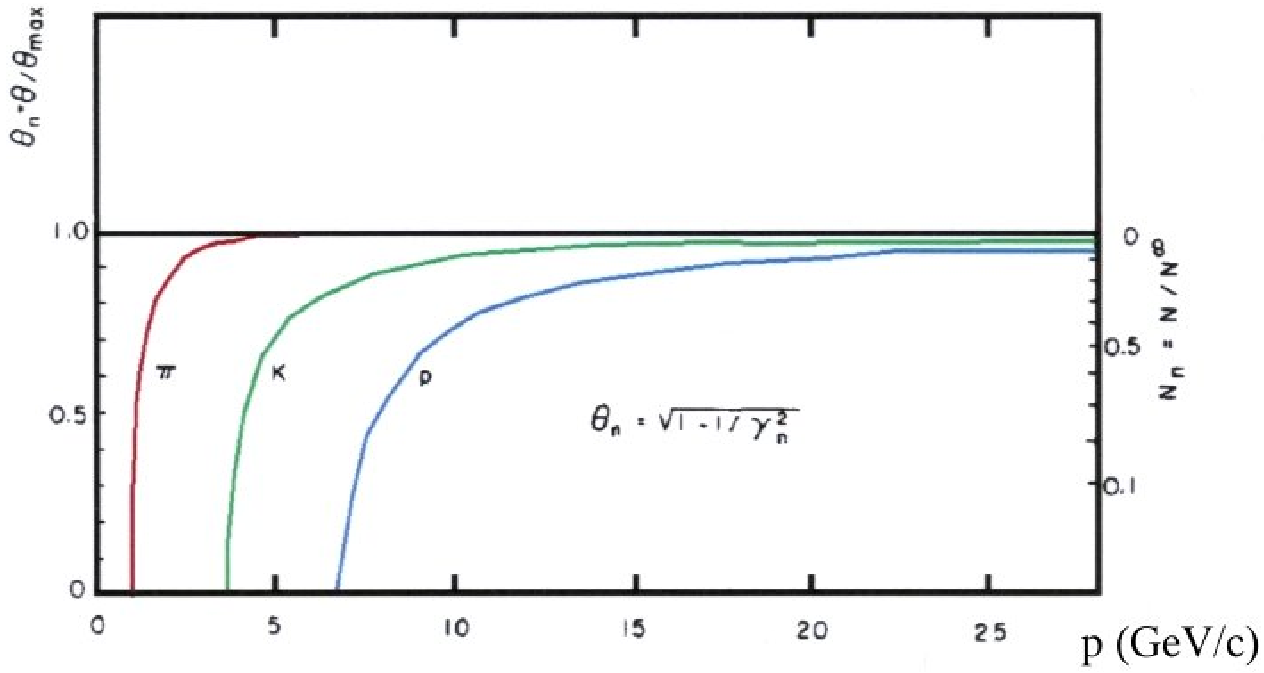
$$P_n = \frac{\langle n_e \rangle^n \exp - \langle n_e \rangle}{n!} \qquad \text{Poisson}$$

$$\text{with} \langle n_e \rangle = 7$$

$$\sum_0^3 P_n = 7.9\% \qquad \text{efficiency for } n \geq 4: \; 92.1\%$$

need about 35-45 Cherenkov photons $\rightarrow$ about 0.5 m freon

Asymptotic Cherenkov angle and number of photons as function of momentum



number of photons grows with β and reaches asymptotic value for $\beta \to 1$

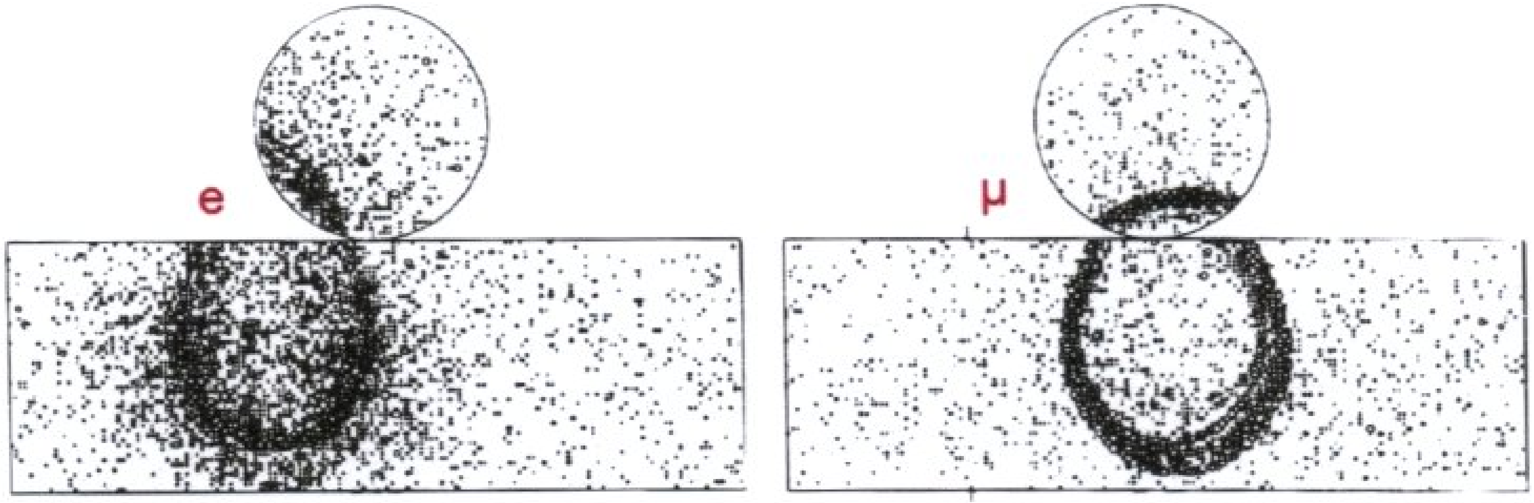
$$\cos\theta_c^\infty = \frac{1}{n} \quad \text{or} \quad \theta_c^\infty = \arccos\frac{1}{n}$$

$$N_\gamma = x \cdot 370/\text{cm}\left(1 - \frac{1}{\beta^2 n^2}\right)$$ for a photon energy interval of 1 eV

$$N_\gamma^\infty = x \cdot 370/\text{cm}\left(1 - \frac{1}{n^2}\right)$$ "

Use of Cherenkov light for neutrino detection

electron neutrinos: charged current events
all neutrinos: neutral current
leading to final state neutrino and energetic electron detected by Cherenkov radiation (typically $E > 5$ MeV to be above background from natural radioactivity)

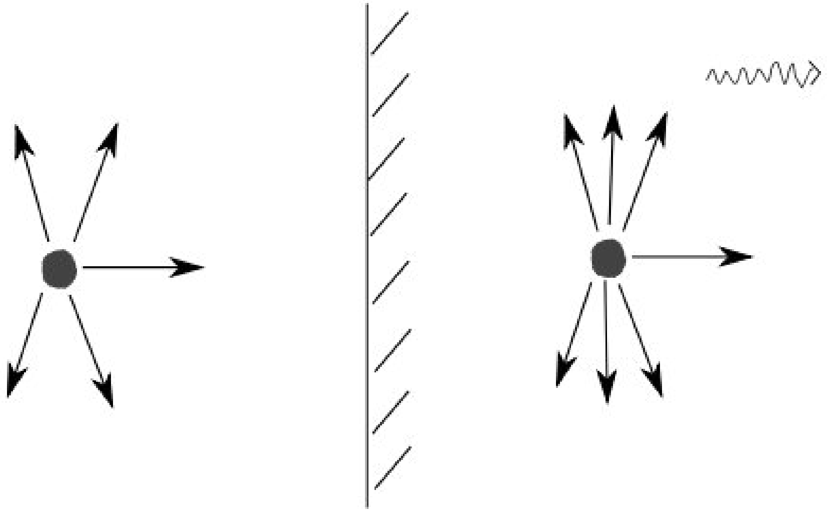


electron and muon Cherenkov rings

electron ring becomes diffuse due to multiple scattering of electron
allows to distinguish electron from muon, important for neutrino detectors (Superkamiokande, SNO)

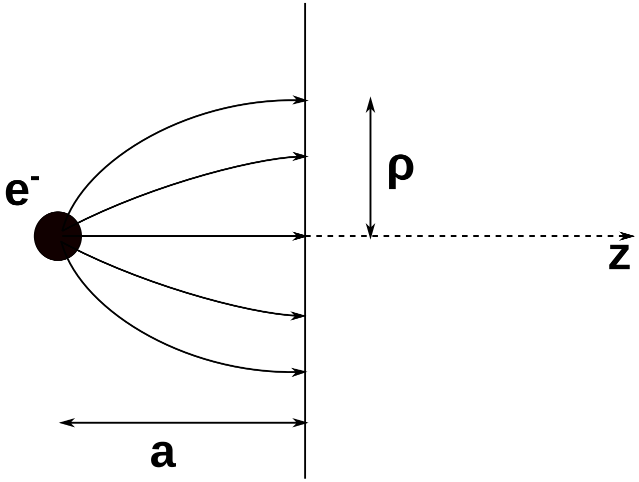
2.6 Transition radiation

a relativistic particle can emit a real photon when traversing the boundary between 2 different dielectrics
predicted: Ginzburg and Frank 1946; confirmed in 1970ies



electric field needs to rearrange

simple model: electron moves in vacuum towards a conducting plate, the E-field can be described by method of mirror charges



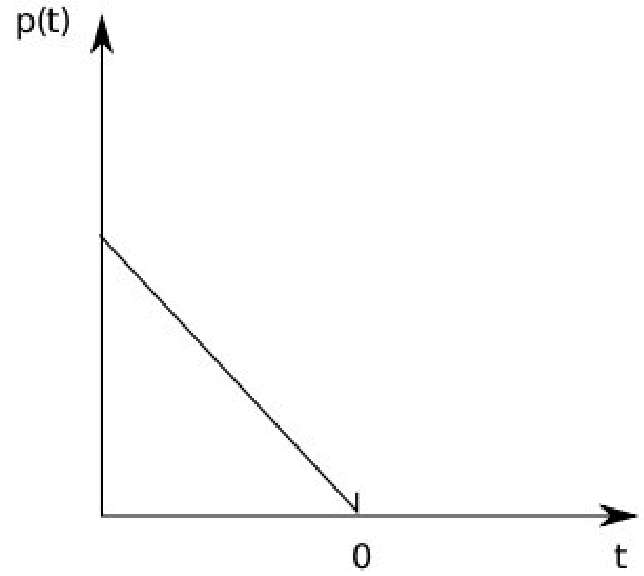
normal component at metal surface

$$|\vec{E}_n| = \frac{a \cdot e}{(a^2 + \varrho^2)^{\frac{3}{2}}}$$

can be generated (Gedankenexperiment) by a dipole $\vec{p} = 2e\vec{a}$

Radiation:

annihilation of dipole as particle enters the metal



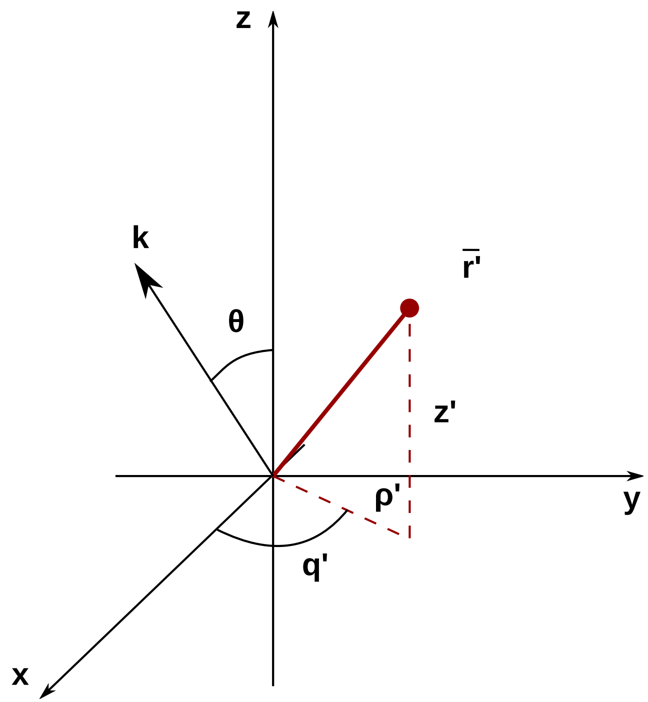
within classical electrodynamics one can show how E-field varies in point $\vec{r}' = (\varrho', z')$ leading to time dependent polarization

at $t = 0$ particle is at origin, it propagates in z-direction, consider radiation in k-direction.

$$E_z = \frac{e\gamma(z' - vt)}{(\varrho'^2 + \gamma^2(z' - vt)^2)^{\frac{3}{2}}}$$

$$E_\perp = \frac{e\gamma\varrho'}{(\varrho'^2 + \gamma^2(z' - vt)^2)^{\frac{3}{2}}}$$

$\rightarrow$ time-dependent polarization $\vec{P}(\vec{r}', t)$



variation of induced dipoles with time leads to radiation of photons

coherent superposition of radiation from neighboring points in vicinity of track
$\rightarrow$ angular range of radiation

θ: large Fourier component of $\vec{P}$ at

$$\varrho^i \leq \frac{\gamma v}{\omega} \simeq \varrho_{\text{max}} \quad \rightarrow \quad \theta \simeq \frac{1}{\gamma}$$

$\rightarrow$ depth from surface up to which contributions add coherently: formation length $D \simeq \gamma \cdot \frac{c}{\omega_p}$

$\rightarrow$ volume element producing coherent radiation $V = \pi \varrho_{\text{max}}^2 D$
characterized by plasma frequency ω_p

$$\sqrt{\epsilon_1} = n(\omega) \simeq 1 - \frac{\omega_p^2}{\omega^2} \quad \text{with} \quad \omega_p = \sqrt{\frac{4\pi\alpha n_e}{m_e c^2}} = 28.8\sqrt{\varrho \frac{Z}{A}} \text{ eV}$$

typical values: $\omega_p^{CH_2}$=20 eV polyethylene ($\varrho \approx 1$ g/cm^3); for $\gamma = 10^3 \rightarrow D \approx 10\mu$m
$\omega_p^{\text{air}} = 0.7$ eV

$\rightarrow$ radiator made out of foils of this typical thickness; for $d > D$ absorption dominates

typical photon energy: $E_\gamma^{\text{max}} \simeq \gamma \hbar \omega_p$ X-Rays for $\gamma \gg 1$

$$\frac{d^2W}{d\omega d\Omega} = \frac{\alpha}{\pi^2}\left(\frac{\theta}{\gamma^{-2}+\theta^2+\xi_1^2} - \frac{\theta}{\gamma^{-2}+\theta^2+\xi_2^2}\right)^2$$

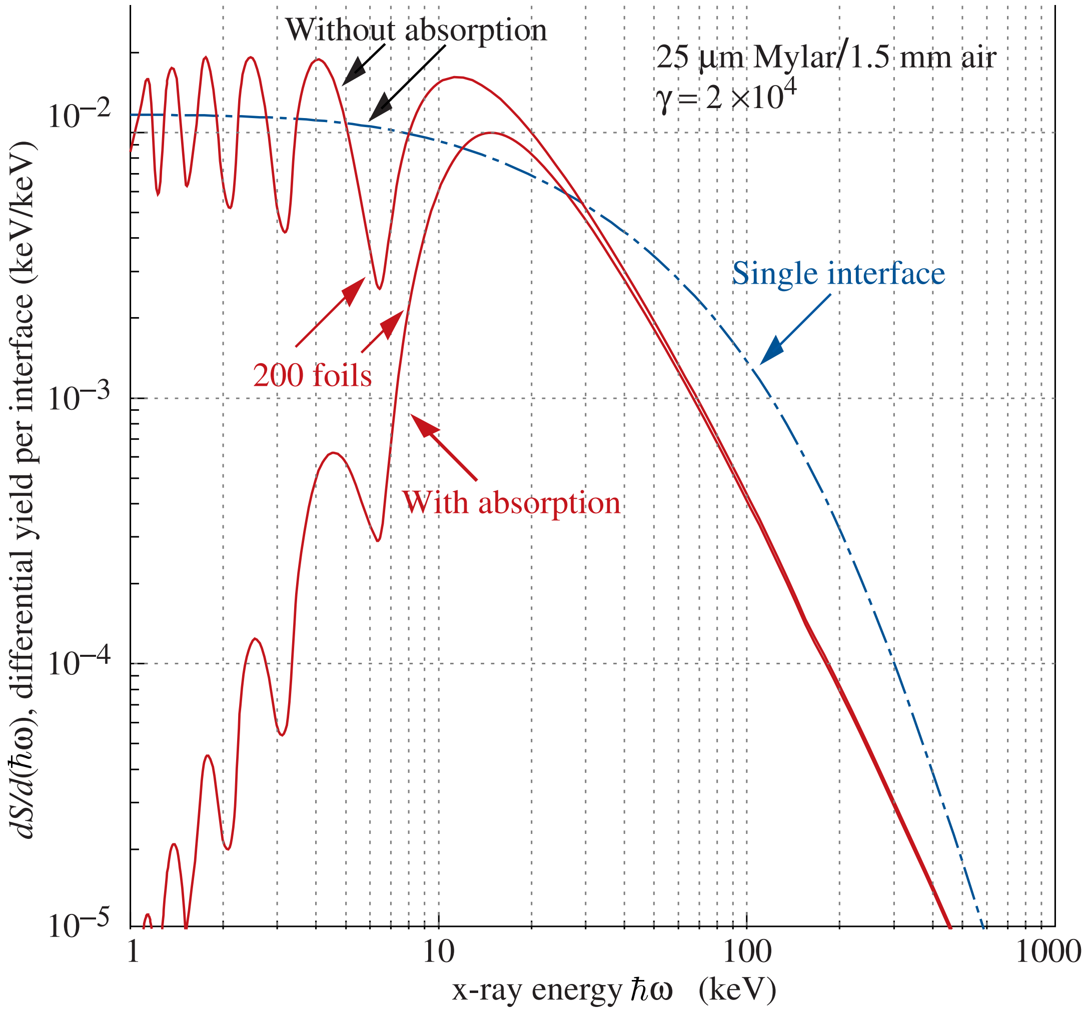
$$\text{with} \quad \xi_i = \frac{\omega_{p_i}^2}{\omega^2} = 1 - \epsilon_{1i}(\omega) \ll 1$$

$\rightarrow$ per boundary

$$\frac{dW}{d\omega} = \frac{\alpha}{\pi}\left(\frac{\xi_1^2+\xi_2^2+2\gamma^{-2}}{\xi_1^2-\xi_2^2}\ln\frac{\gamma^{-2}+\xi_1^2}{\gamma^{-2}+\xi_2^2} - 2\right)$$

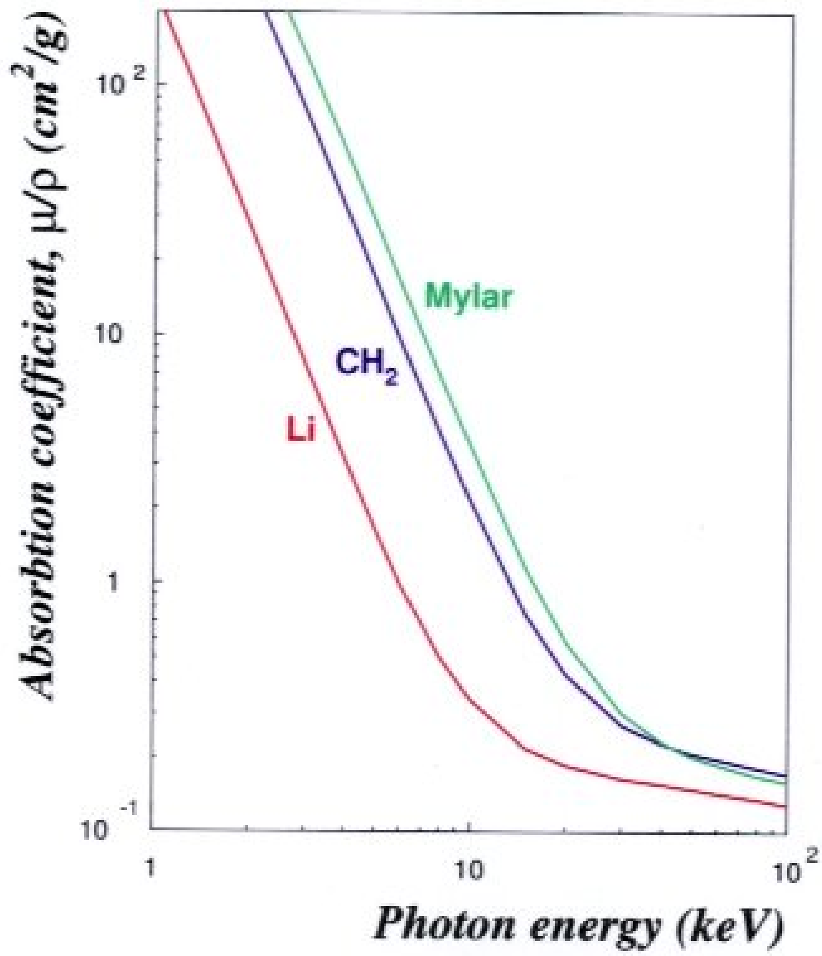
foil: contribution from both surfaces, depending on photon interference

typical number of photons per foil $\simeq \alpha$
$\rightarrow$ need many (!) foils
O(100) $\rightarrow \langle n_\gamma \rangle = 1 - 2$

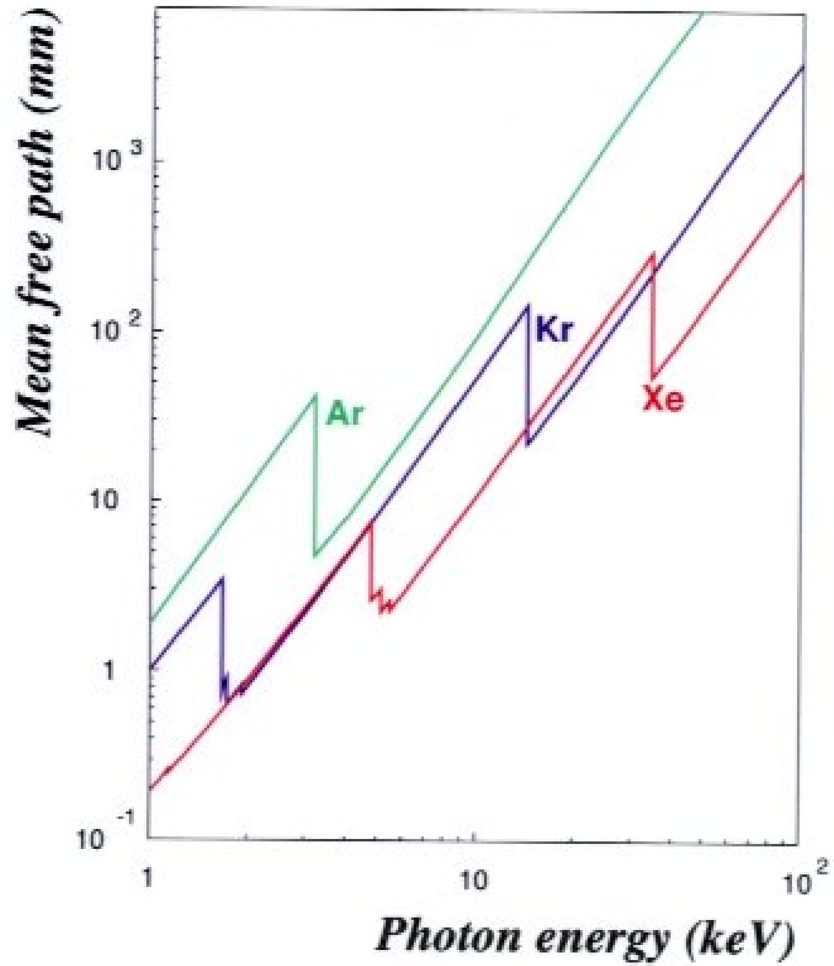


TR spectrum for single interface and multiple foil configurations.

photons generated in e.g. mylar foils and absorbed in gas with high Z (xenon)

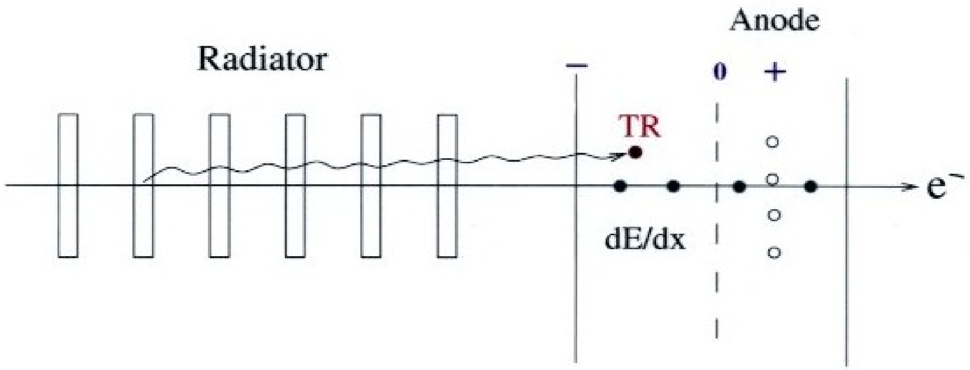


X-Rays absorption coefficient for Li, CH_2 and mylar

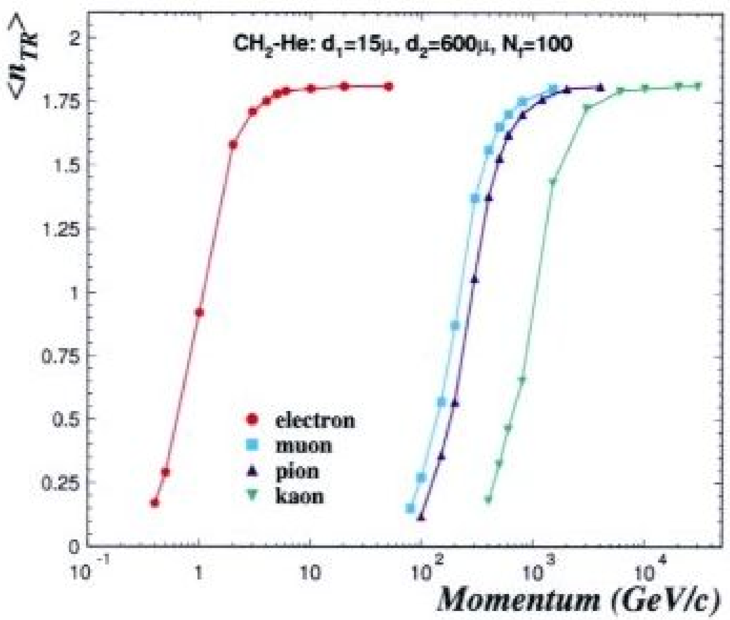


mean free path of X-rays in different gases

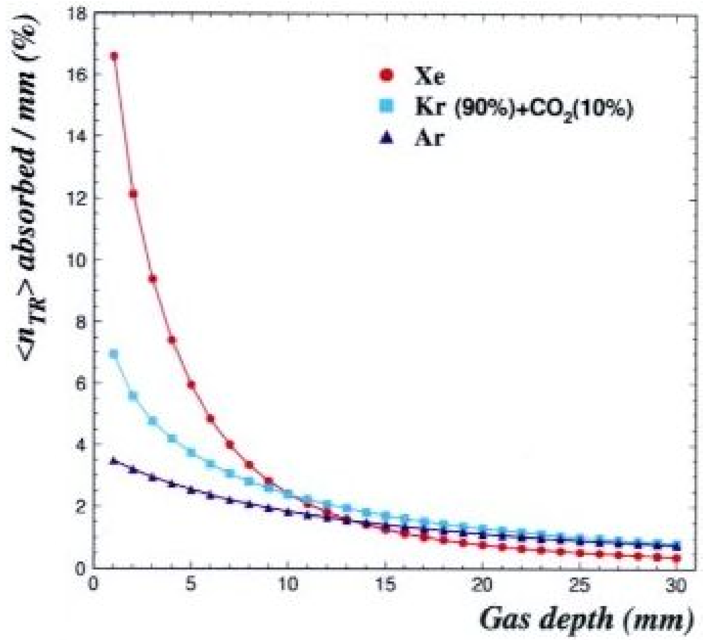
Principle of a transition radiation detector



photons generated in e.g. mylar foils and absorbed in gas with high Z (xenon)

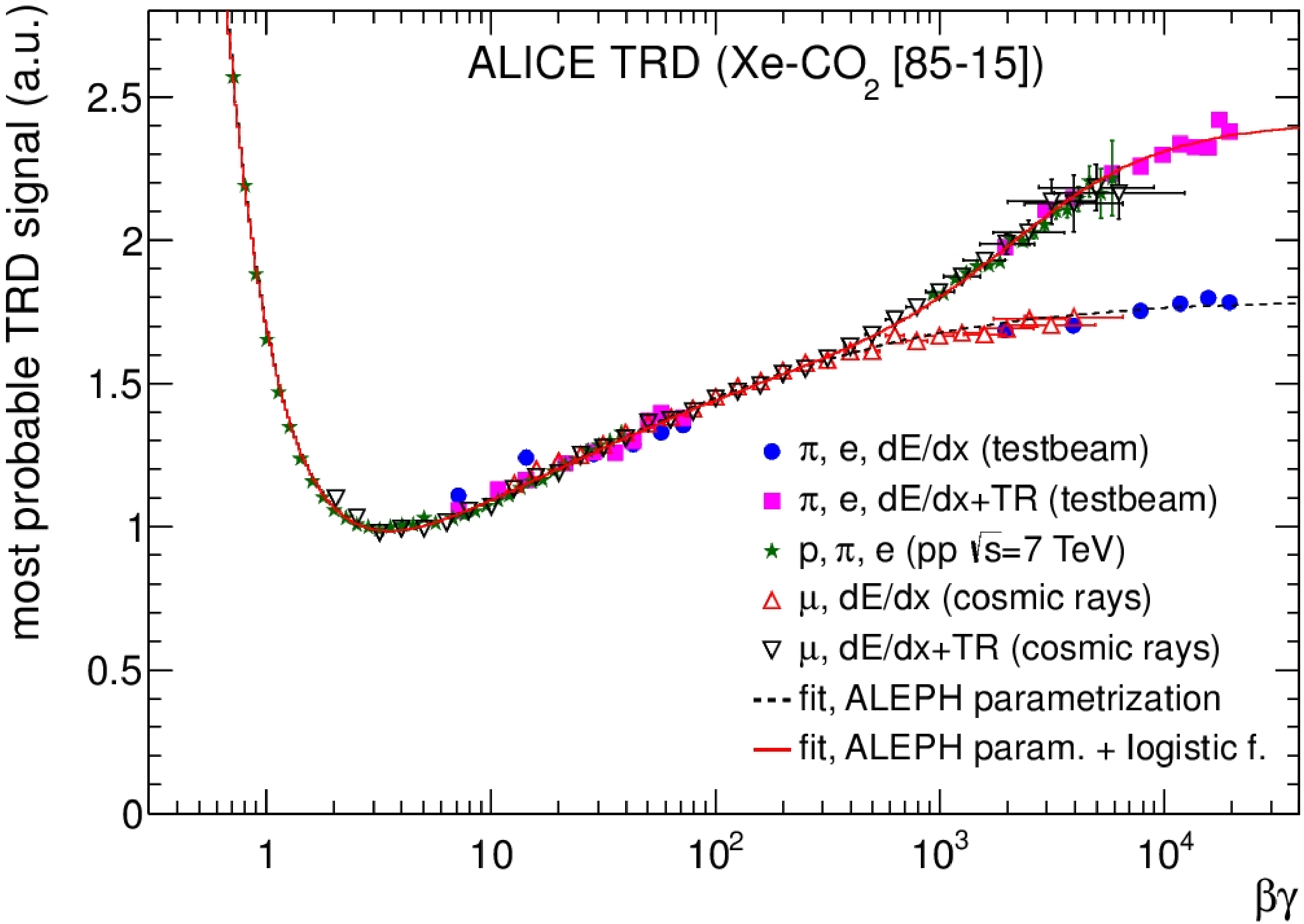


onset of TR production for electrons, muons, pions and kaons. Radiator of 100 foils, thickness d1, spacing d2



fraction of absorbed TR photons as a function of detector depth. For good absorption probability preferential use of Xe gas, typical dimension cm

the ALICE transition radiation detector TRD



demonstration of the onset of TR at $\beta\gamma \approx 500$
(doctoral thesis Xian-Guo Lu, U. Heidelberg, Oct. 2013)