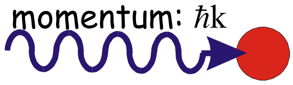
Cold Atoms

mechanical effects of light

Scattering of a photon by an atom

photon momentum: $\hbar k$

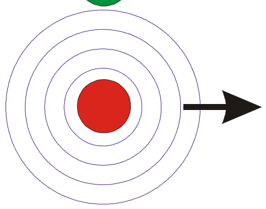


atom momentum: $0\hbar k$

after excitation of atom:

atom momentum: $1\hbar k$

after spontaneous decay of atom:



mean atom momentum: $1\hbar k$

Mean force on atom:

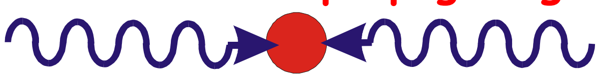
$$F = \frac{dp}{dt} \approx \frac{\Delta p}{\Delta t} = \hbar k \Gamma \rho_{22} = \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + \left(2\frac{\omega_L - \omega_a - \vec{k}\vec{v}}{\Gamma}\right)^2}$$

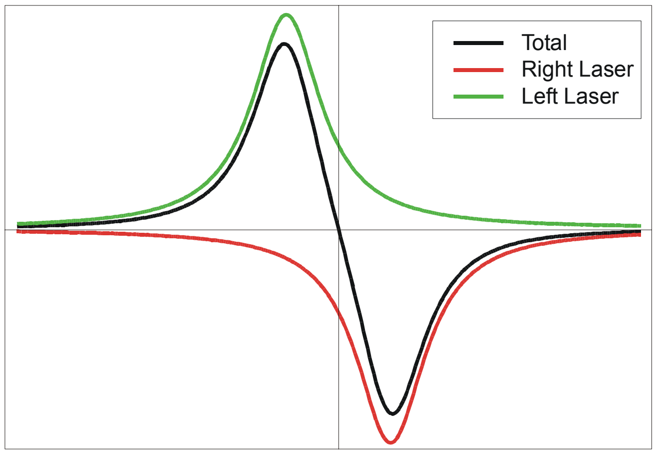
typical forces on the atom can lead to accelerations of 10^4-10^6 m/s^2

Cold Atoms

laser cooling

Atom in counter propagating laser field: optical molasses





Close to velocity zero: force is linear in velocity

$$\mathbf{F} = -\alpha \mathbf{v}$$

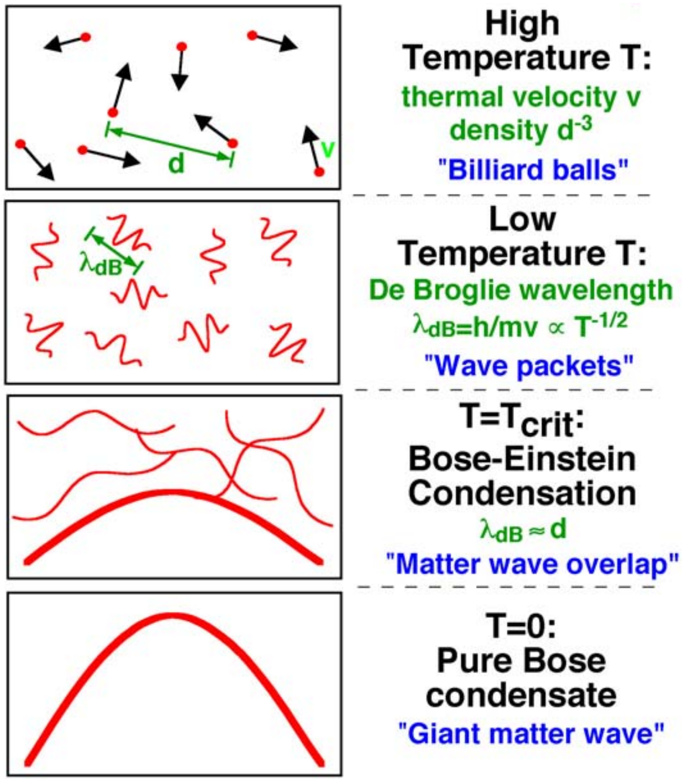
For a detuning

$$\delta = \omega_{laser} - \omega_{atom} < 0$$

(red from resonance) $\alpha > 0$ and the force is a damping force

Heating due to randomness of the photon scattering
typical temperature: $k_B T = \hbar\Gamma/2$ (Doppler limit)
140 μK for Γ=5 MHz

BEC
basic introduction



What is BEC? What is its underlying Physics? What the fundamental concept?

Colloquial: *'all particles are in the same state'*

- Broken Gauge Symmetry,
- Off-diagonal long range order (ODLRO)
- Long range phase coherence
- Macroscopic wave function of the condensate

These concepts were first introduced in studying superconductivity and superfluidity

What is the signature?

- Delta function of the occupation number of particles with zero momentum associated with long range phase coherence
- Bose narrowing (decrease in average energy as density gets higher). For fermions it is the opposite.
- Process of stimulated scattering: The scattering rate contains a factor $(1+N_f)$ where N_f is the occupation number of the final state

BEC
basic introduction

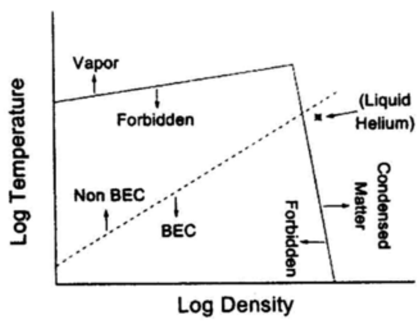


FIG. 1. Generic phase diagram common to all atoms: dotted line, the boundary between non-BEC and BEC; solid line, the boundary between allowed and forbidden regions of the temperature-density space. Note that at low and intermediate densities, BEC exists only in the thermodynamically forbidden regime.

Strongly interacting vs. weakly interacting Bose gas

- Liquid Helium is dominated by interactions. The BEC fraction is in the order of 10%. Many phenomena are masked by the strong interactions
- A weakly interacting gas (Atoms, Excitons): theoretic description is easier

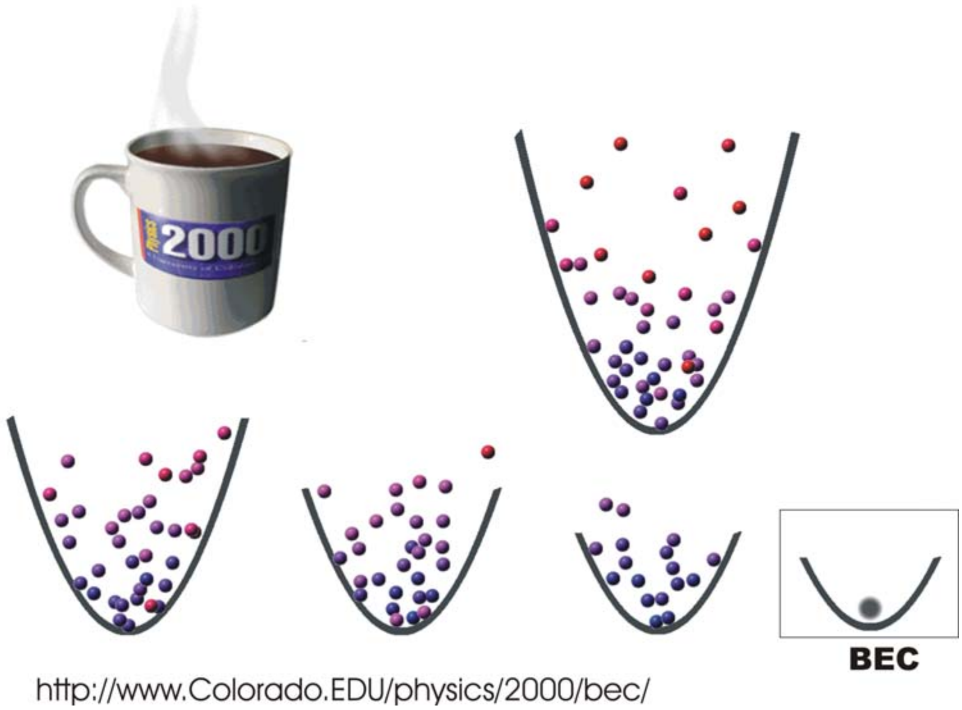
Condensation in free space vs. trapped condensates

- Free space one gets the classic formulas for BEC and its thermodynamic properties.
- Trapped gases: one has to look at the density of states in the trap.
  - o isotropy of trap potential
  - o dimensionality: 3d, (quasi) 2d, 1d
  - o disordered potentials
- small number of particles vs. continuum in thermodynamics
  - o what is the minimal size of a system we still can call a Bose condensate?

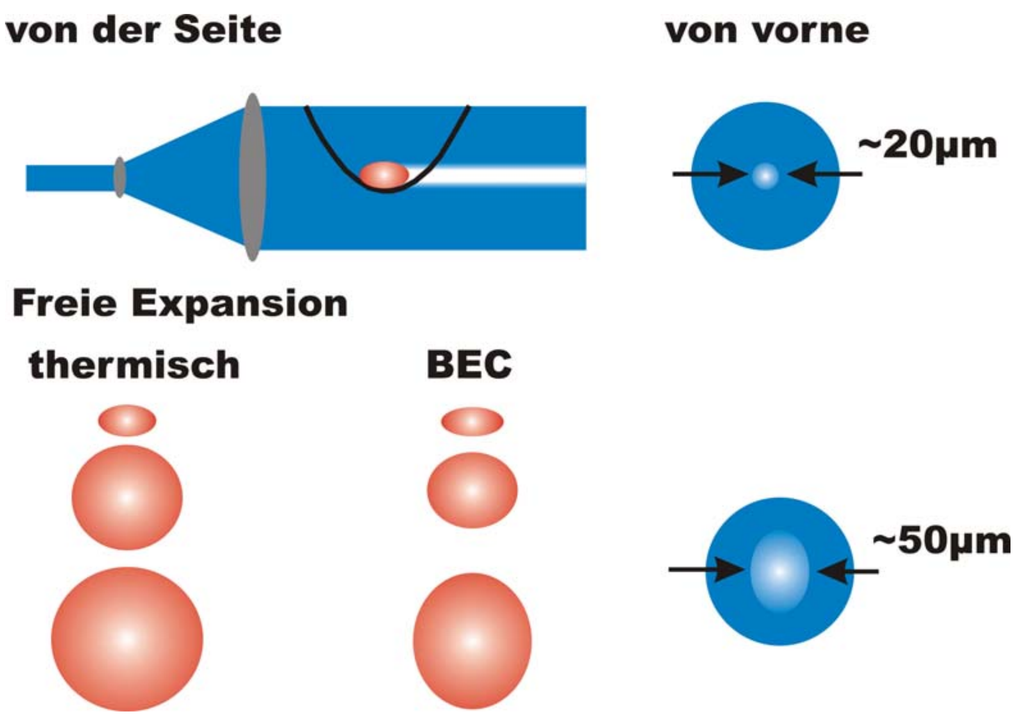
Fermions

- Pauli principle, FD statistics
- At low temperatures: BEC vs. BCS
  - o BEC: particle correlation length is very short compared to particle spacing
  - o BCS: particle correlation length is larger than the inter particle spacing

Wie macht man BEC mit Atomen

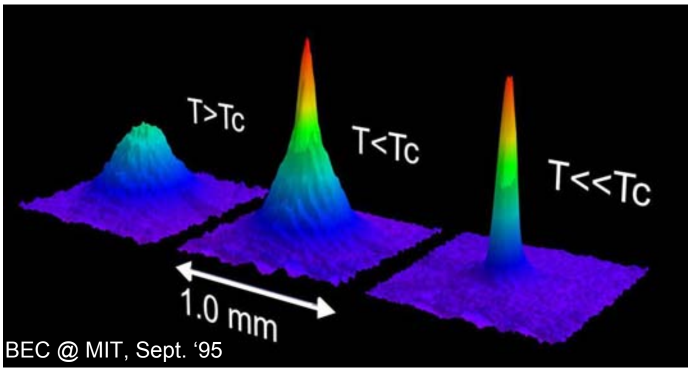


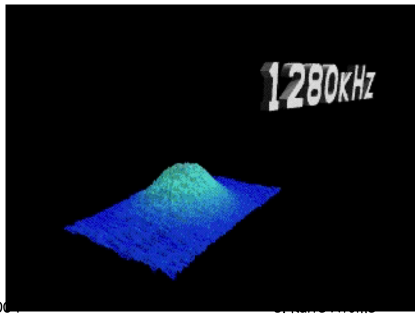
Wie sieht man die Kondensation



Observing BEC
phase transition, expansion

phase transition

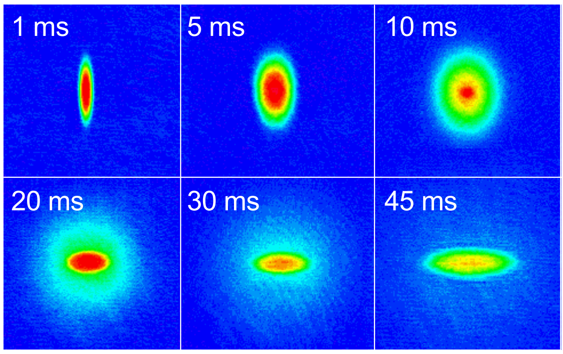


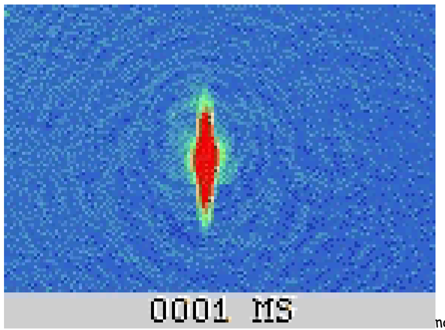


Expansion
man "sieht" den Grndzustand und in der Expansion erkennt man die Unschärferelation

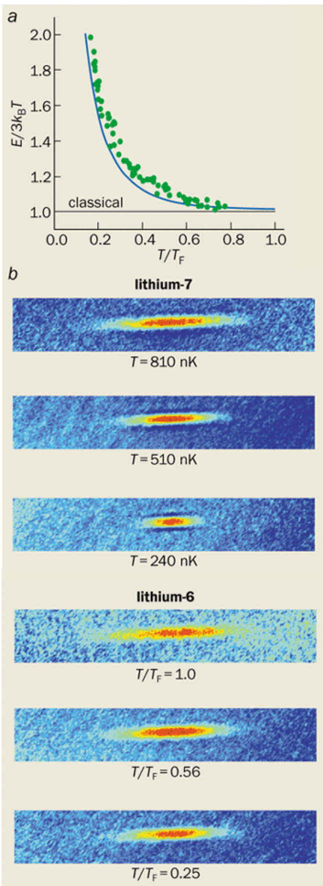
Kurze Zeiten: sieht δx
Lange Zeiten: sieht δp

kleines δx -> großes δp (schnelle Expansion)
großes δx -> kleines δp (langsame Expansion)





Hulet,Rice University 2001



Bosonen

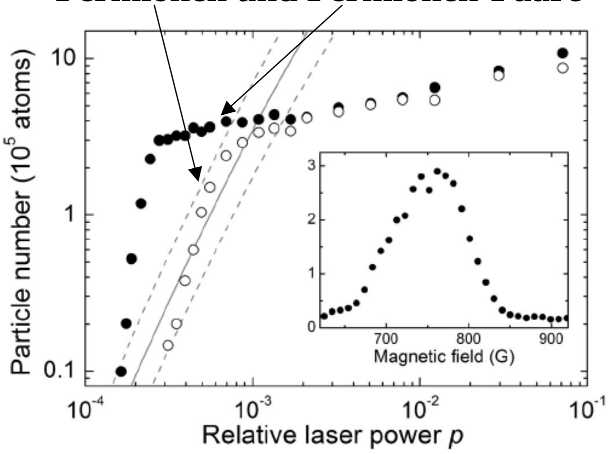
Fermionen

Ultrakalte Fermionen

$$\frac{1}{e^{(E-\mu)/k_BT}+1} \qquad \mu = E_F$$

Fermi-Dirac statistics

Fermionen und Fermionen-Paare



Grimm, Innsbruck Nov. 2003

BEC
basic introduction

BEC is a common phenomenon occurring in physics on all scales

- Condensed matter
- atomic physics
- nuclear and elementary particle physics
- astrophysics

Bosonic degrees of freedom are composite, they originate from Fermionic degrees of freedom (in most cases)**.**

- Fundamental Bosons:
  gauge Bosons : Photon, W,Z
- Fundamental Fermions:
  p,n,e ….

Table 1. *Bosons under study*

Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	metals	superconductivity
Cooper pair	h^+, h^+	copper oxides	high-T_c superconductivity
exciton	e^-, h^+	semiconductors	luminescence and drag-free transport in Cu_2O
biexciton	$2(e^-, h^+)$	semiconductors	luminescence and optical phase coherence in CuCl
positronium	e^-, e^+	crystal vacancies	(proposed)
hydrogen	e^-, p^+	magnetic traps	(in progress)
^{4}He	$^4He^{2+}$, $2e^-$	He-II	superfluidity
^{3}He pairs	$2(^3He^{2+}, 2e^-)$	^{3}He-A,B phases	superfluidity
cesium	$^{133}Cs^{55+}$, $55e^-$	laser traps	(in progress)
interacting bosons	nn or pp	nuclei	excitations
nucleonic pairing	nn or pp	nuclei neutron stars	moments of inertia superfluidity and pulsar glitches
chiral condensates	$\langle \bar{q}q \rangle$	vacuum	elementary particle structure
meson condensates	pion condensate $= \langle \bar{u}d \rangle$, etc. kaon condensate $= \langle \bar{u}s \rangle$	neutron star matter	neutron stars, supernovae (proposed)
Higgs boson	$\langle \bar{t}t \rangle$ condensate (proposed)	vacuum	elementary particle masses

Bose Verteilung I
Planck'sches Strahlungsgesetz

Behandlung relativistisch (s. 3.2) $\Rightarrow E = h\nu$

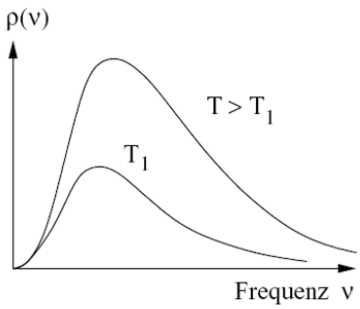
2 Polarisationszustände des Photons $\Rightarrow g_s = 2$

Mit $g(E)dE = g(h\nu)d(h\nu)$ $\quad g(\nu)d\nu = V\frac{8\pi}{h^3}\frac{(h\nu)^2}{c^3}d(h\nu) = \frac{8\pi V}{c^3}\nu^2 d\nu$

Totale Photonenzahl nicht fest, sondern massiv temperaturabhängig
$\Rightarrow \alpha = 0$, $e^\alpha = 1$ (α nicht durch N_0 fixierbar)

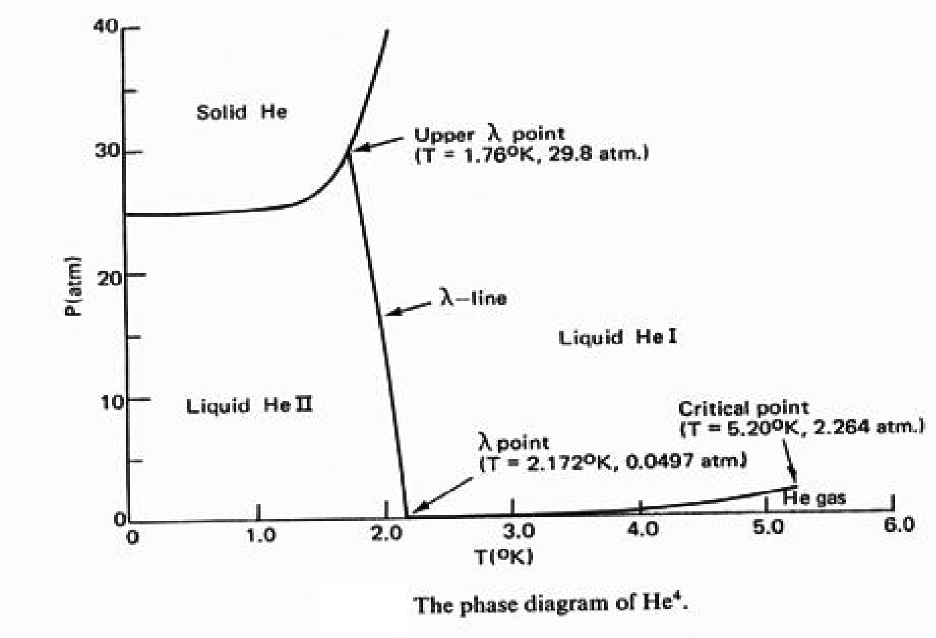
Anzahldichte der Photonen $\quad \frac{dN}{d\nu} = \frac{8\pi V}{c^3}\nu^2 \frac{1}{e^{h\nu/kT} - 1}$

Energiedichte des Photonen„gases" $\quad \rho(\nu) = \frac{1}{V}\frac{dN}{d\nu}h\nu$



$$\Rightarrow \quad \boxed{\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT} - 1}d\nu} \quad \textit{Planck}$$

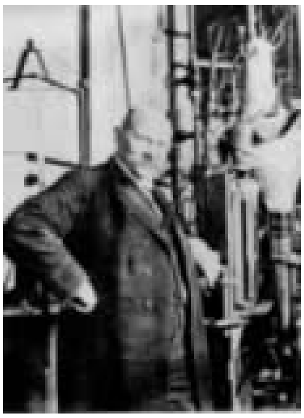
Boseverteilung II
Bose Flüssigkeit: Helium

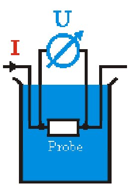


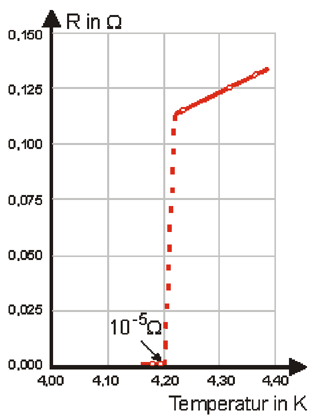
The phase diagram of He^4.

Bose Verteilung III
Supraleitung

Kamerlingh Onnes 1911:
elektr. Widerstand von Hg bei 4.2K



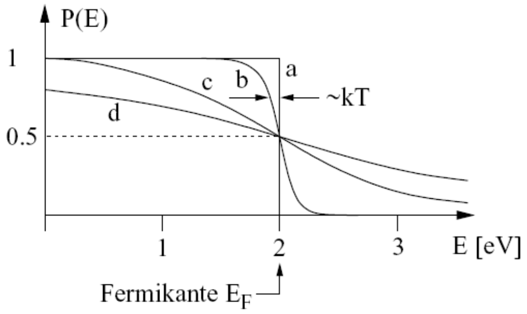




Cooper-Paar {p↑ , -p↓}

Fermi-Verteilung I

Annahme: E_F fest (s.u.); numerisches Beispiel für $E_F = 2$ eV, $T_F = E_F/k = 23\,000$ K



	T (K)	kT (eV)
a	0	0
b	1000	0.085
c	5000	0.43
d	20000	1.7

Diskussion der Kurven:

a $T = 0$: alle Phasenraumzellen der Zustände E_i bis zur Grenze E_F mit je **1** Fermion besetzt, also *voll*; $P(E) = 1$. Für $E_i > E_F$ alle Zellen leer, d.h. $P(E) = 0$.

b, c $T > 0$, aber $kT \ll E_F$: Abrundung, Verschmierung der Fermikante

d $T \gg 0$, $kT \approx E_F$: selbst für die niedrigsten Zustände E_i gibt es unbesetzte Zellen

⇒ irgendwann Grenzfall Boltzmann

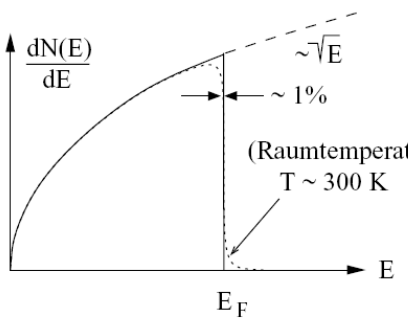
Allgemeine Regel: Im Bereich $E \gg E_F$ Boltzmann-„Schwanz": dort $e^{-E/kT}$ immer ausreichend!

Fermi-Verteilung II
Leitungselektronen

$$\frac{dN}{dE} = \frac{V}{h^3} 8\pi\sqrt{2m^3}\sqrt{E}\frac{1}{e^{(E-E_F)/kT}+1}$$

Totale Teilchenzahl N_0 fest: E_F aus $N_0 = \int_0^\infty \frac{dN}{dE}\,dE\Big|_{T=0}$

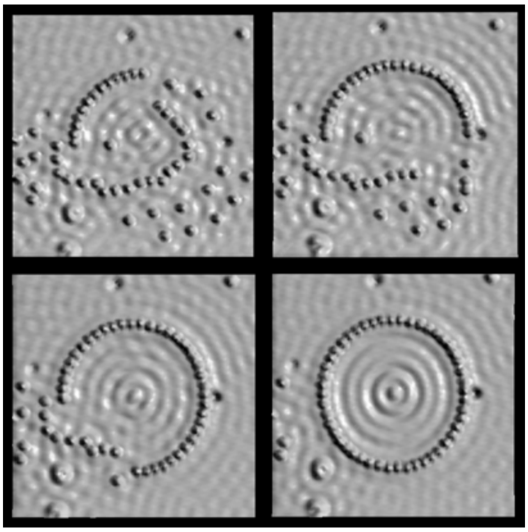
$$E_F = \frac{h^2}{8m}\left(\frac{3N_0}{\pi V}\right)^{2/3}$$



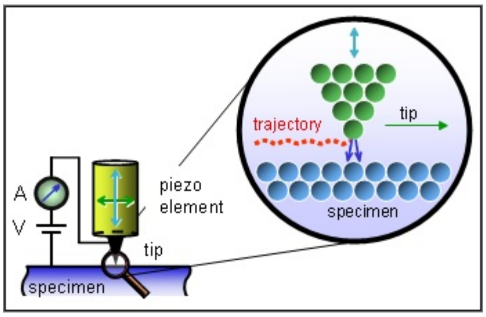
Typische Zahlen: $E_F = 2\ldots7$ eV $\gg kT$!
„Entartungstemperatur":
$T_F = E_F/k = 23\,000 - 82\,000$ K!

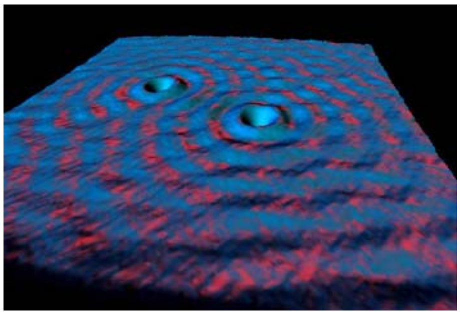
⇒ Fermi-Verteilung extrem scharfkantig bei Raumtemperatur

Fermi-Verteilung III
Elektronenwellen mit STM



STM-Scanning Tunneling Microscope

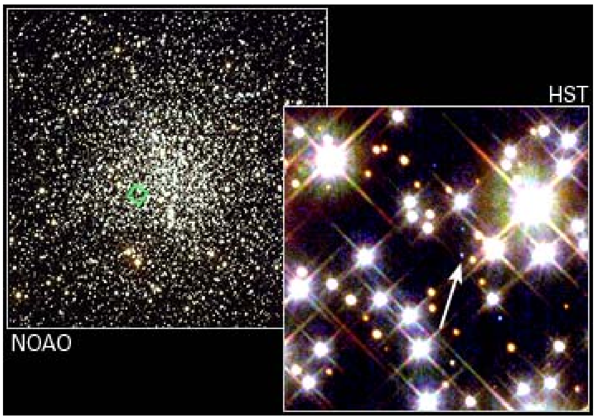




M.F. Crommie, C.P. Lutz, D.M. Eigler.
Imaging standing waves in a two-dimensional electron gas.
Nature 363, 524-527 (1993).

M.F. Crommie, C.P. Lutz, D.M. Eigler.
Confinement of electrons to quantum corrals on a metal surface.
Science 262, 218-220 (1993).

Fermi-Verteilung IV
Weiße Zwerge



http://hubblesite.org/newscenter/newsdesk/archive/releases/2003/19/image/b

Gleichgewichtsbedingung

$$\boxed{P_0 = P_G}$$

$$\frac{2}{5}\cdot n \cdot E_F = -\frac{G}{4\pi}\cdot\frac{M^2}{R^4}$$

$$\frac{M^2}{R^4} = \frac{4\pi}{5m_e \cdot G}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}}\cdot n^{\frac{5}{3}}\cdot h^2$$